

STATISTICAL DETECTION AND CLASSIFICATION OF TRANSIENT SIGNALS IN LOW-BIT SAMPLING TIME-DOMAIN SIGNALS

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*NEW JERSEY INSTITUTE OF TECHNOLOGY

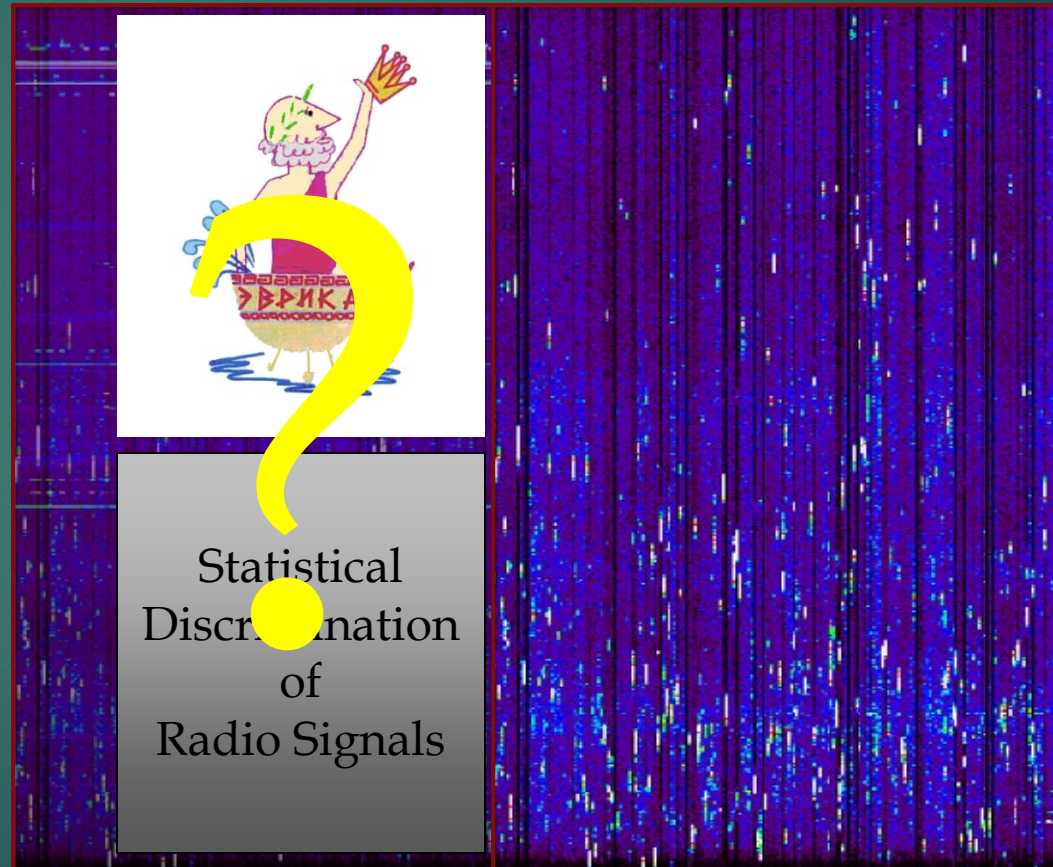
**JOINT INSTITUTE FOR VLBI ERIC

A well-known radio data analysis challenge

2/20



And its not so widely-known statistical solution...



Radio Frequency Interference

Astrophysical Signal

The Generalized Spectral Kurtosis Estimator

Nita and Gary 2010, MNRAS 406 L60-L64

4/20

Theorem: Given that, for a particular signal, the set of its power estimates P_k obeys a gamma distribution characterized by the shape parameter d , the infinite series of statistical moments MS_2/S_1^2 , where $S_1 = \sum_{k=1}^M P_k$ and $S_2 = \sum_{k=1}^M P_k^2$ is given by

$$E \left[\left(\frac{MS_2}{S_1^2} \right)^n \right] = \frac{M^n \Gamma(Md)}{\Gamma(d)^M \Gamma(Md + 2n)} \times \frac{\partial^n}{\partial t^n} \left[\sum_{r=0}^n \frac{1}{r!} \Gamma(2r + d) t^r \right]^M \Bigg|_{t=0}$$

The Generalized Spectral Kurtosis Estimator:

$$SK = \frac{Md + 1}{M - 1} \left(\frac{MS_2}{S_1^2} - 1 \right)$$

Statistical properties of the SK estimator:

- Has an **unbiased** unity expectation $E[SK] = 1$, independent of the integrated power S_1
- The infinite series of statistical moments of its PDF are analytically defined only in terms of M and d

The SK estimator is well suited for detecting mixed signals not obeying the same gamma probability distribution:
Detection thresholds of deviation from unity characterized by **analytically defined probabilities of false alarm (PFA)**

Practical cases well suited for SK analysis

5/20

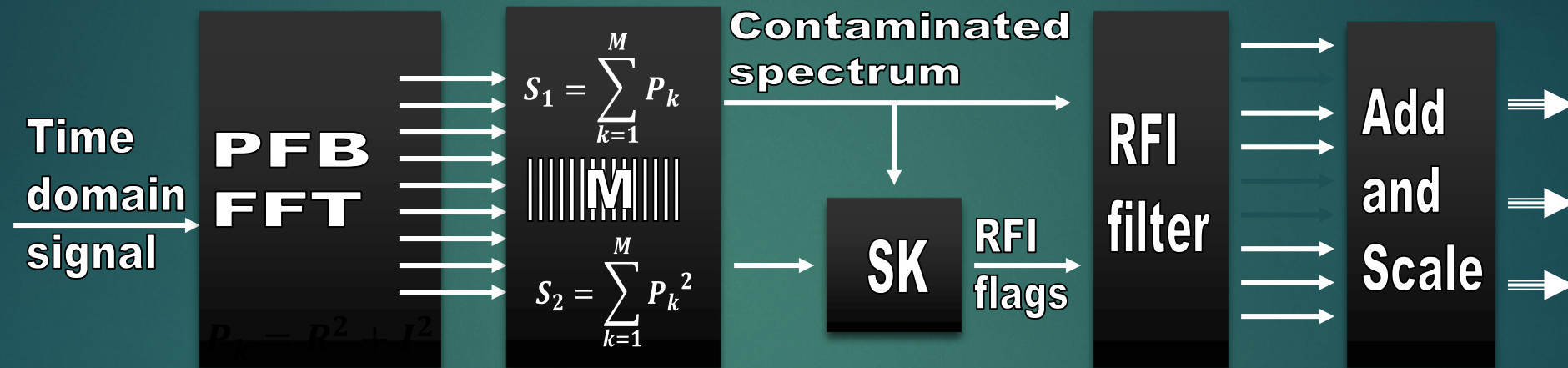
- ▶ Raw power estimates based on time domain real signals
 - ▶ Gamma distribution of shape factor $d=0.5$ (Chi-Square distribution)
- ▶ Raw power estimates based on time or frequency domain complex signals
 - ▶ Gamma distribution of shape factor $d=1$ (Exponential distribution)
- ▶ Accumulations of N raw power estimates of shape factor δ
 - ▶ Gamma distribution of shape factor $d=N\delta$
- ▶ Power estimates based on quantized time domain signals or quantized frequency domain power estimates (Nita, Gary, and Hellbourg 2017, IEEE)
 - ▶ Gamma distribution having an instrument-dependent shape factor d

The Spectral Kurtosis Spectrometer

Nita et al. 2007 PASP, 119, 805

Gary, Liu & Nita 2010 PASP, 122, 560

6/20



The unbiased
Spectral Kurtosis
Estimator

$$SK \equiv \frac{M+1}{M-1} \left(\frac{MS_2}{S_1^2} - 1 \right)$$

$$E(SK) = 1$$

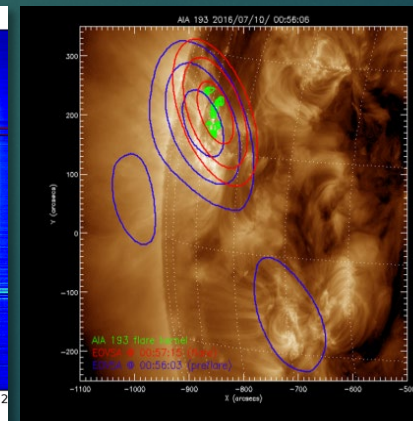
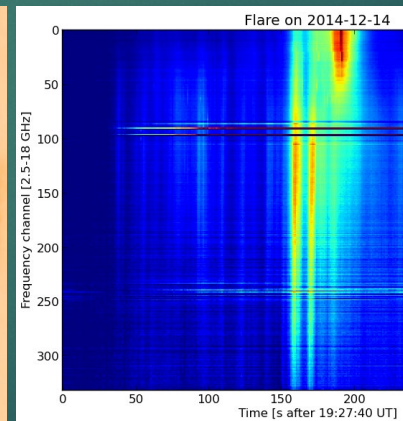
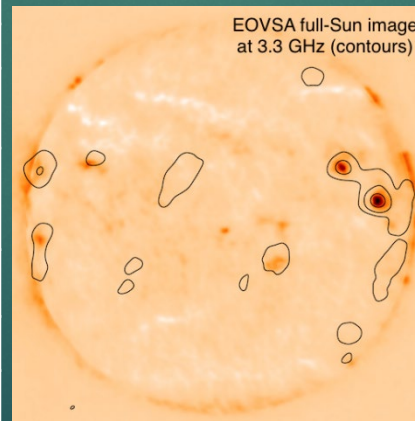
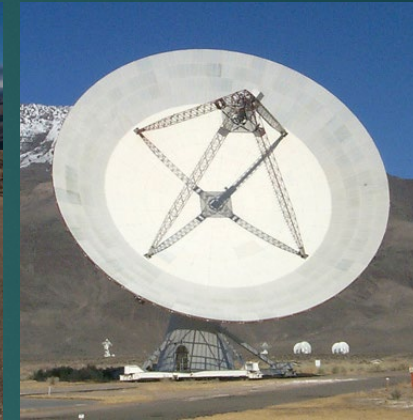
$$\sigma^2(SK) \approx \frac{4}{M}$$

