On Optimal Sensing and Capacity Trade-off in Cognitive Radio Systems with Directional Antennas

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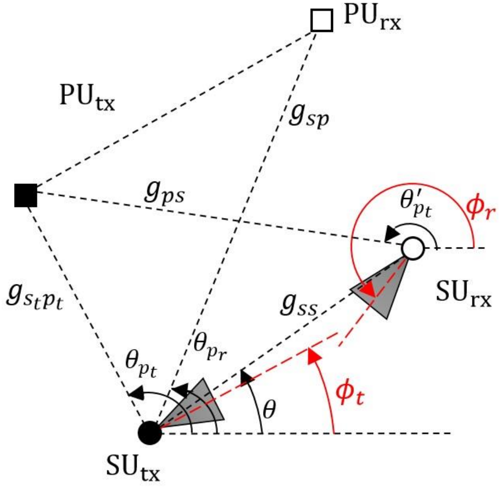
Outline

- ✓ System Model
- ✓ Spectrum Sensing
- ✓ Data Communication Channel
- ✓ Sensing-Capacity Trade-off
- ✓ Constrained Optimization Problem
- ✓ Solution
- ✓ Simulation Results

System Model

Geometry

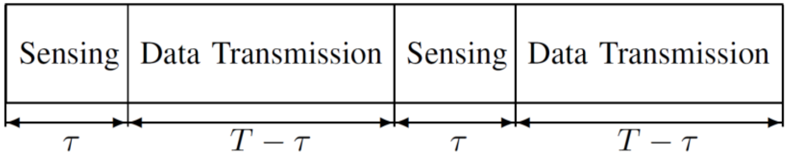
- Secondary users (SUs) and primary users (PUs) coexist.
- SUs are equipped with steerable directional antennas.
- The directional antennas can identify and enable transmission and reception across spatial domain and enhance spectrum utilization, compared with omni-directional antennas.
- $\mathrm{SU_{tx}}$ first senses the spectrum, and, then transmits data to $\mathrm{SU_{rx}}$ with power P if spectrum is sensed idle.
- $\mathrm{SU_{tx}}$ knows the geometry of CR network.
- $\mathrm{SU_{tx}}$ knows only the CSI of $\mathrm{SU_{tx}}$-$\mathrm{SU_{rx}}$ link, and the statistics of the other links.
- θ, θ_{pr} and θ_{pt} are the orientations of $\mathrm{SU_{rx}}$, $\mathrm{PU_{rx}}$ and $\mathrm{PU_{tx}}$ w.r.t. $\mathrm{SU_{tx}}$.
- ϕ_t and ϕ_r are the boresight of $\mathrm{SU_{tx}}$ and $\mathrm{SU_{rx}}$ antennas (to be optimized).



System Model

Frame Structure of SUs

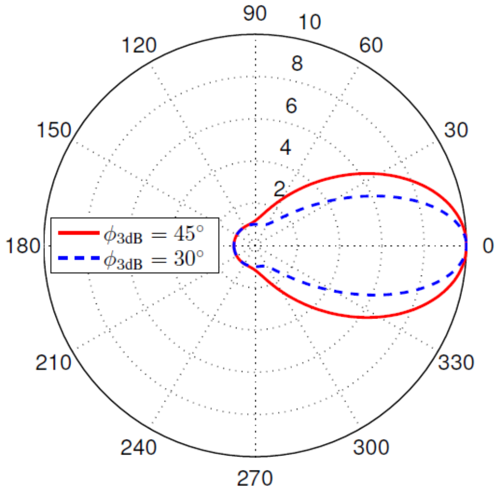
- SU_{tx} employs a frame with duration T seconds.
- SU_{tx} senses the spectrum for a duration of τ seconds, to decide whether it is busy or idle.
- The remaining frame of duration $T-\tau$ seconds is used for data transmission if the channel is sensed idle.



Antenna Model

Gaussian Pattern $$A(\phi) = A_1 + A_0 \exp\left(-B\left(\frac{\phi}{\phi_{3\text{dB}}}\right)^2\right)$$

$\phi_{3\text{dB}}$ is the half-power beam-width
A_1 and A_0 are two constant parameters



Spectrum Sensing

- We formulate the spectrum sensing at SU$_{\text{tx}}$ as a binary hypothesis testing problem.

