Consensus Optimization for Distributed Registration

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Registration Problem

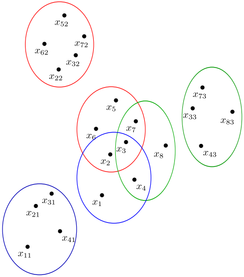
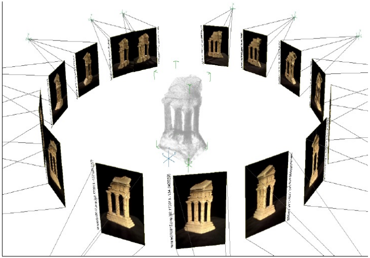


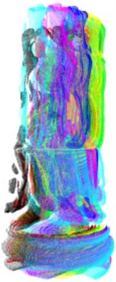
Figure: Registration problem for three point clouds.

- Given: Local coordinates $\mathbf{x}_{k,i}$, point correspondence.
- Unknowns: Global coordinates $\mathbf{z}_1, \ldots, \mathbf{z}_N$, rigid transformations $(\mathbf{O}_1, \mathbf{t}_1), \ldots, (\mathbf{O}_M, \mathbf{t}_M)$ where $\mathbf{O}_i \in \mathbb{O}(d), \mathbf{t} \in \mathbb{R}^d$, and $\mathbb{O}(d) = \{\mathbf{O} \in \mathbb{R}^{d \times d} : \mathbf{O}^\top \mathbf{O} = \mathbf{I}_d\}$.
- Noiseless scenario: $\mathbf{z}_k = \mathbf{O}_i \mathbf{x}_{k,i} + \mathbf{t}_i$.

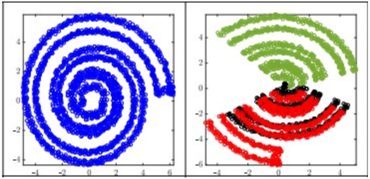
Applications



(a) Multiview registration.

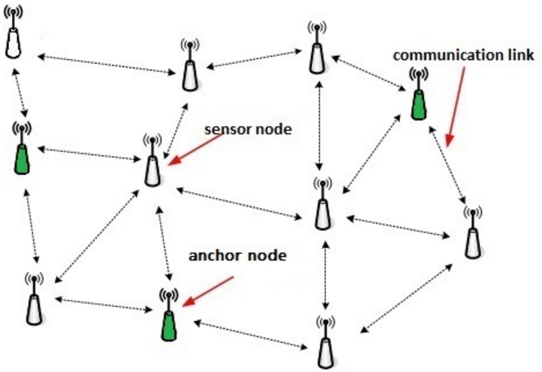


(b) 3D scan.



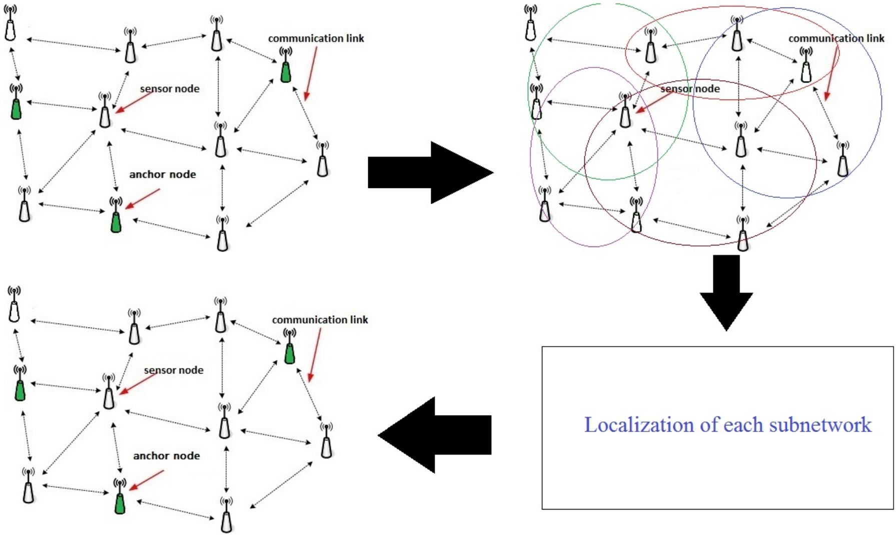
(c) Sensor network localization.