Expanded Owens Valley Solar Array

7/20

World-first frequency agile interferometer equipped with an hardware embedded SK real-time computation engine

Table 1: EOVSVA Specifications	
Frequency range	2.5 – 18 GHz
Number of data channels/antenna	2 (dual polarization)
IF bandwidth	500 MHz single sideband
Frequency resolution	4096 spectral channels per 600 MHz band) 500 science channels variable ~1-40 MHz
Time resolution	Sample time: 20 ms Full Sweep: 1 s
Polarization	Full Stokes (IQUV)
Number correlator inputs per poln	16
Number and type of antennas	Thirteen 2.1-m Two 27-m equatorial (cal. only)
System Temperature	570 K (2 m); 35 K (27 m)
Baselines for imaging	78
Angular resolution	$56/n_{\text{GHz}} \times 51/n_{\text{GHz}}$ arcsec

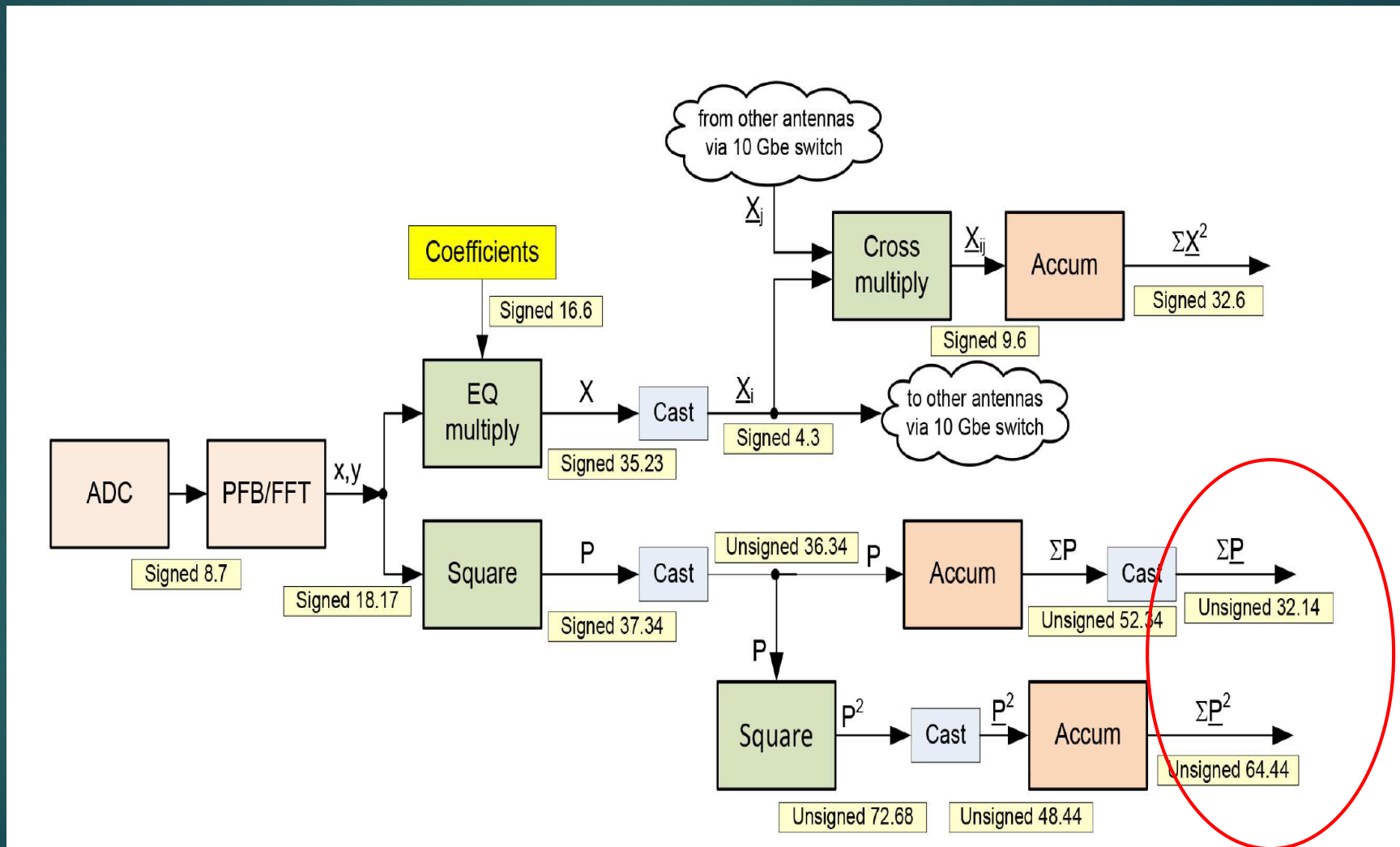


The EOVSVA correlator outputs integrated power and squared power for all 15 antennas and R and L circular polarizations with 20ms-0.125MHz time-frequency resolution

EOVSA CORRELATOR

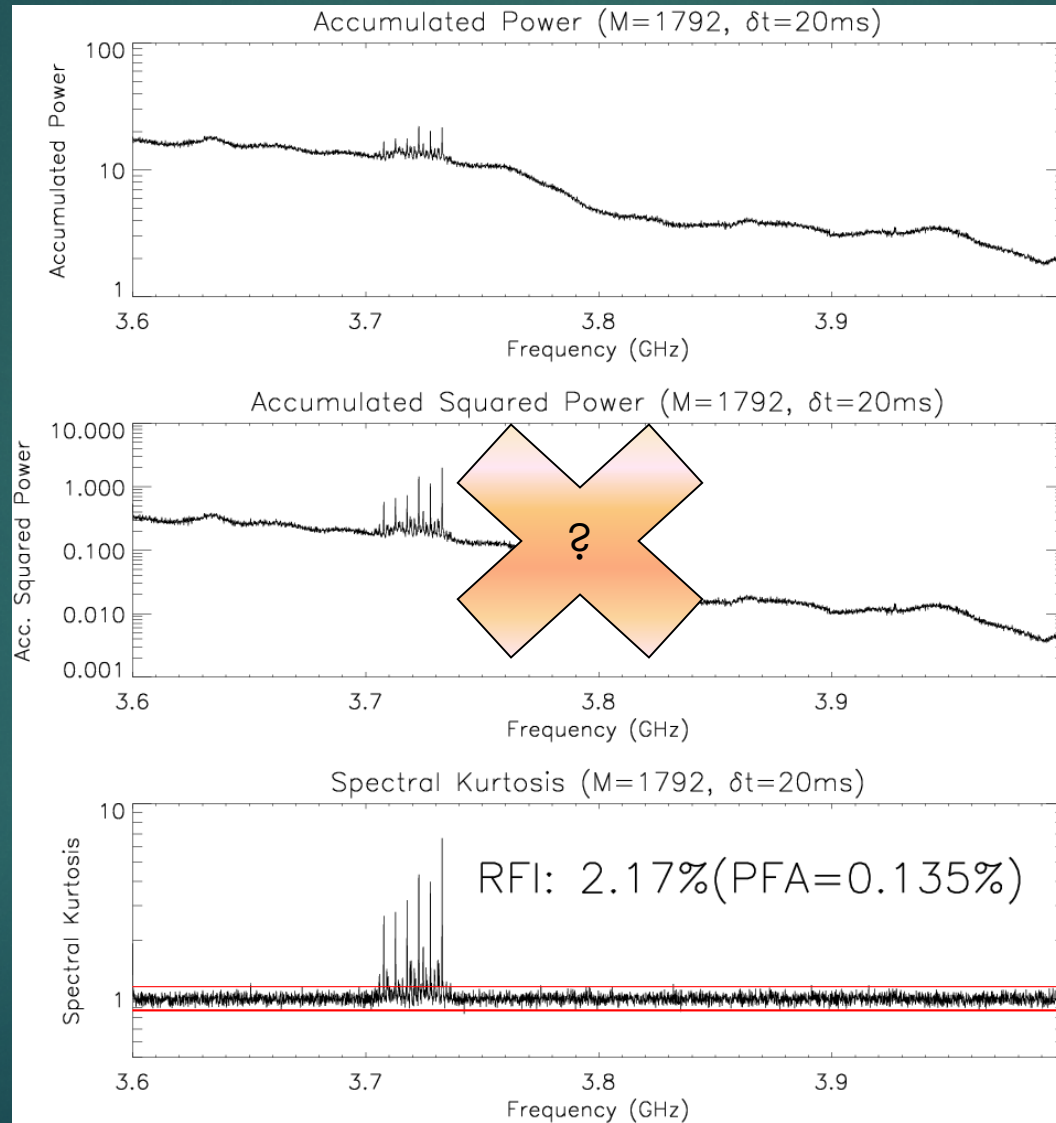
High bit resolution POWER AND SQUARED POWER outputs