➢ The Binary Hypothesis Testing Problem

$$\begin{cases} \mathcal{H}_0: \quad r[k] = w[k] & P(\mathcal{H}_0) = \pi_0 \\ \mathcal{H}_1: \quad r[k] = \sqrt{g_{s_t p_t} A(\phi_t - \theta_{p_t}) L_{s_t p_t}}\; p[k] + w[k] & P(\mathcal{H}_1) = \pi_1 \end{cases}$$

- $E(p^2) = P_p$

- Considering energy detection, the decision rule at SU$_{\text{tx}}$ is $Z = \frac{1}{N_s}\sum_{k=1}^{N_s} |r[k]|^2 \gtrless_{\hat{\mathcal{H}}_0}^{\hat{\mathcal{H}}_1} \xi$

$$P_f(\phi, \tau) = \Pr\{\hat{\mathcal{H}}_1 | \mathcal{H}_0\} = Q\left(\left(\frac{\xi}{\sigma_n^2} - 1\right)\sqrt{\tau f_s}\right)$$

$$P_d(\phi, \tau) = \Pr\{\hat{\mathcal{H}}_1 | \mathcal{H}_1\} = Q\left(\left(\frac{\xi}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma + 1}}\right)$$

- $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_0$with probabilities $\hat{\pi}_1$ and $\hat{\pi}_0$ denote that the result of spectrum sensing is busy and idle.

Data Communication Channel

- When the spectrum is sensed idle, the SU$_{\text{tx}}$ uses power P to transmit signal to SU$_{\text{rx}}$.

$$y[m] = \sqrt{g_{ss} L_{ss} G(\theta, \phi_t, \phi_r)}\; s[m] + n[m]$$

$$G(\theta, \phi_t, \phi_r) = A(\phi_t - \theta) A(\phi_r - \pi - \theta) \qquad s \sim \mathcal{N}(0, P) \qquad n \sim \mathcal{N}(0, \sigma_n^2)$$

Ergodic Capacity

- Spectrum sensing is imperfect and the ergodic capacity would depend on the true status of the PU and the spectrum sensing result.
- The false alarm and detection probabilities should be incorporated in the design and performance analysis.

$$C = D\, E\{\alpha_0\, c_{0,0} + \beta_0\, c_{1,0}\}$$

$$\alpha_0 = \Pr\{\mathcal{H}_0, \widehat{\mathcal{H}}_0\} \qquad \beta_0 = \Pr\{\mathcal{H}_1, \widehat{\mathcal{H}}_0\} \qquad D = \frac{T-\tau}{T}$$

$$c_{0,0} = \log_2\left(1 + \frac{g_{ss} L_{ss} G\, P}{\sigma_n^2}\right)$$

$$c_{1,0} = \log_2\left(1 + \frac{g_{ss} L_{ss} G\, P}{\sigma_n^2 + P_p g_{ps} L_{ps} A(\phi_r - \theta'_{p_t})}\right)$$

Sensing-Capacity Trade-off

- If we increase the sensing time τ, the spectrum sensing becomes more accurate. On the other hand, data transmission duration decreases. Therefore, a trade-off exists between the sensing time and the transmission capacity of our CR network.

- To increase probability of detection $P_d(\phi,\tau)$, $\mathrm{SU_{tx}}$’s antenna should be pointed to $\mathrm{PU_{tx}}$’s direction to receive the maximum power. On the other hand, the $\mathrm{SU_{tx}}$’s antenna should be pointed to $\mathrm{SU_{rx}}$’s direction to maximize the transmission capacity. Thus, there is a sensing-capacity trade-off in terms of the $\mathrm{SU_{tx}}$’s antenna orientation.

- There are trade-offs between sensing and capacity in terms of sensing duration τ and $\mathrm{SU_{tx}}$'s antenna orientation.

Constrained Optimization Problem

- **Outage Interference Probability Constraint**
- We define the interference outage probability as the probability that the interference exceeds a maximum threshold I_{pk} be smaller than a maximum value ε.

$$Pr\left\{D\beta_0 g_{sp} L_{sp} P\, A\left(\phi_t - \theta_{p_r}\right) > I_{pk} \mid g_{ss}\right\} \le \varepsilon \qquad (1)$$