Nita, Hickish, MacMahon, and Gary 2016, J. Astronomical Instrumentation 5(4)



EOVSA SK RFI EXCISION EXAMPLE

9/20



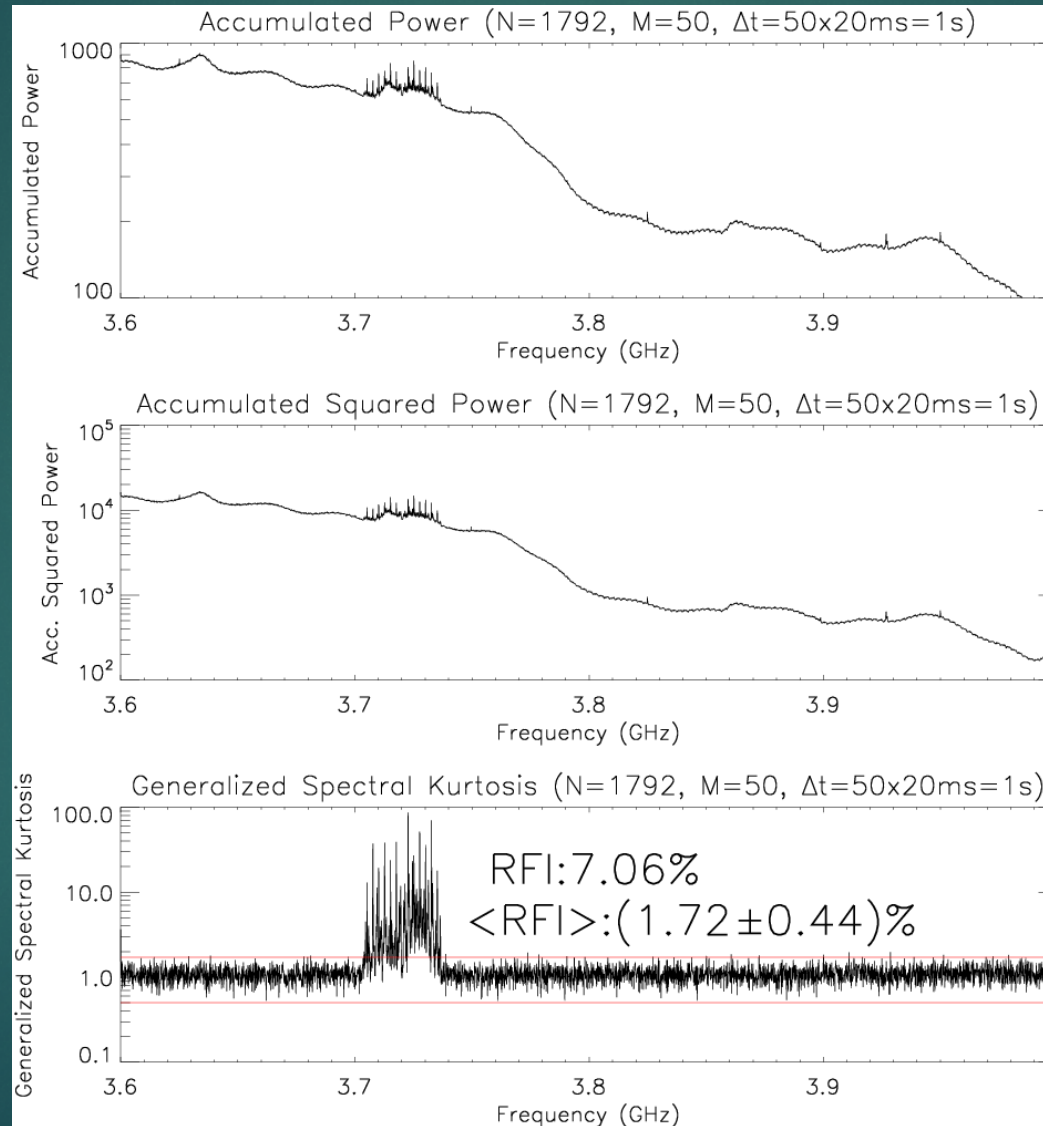
$$S_1 = \sum_{k=1}^M P_k$$

$$S_2 = \sum_{k=1}^M P_k^2$$

$$SK = \frac{M+1}{M-1} \left(\frac{MS_2}{S_1^2} - 1 \right)$$

EOVSA Generalized SK Analysis

10/20



$$s_1 = \sum_{k=1}^N P_k$$
$$S_1 = \sum_{i=1}^M s_1$$

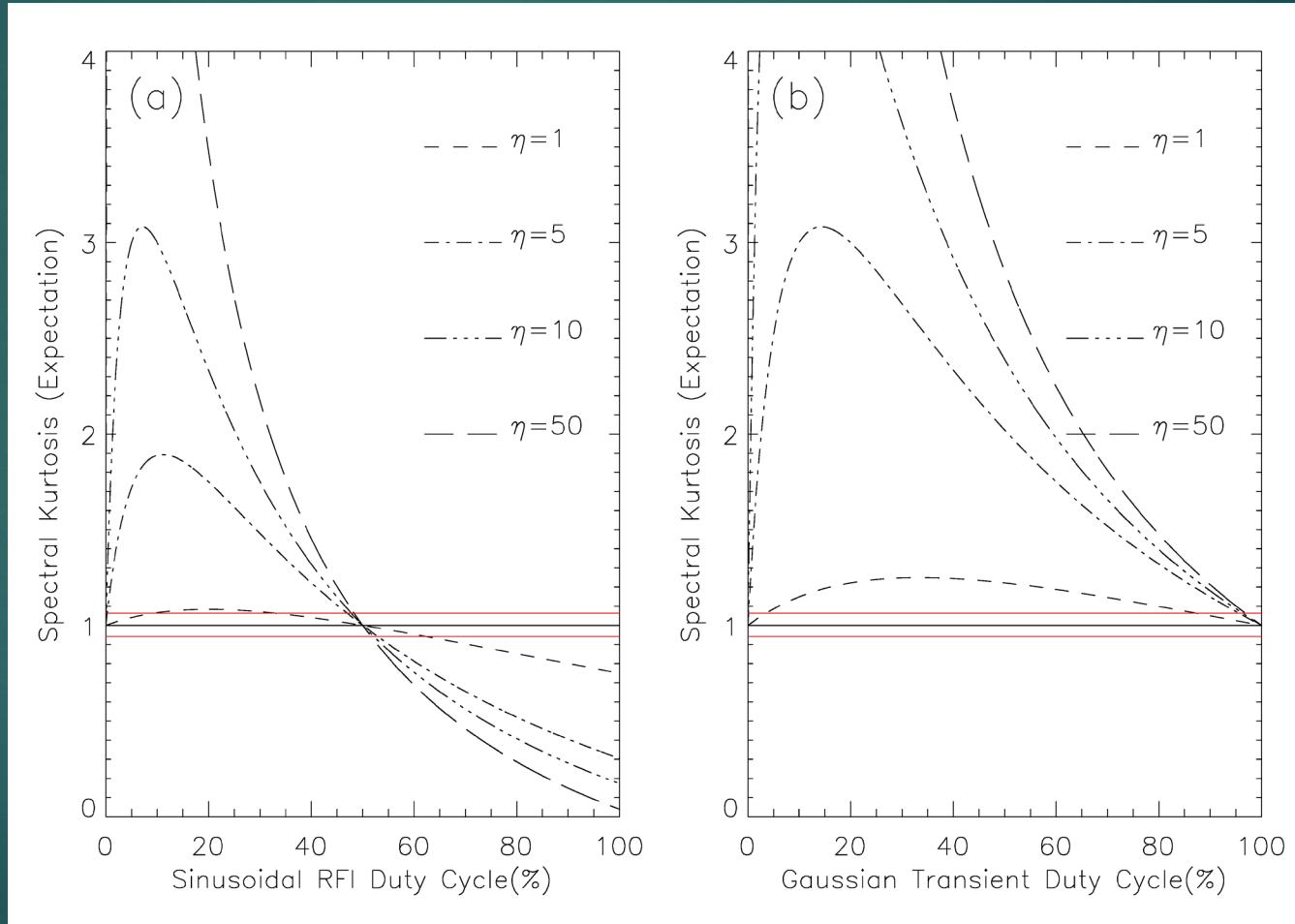
$$S_2 = \sum_{i=1}^M s_1^2$$

$$SK = \frac{MN + 1}{M - 1} \left(\frac{MS_2}{S_1^2} - 1 \right)$$

SK Dependence on the Integration–Relative Duty-Cycle RFI and Gaussian Transient Signals

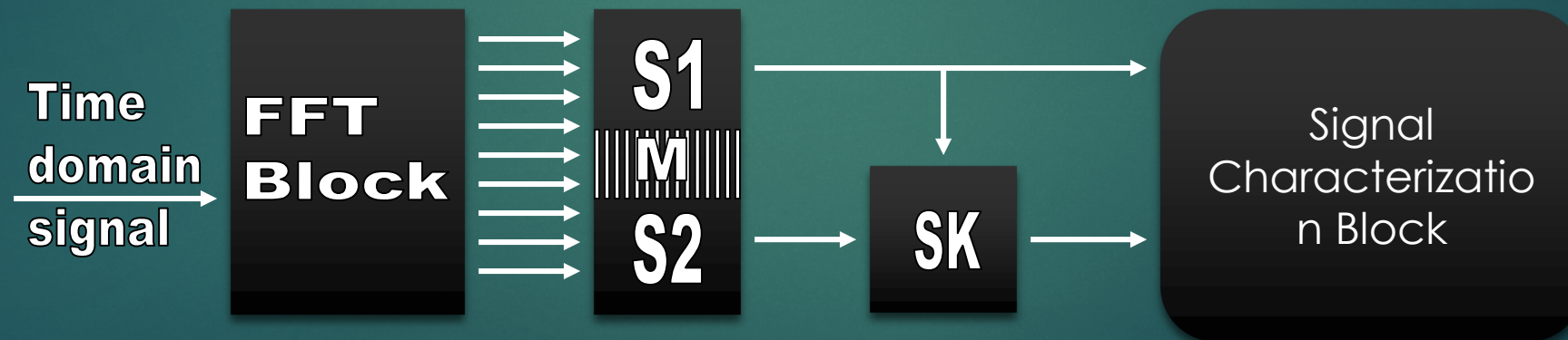
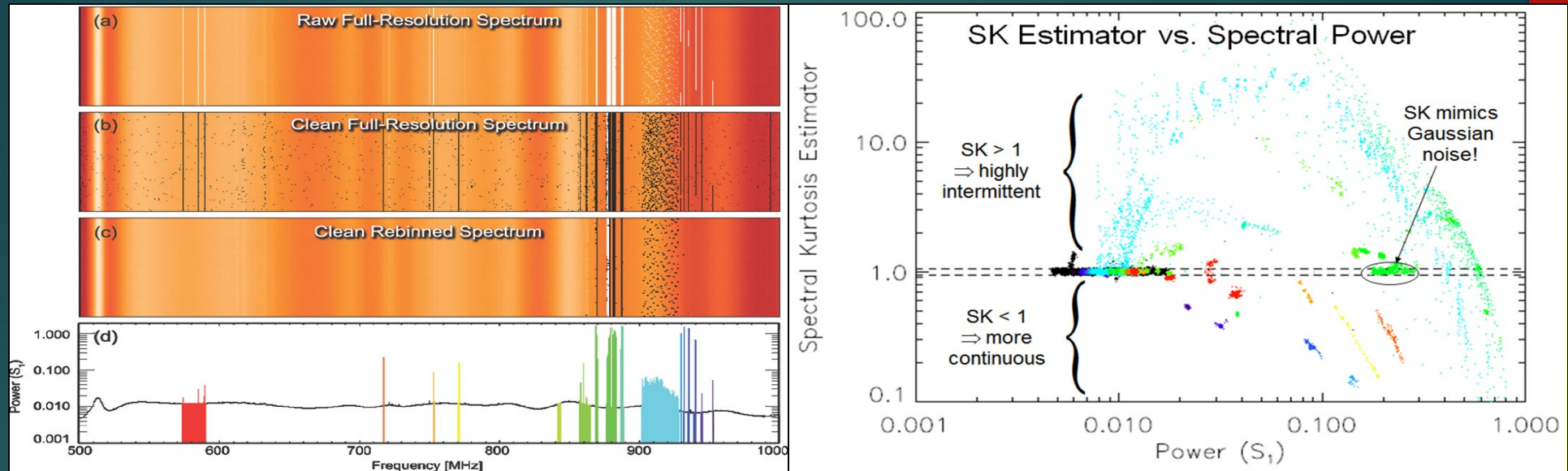
(Nita et al. 2007, PASP, 119; Nita 2016, MNRAS, 458)

11/20

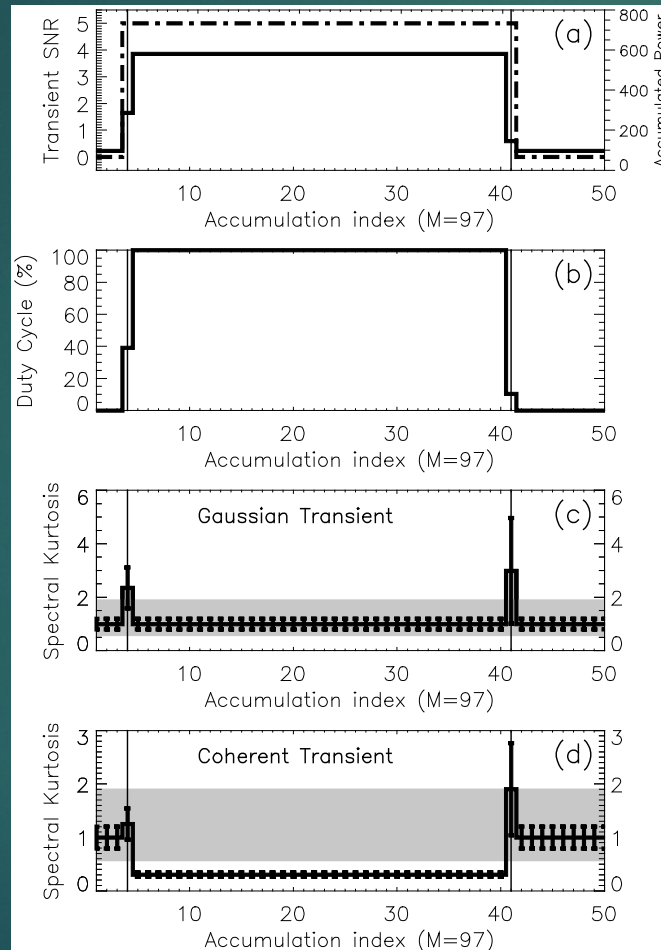


SPECTRAL KURTOSIS: A POWERFUL SIGNAL CLASSIFICATION TOOL

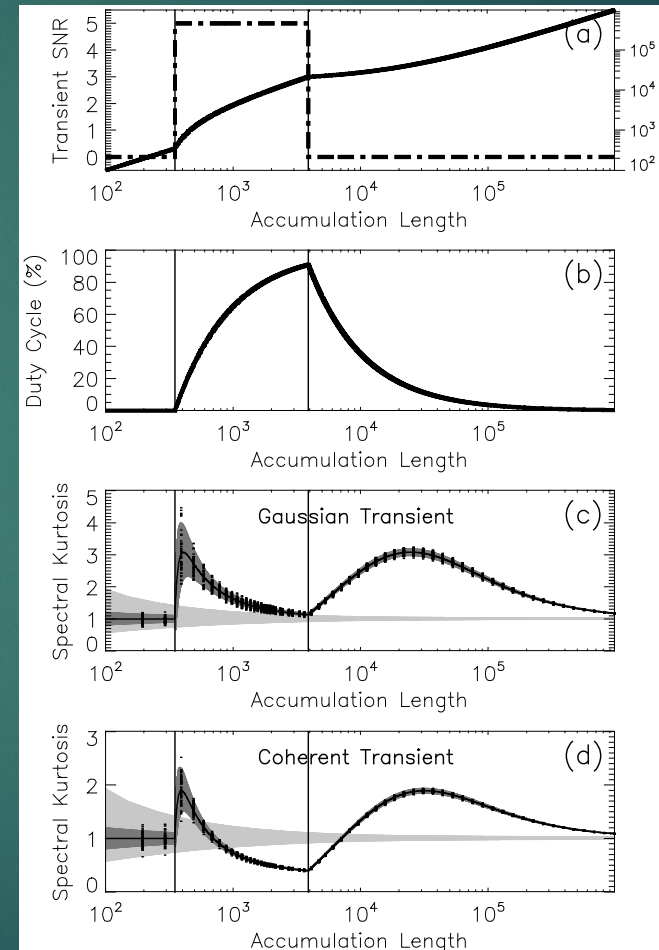
12/20



Multiscale SK Analysis : Real-Time Detection and Discrimination Transients (Nita 2016, MNRAS, 458)