- **Peak Transmit Power Constraint**

$$D\,\hat{\pi}_0\,P \le P_{pk} \qquad (2)$$

- **Constraints on Angles**

$$|\phi_t - \theta| \le \phi_{3\mathrm{dB}} \qquad (3a)$$

$$|\phi_r - \pi - \theta| \le \phi_{3\mathrm{dB}} \qquad (3b)$$

- **Optimization Problem**

$$\underset{P,\tau,\phi_t,\phi_r}{\mathrm{Max}} \quad C = D\,E\left\{\alpha_0\, c_{0,0} + \beta_0\, c_{1,0}\right\}$$

$$\mathrm{s.t.:}\ (1), (2) \text{ and } (3) \text{ are satisfied.}$$

Solution

- Taking the first derivative of C with respect to τ, we get

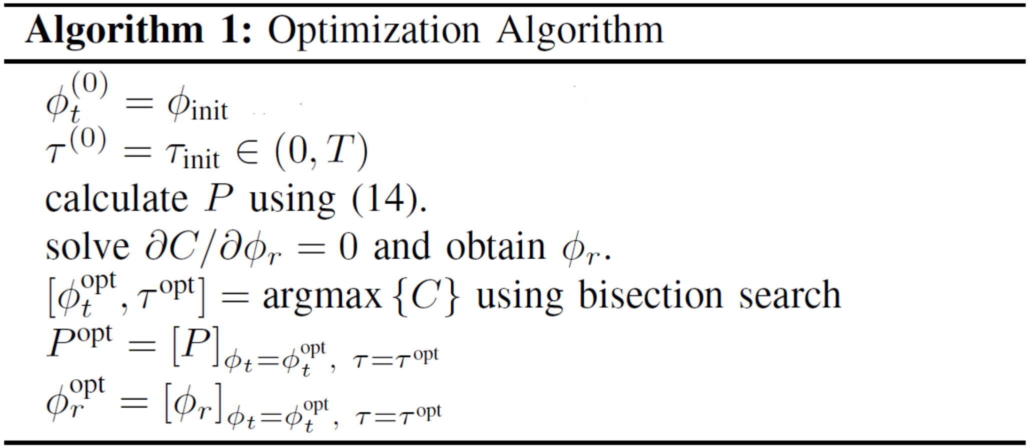
$$\lim_{\tau\to 0}\frac{\partial C}{\partial \tau} \to +\infty \qquad \lim_{\tau\to T}\frac{\partial C}{\partial \tau} < 0$$

$$\xi \geq \sigma_n^2(1+m\gamma) \qquad m = \frac{\pi_1}{\pi_1+\pi_0\sqrt{2\gamma+1}} < 1$$

- Hence, C has a maximum point with respect to τ within the interval $(0, T)$.
- The capacity is concave with respect to P and φ_r. However, in general, it is not concave with respect to φ_t and τ.

$$P^{\text{opt}} = \min\left\{\frac{P_{\text{pk}}}{D\hat{\pi}_0}, \frac{-I_{\text{pk}}}{D\bar{b}_0\ln(\varepsilon)}\right\} \qquad (14)$$