Monoscale SK Analysis

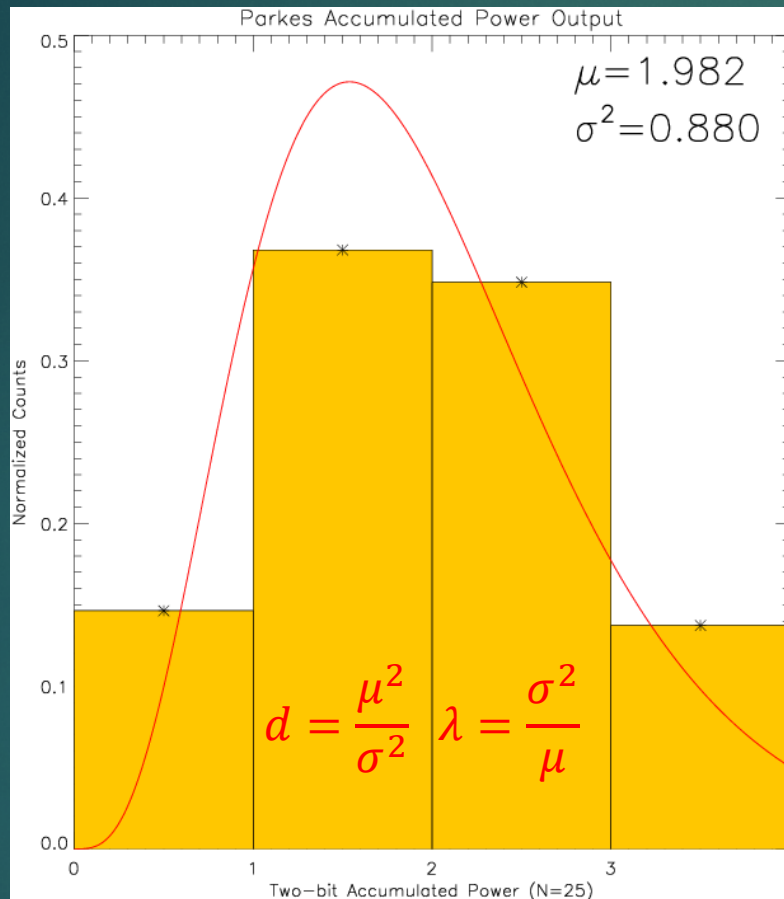


Multiscale SK Analysis

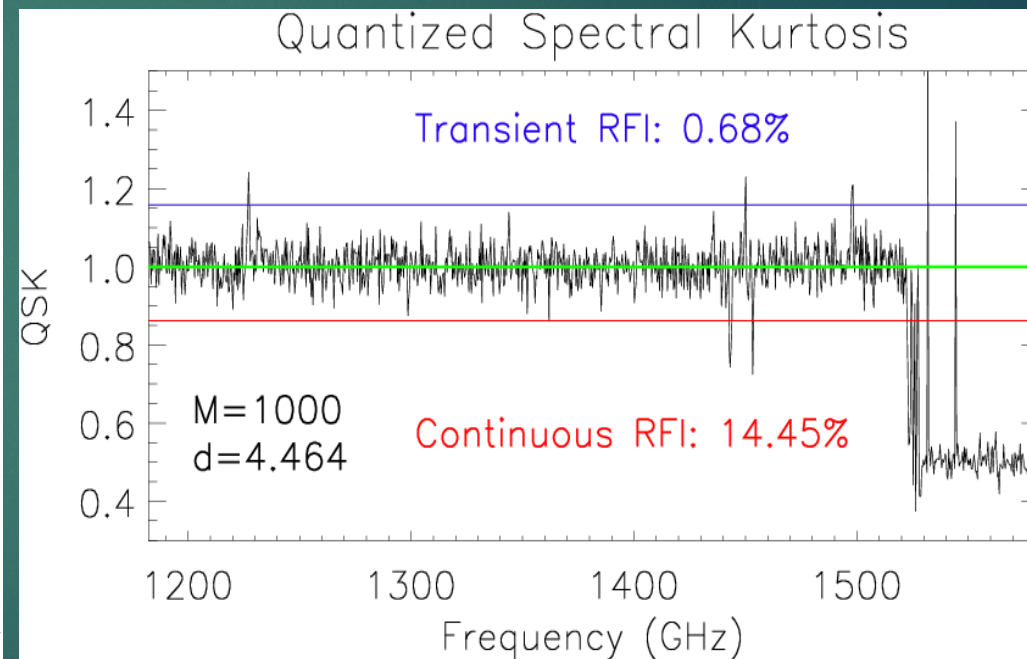
Spectral Kurtosis Statistics of Quantized Signals