$$\bar{b}_0 = \beta_0\gamma_{sp}L_{sp}A(\phi_t - \theta_{p_r})$$



Algorithm 1: Optimization Algorithm

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$\phi_t^{(0)} = \phi_{\text{init}}$
$\tau^{(0)} = \tau_{\text{init}} \in (0, T)$
calculate $P$ using (14).
solve $\partial C/\partial\phi_r = 0$ and obtain $\phi_r$.
$[\phi_t^{\text{opt}}, \tau^{\text{opt}}] = \text{argmax}\,\{C\}$ using bisection search
$P^{\text{opt}} = [P]_{\phi_t=\phi_t^{\text{opt}},\ \tau=\tau^{\text{opt}}}$
$\phi_r^{\text{opt}} = [\phi_r]_{\phi_t=\phi_t^{\text{opt}},\ \tau=\tau^{\text{opt}}}$
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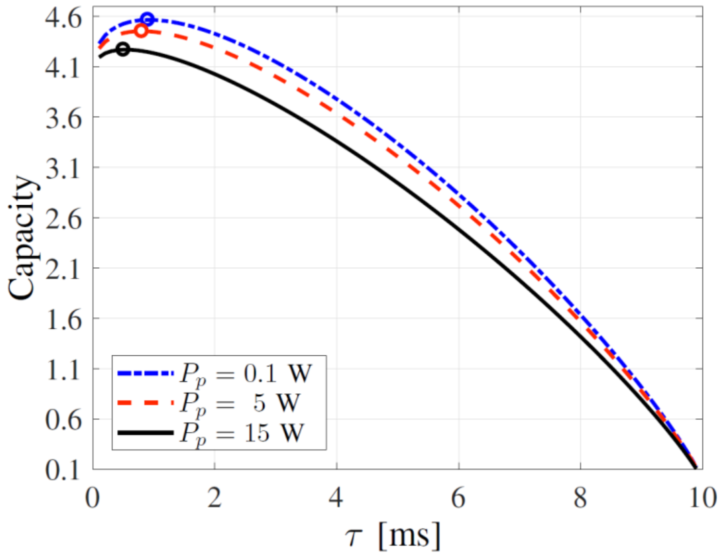
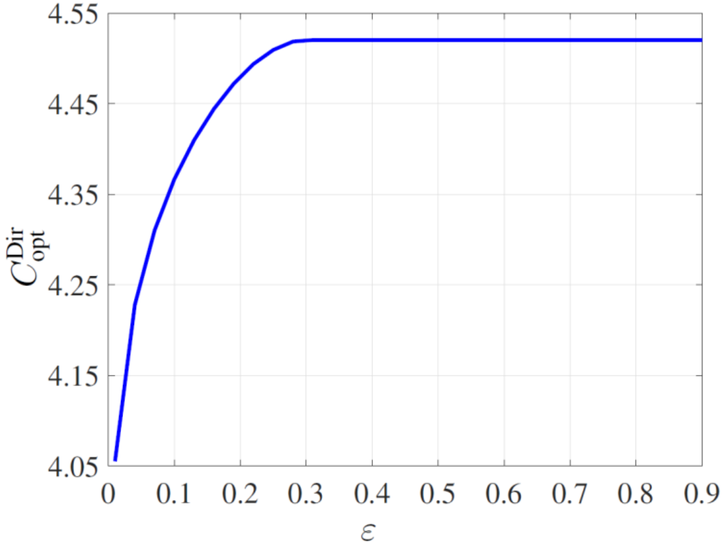
Simulation Results

$\pi_1 = 0.3,\ T = 10\ \text{ms},\ f_s = 20\ \text{KHz},$



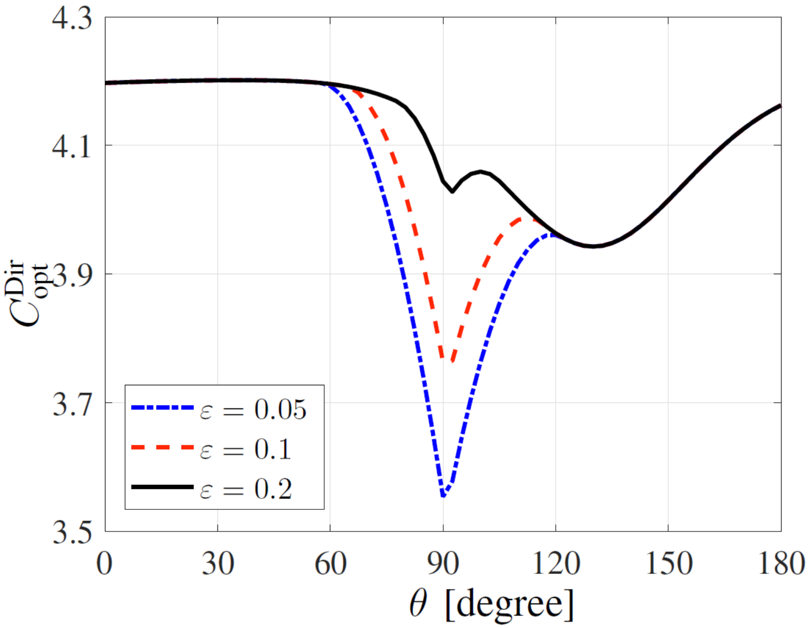
Variations of C versus τ.