Nita, Gary and Hellborg 2016, IEEE RFI Workshop

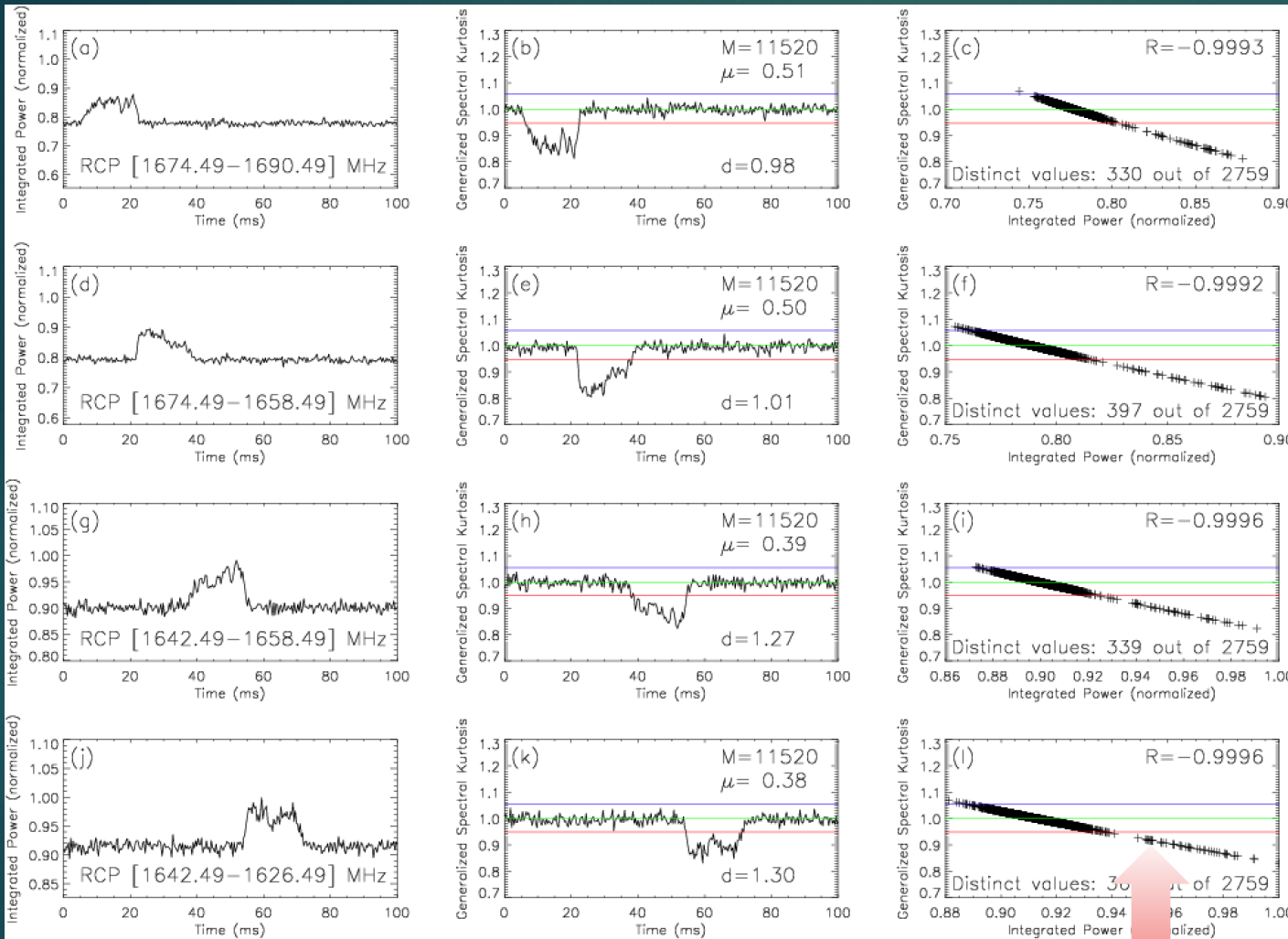
14/20



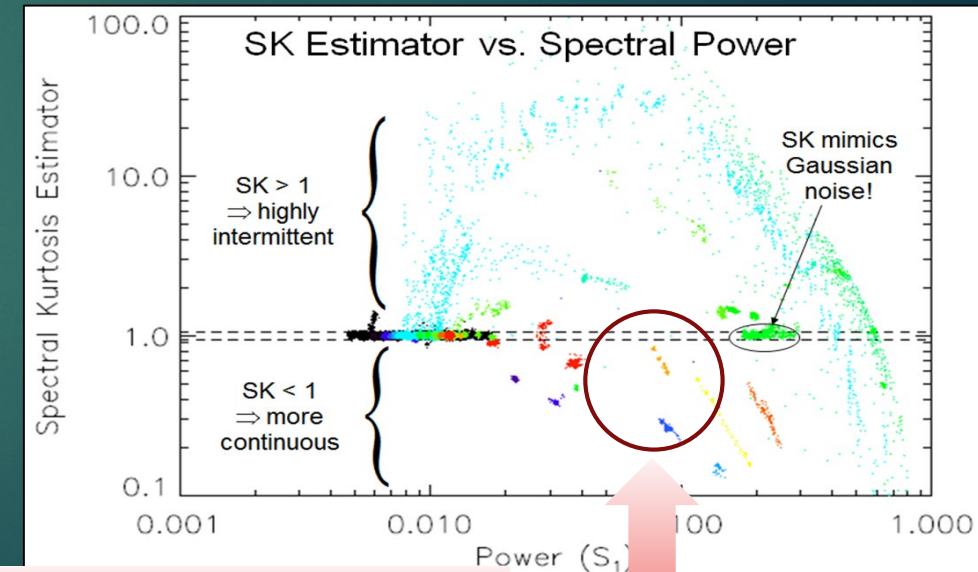
$$SK = \frac{Md + 1}{M - 1} \left(\frac{MS_2}{S_1^2} - 1 \right)$$



The distribution of the Parkes Telescope quantized accumulated power output corresponding to a Gaussian time domain signal can be approximated by a Gamma distribution of shape parameter $d < N$, to which the Generalized Spectral Kurtosis theory may be applied.

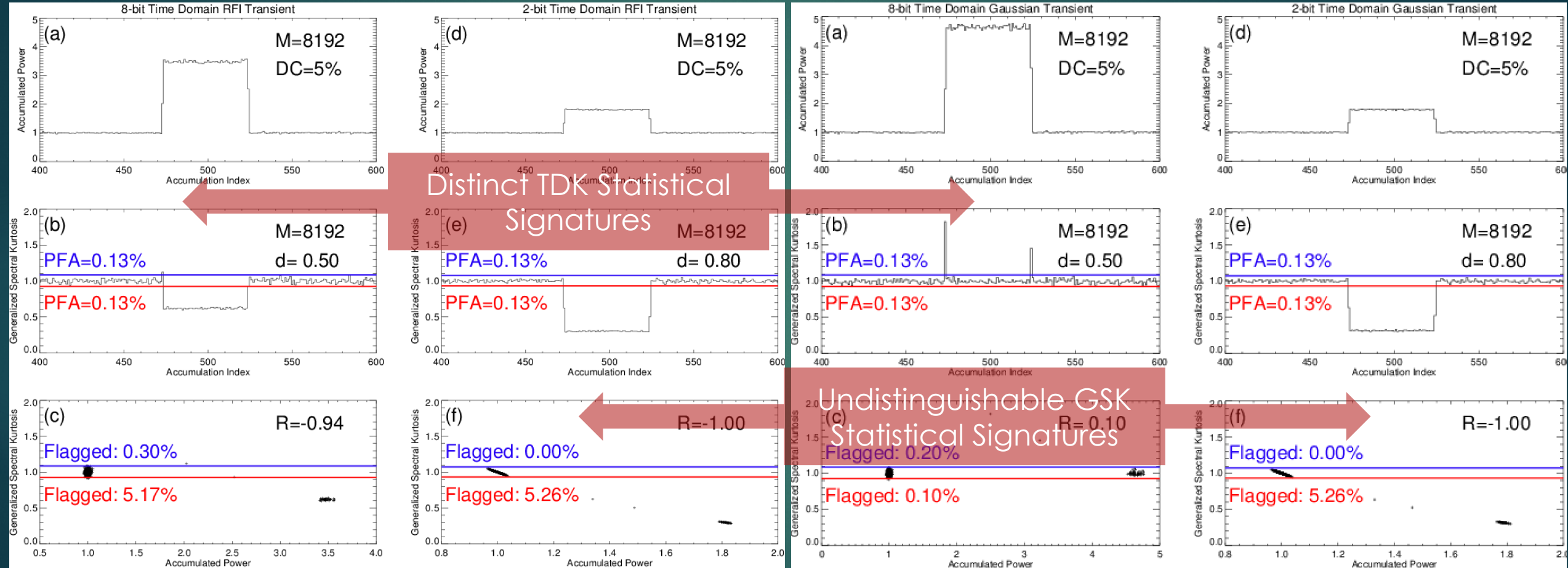


Time Domain GSK analysis of the VLBI 2-bit sampling RCP voltage data containing the FRB 121102 signal