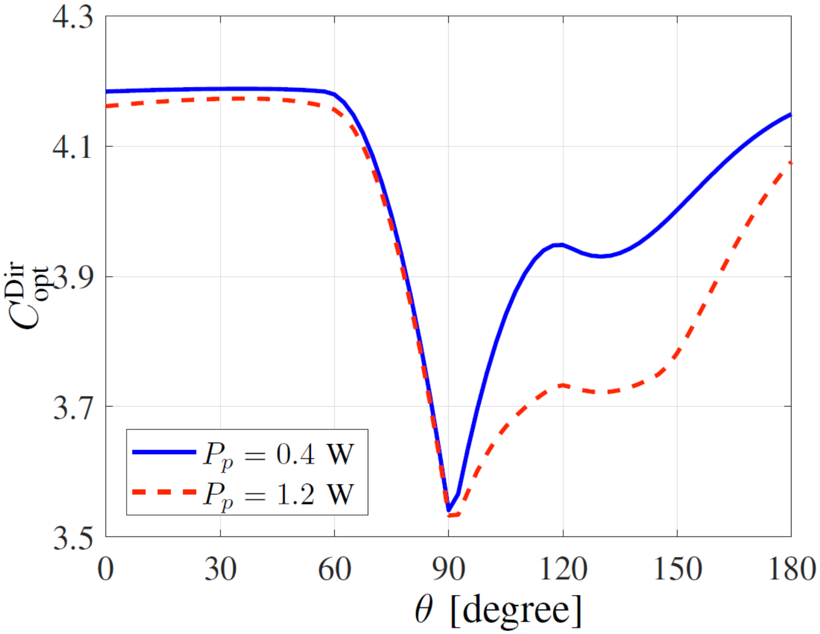
Variations of $C_{\mathrm{opt}}^{\mathrm{Dir}}$ versus ε.

$I_{pk} = 2\ \text{dB}, \qquad P_{pk} = 10\ \text{dB}, \qquad \varphi_{3dB} = 30°, \qquad \varepsilon = 0.05$

Simulation Results



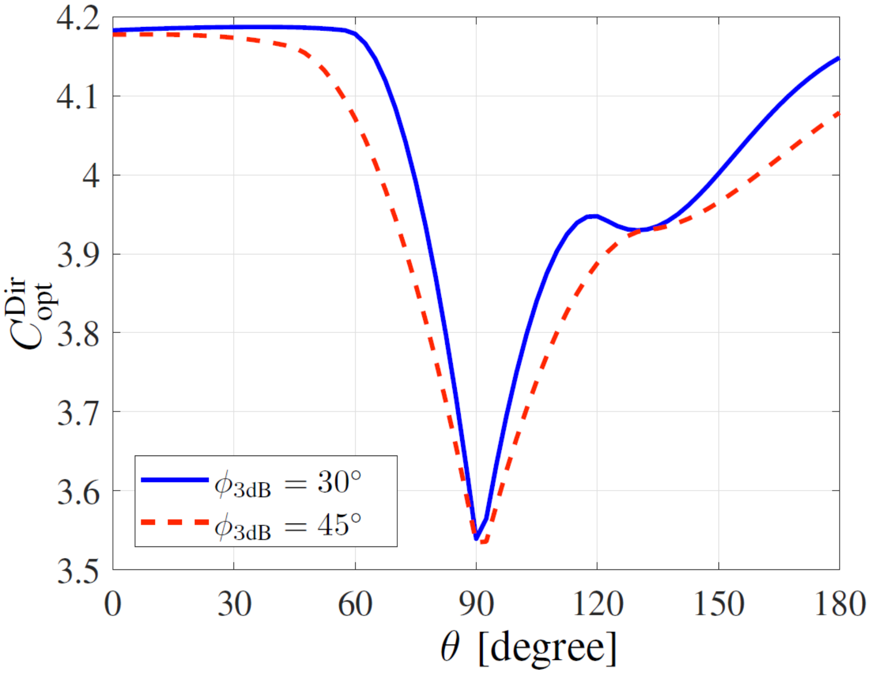
$C_{\mathrm{opt}}^{\mathrm{Dir}}$ versus θ for different ε.



$C_{\mathrm{opt}}^{\mathrm{Dir}}$ versus θ for different P_p.