RFI-like statistical signature of the FRB 121102 2-bit signal

8-bit vs 2-bit Time-Domain GSK Analysis of RFI and Gaussian Signals



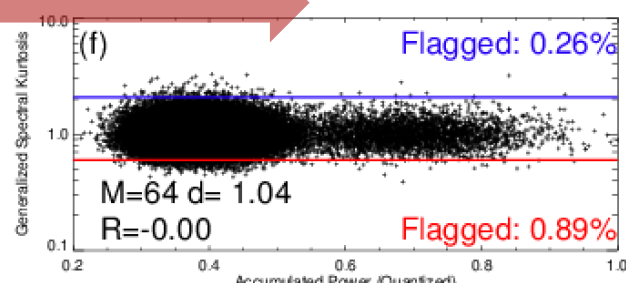
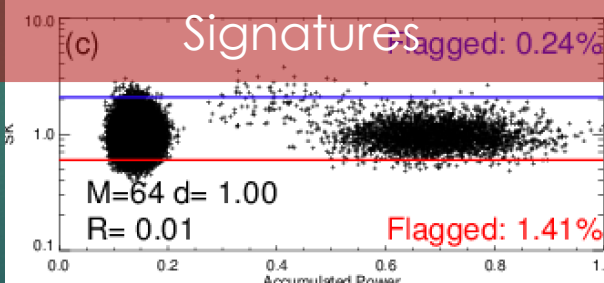
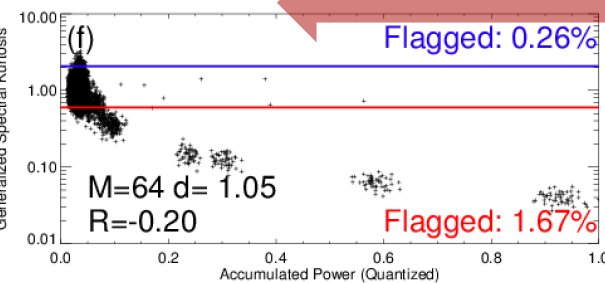
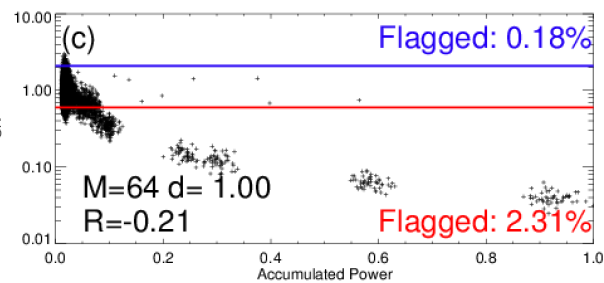
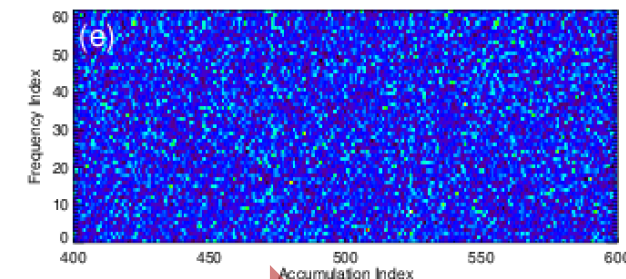
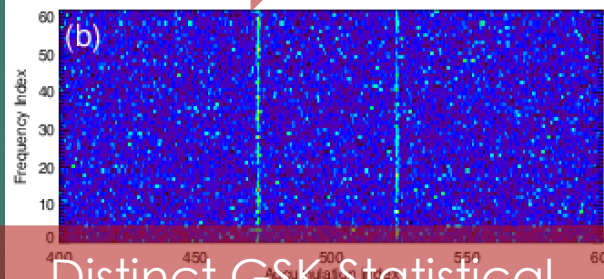
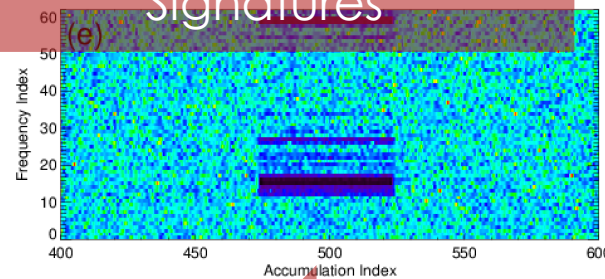
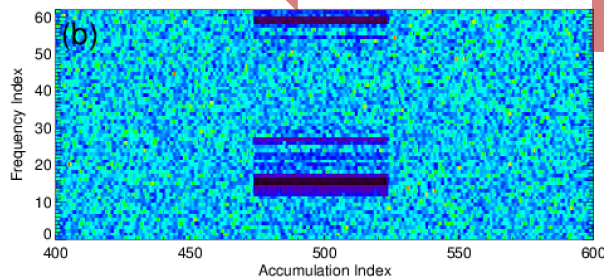
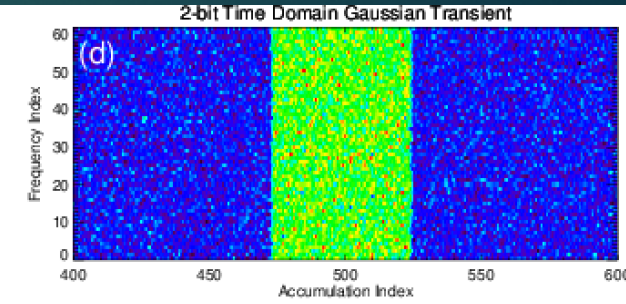
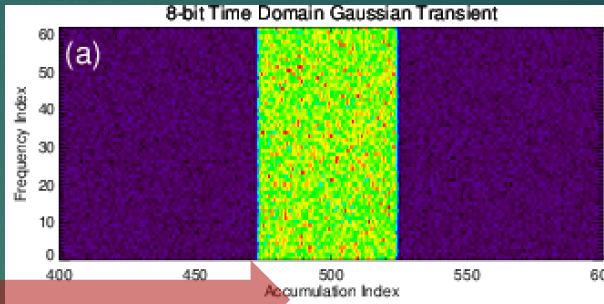
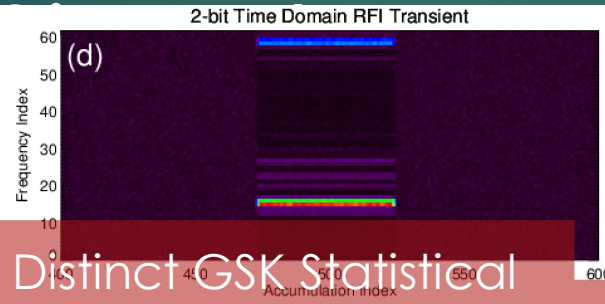
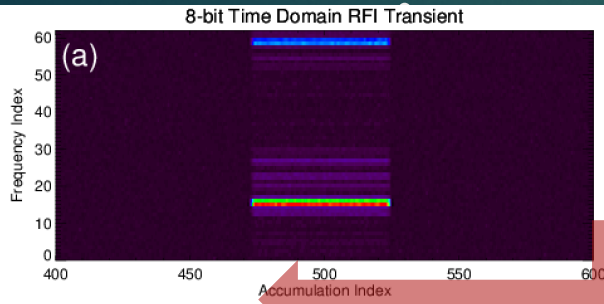
RFI Transient Signal

Gaussian Transient Signal

8-bit vs 2-bit Spectral-Domain GSK Analysis of RFI and Gaussian Signals

Distinct GSK Statistical Signatures

Distinct GSK Statistical Signatures

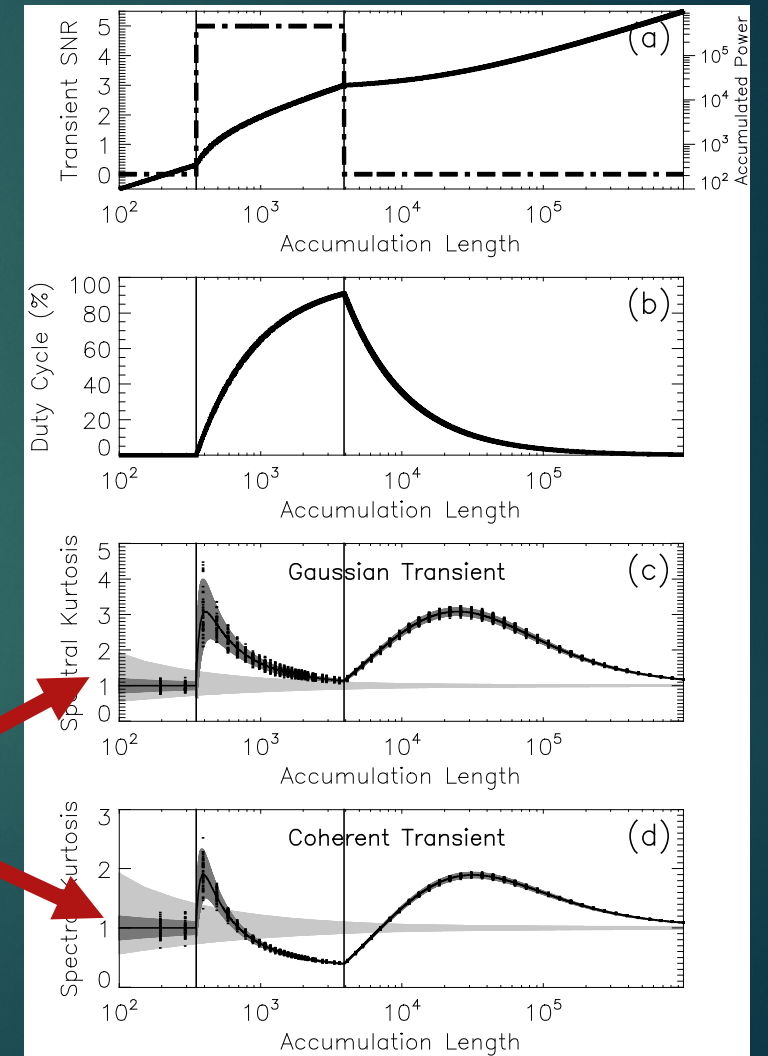
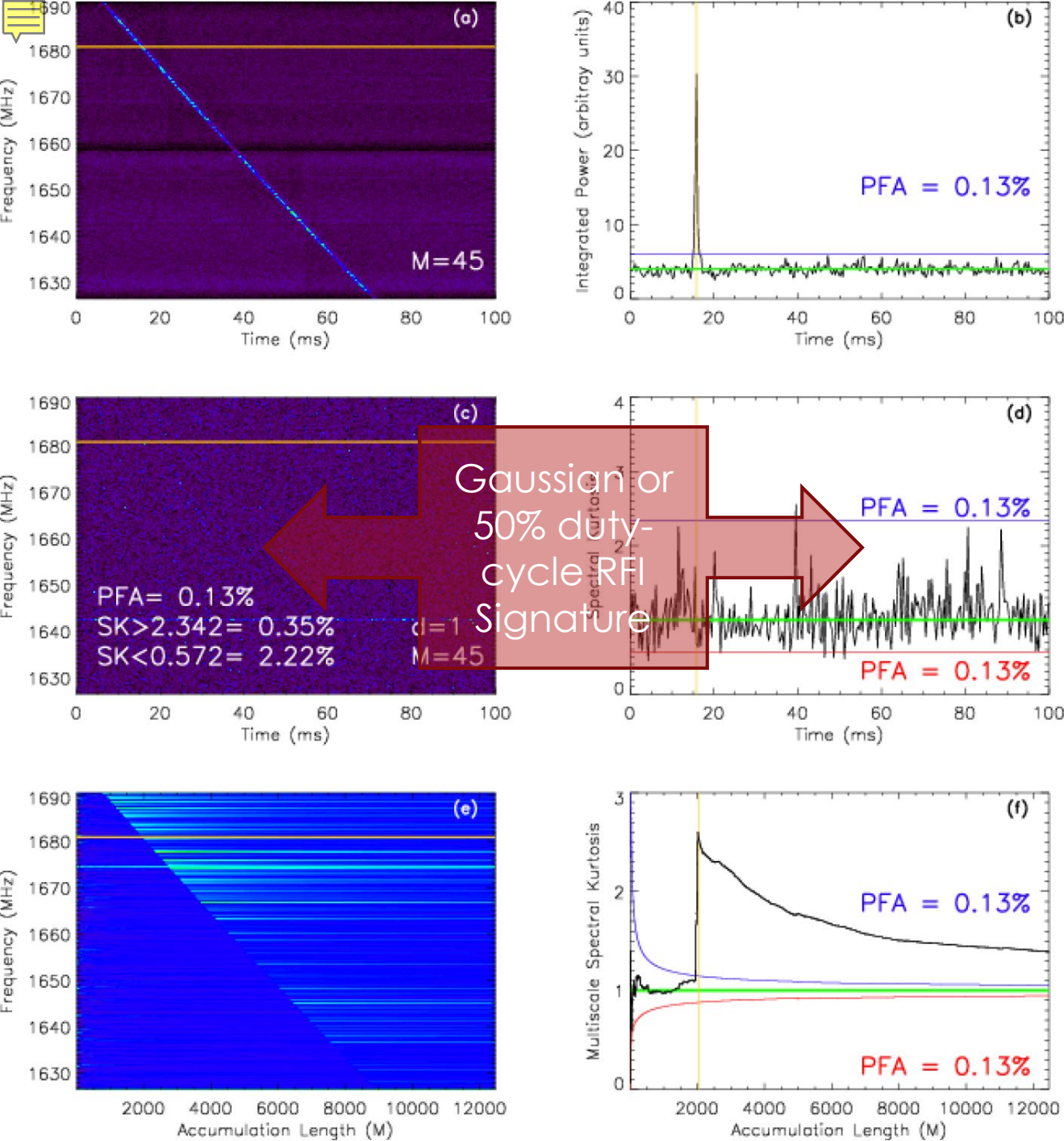