$\phi_{p_r} = 90^o$

Simulation Results



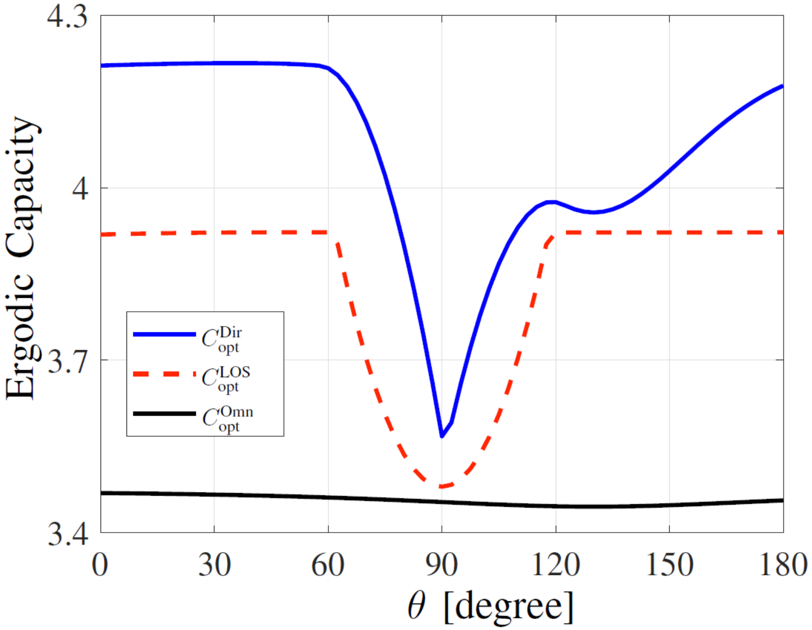
$C_{\mathrm{opt}}^{\mathrm{Dir}}$ versus θ for $\phi_{3\mathrm{dB}} = 30°, 45°$.

Simulation Results

$C_{\text{opt}}^{\text{Dir}} \rightarrow P, \tau, \phi_t, \phi_r$

$C_{\text{opt}}^{\text{Omn}} \rightarrow P, \tau$

$C_{\text{opt}}^{\text{LOS}} \rightarrow P, \tau \qquad (\phi_t = \theta,\ \phi_r = \pi + \theta)$

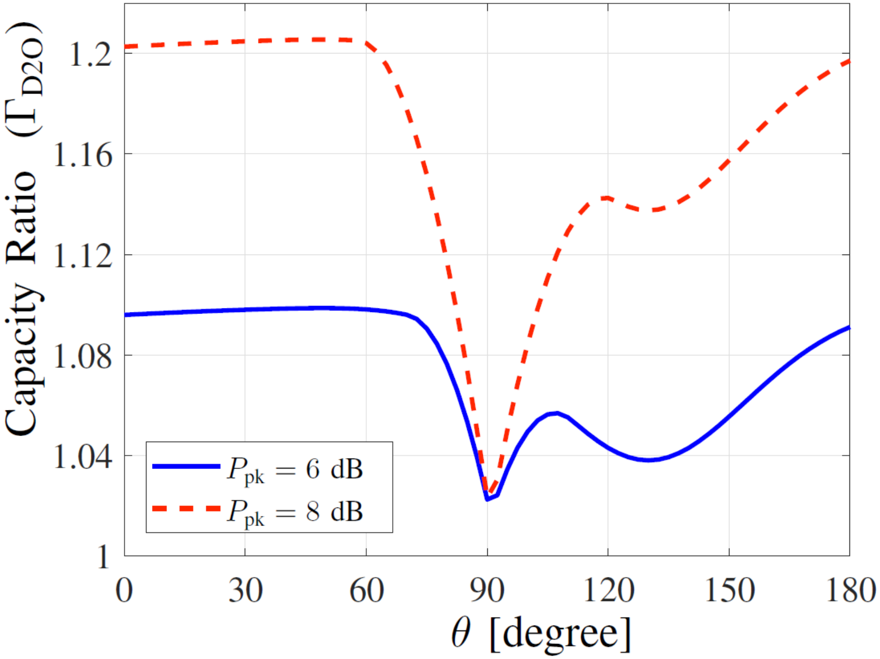


$C_{\text{opt}}^{\text{Dir}}$, $C_{\text{opt}}^{\text{LOS}}$ and $C_{\text{opt}}^{\text{Omn}}$ versus θ.

$\varphi_{3dB} = 30°$

Simulation Results

capacity ratio $\Gamma_{\mathrm{D2O}} = C_{\mathrm{opt}}^{\mathrm{Dir}} / C_{\mathrm{opt}}^{\mathrm{Omn}}$



Γ_{D2O} versus θ for $P_{\mathrm{pk}} = 6, 8$ dB.

Conclusion

- The directional antennas can identify and enable transmission and reception across spatial domain and enhance spectrum utilization, compared with omni-directional antennas.

- There are trade-offs between sensing and capacity in terms of sensing duration τ and $\mathrm{SU}_{\mathrm{tx}}$'s antenna orientation.

- An antenna with narrower half-power beam-width always yields higher capacity since it can cancel more interference from $\mathrm{PU}_{\mathrm{tx}}$.

- The capacity ratio Γ_{D2O} is higher for large P_{pk}.

- We showed the effectiveness of using directional antennas and as well as optimizing their orientation on the capacity of CR network.

Thank you for your attention