RFI Transient Signal

Gaussian Transient Signal

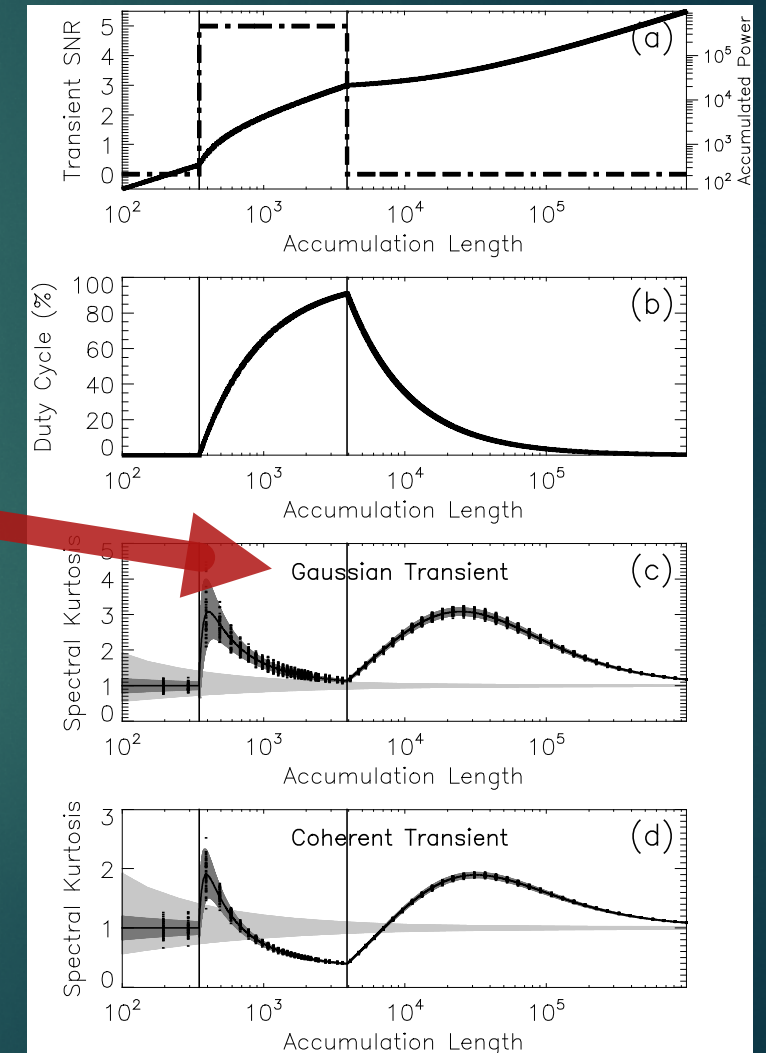
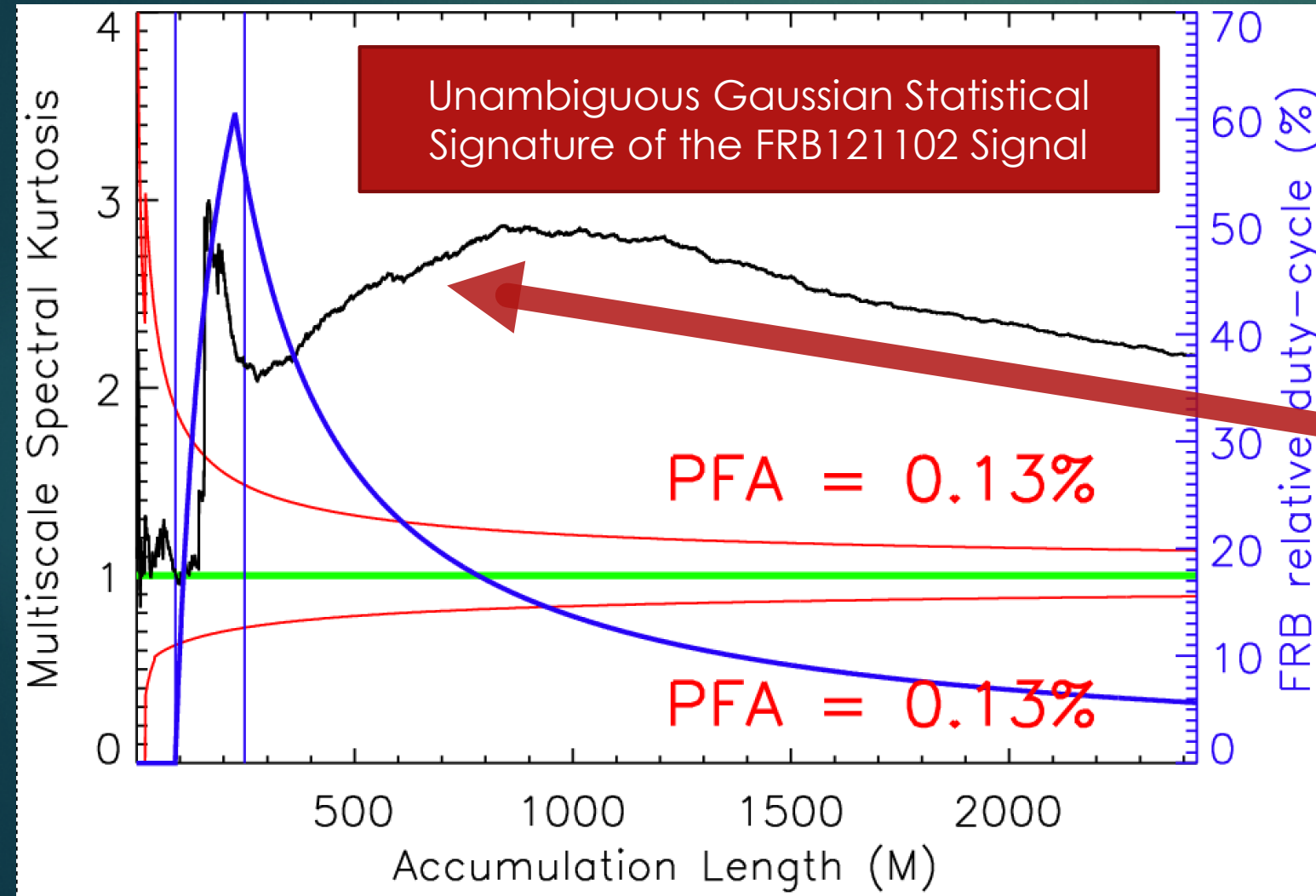
SK and Multi-scale SK Analysis

18/20



Multi-scale SK Analysis using an adaptive starting point of integration

19/20



Conclusions

20/20

- ▶ Time domain Kurtosis analysis of the 2-bit quantized VLBI signals can detect both RFI and natural astronomical transients but is not capable of distinguish them. **Therefore, astronomical transients may be mistakenly flagged as RFI.**
- ▶ Spectral Kurtosis analysis of the 32-bit FFT-transformed 2-bit time domain quantized VLBI signals can detect RFI while remaining blind to the presence of Gaussian transients (but may still detect sharp edges of such gaussian signals). **Therefore it is generally safe to employ spectral domain SK analysis for the purpose automatic RFI excission.**
- ▶ Multi-scale SK analysis of the 32-bit FFT-transformed 2-bit time domain quantized VLBI signals can detect both RFI and Gaussian transient signals and discriminate them based on their statistical signature. **Therefore it is generally safe to employ spectral domain SK analysis for the purpose of detecting and discriminating RFI and natural transients.**
- ▶ **Using multiscale SK analysis we unambiguously established, for the first time, the Gaussian nature of an FRB 121102 signal detected by VLBI**