Sensor Network Localization (SNL)



- Available information: Inter-sensor distances.
- Aim: Estimate the original location of the sensors, or up to some rigid transformation (rotation, reflection, translation) of the original locations.

Divide-and-Conquer Based SNL Algorithm



Least Square Formulation

$$\min_{\mathbf{z}_k,\mathbf{t}_i\in\mathbb{R}^n,\mathbf{O}_i\in\mathbb{O}(d)} \sum_{i=1}^{M}\sum_{k\in\mathcal{P}_i} \|\mathbf{z}_k - (\mathbf{O}_i\mathbf{x}_{k,i} + \mathbf{t}_i)\|^2. \tag{1}$$

- Non-convex problem.
- Convex relaxation proposed by Chaudhury et al., SIOPT 2015[1]:
  - Fix $\mathbf{O}_i$'s, jointly optimizes over $\mathbf{x}_k$ and $\mathbf{t}_i$.
  - Leads to the following problem

$$\begin{aligned} \min_{\mathbf{G}\in\mathbb{S}^n_+} \quad & \mathrm{Tr}(\mathbf{CG}) \\ \text{s.t.} \quad & [\mathbf{G}]_{ii} = \mathbf{I}_d,\ \forall\, i \in [1:M],\ \mathrm{rank}(\mathbf{G}) = d. \end{aligned} \tag{2}$$

  - Drop the rank and solve the semidefinite programming.

[1] K. N. Chaudhury, Y. Khoo, and A. Singer, "Global registration of multiple point clouds using semidefinite programming," SIAM Journal on Optimization, vol. 25, no. 1, pp. 468-501, 2015.

What is the issue then?

- Computing $\mathbf{C}$.

$$\boxed{\mathbf{C} = \mathbf{D} - \mathbf{B}\mathbf{L}^{\dagger}\mathbf{B}^{\top}} \tag{3}$$

  $\mathbf{L}$ is a symmetric matrix of size $(N + M)$.

- Large number of point clouds.

- Rank of $\mathbf{G}^{\star}$ may not be d.

Repose the Registration Problem

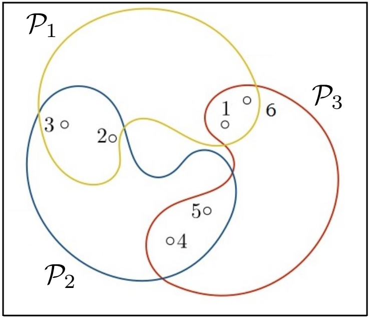
- The least-square formulation of the registration problem:

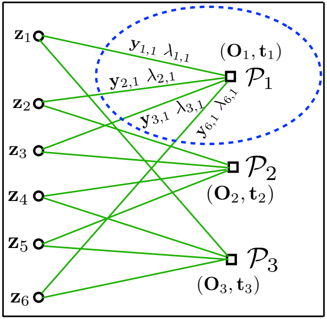
$$\min_{\mathbf{z}_k,\mathbf{t}_i\in\mathbb{R}^n,\mathbf{O}_i\in\mathbb{O}(d)} \sum_{i=1}^{M}\sum_{k\in\mathcal{P}_i}\|\mathbf{z}_k-(\mathbf{O}_i\mathbf{x}_{k,i}+\mathbf{t}_i)\|^2. \tag{4}$$

- Reformulate the registration problem:

$$\begin{aligned} \min_{\mathbf{y}_{k,i},\mathbf{z}_k,\mathbf{t}_i\in\mathbb{R}^n,\mathbf{O}_i\in\mathbb{O}(d)} \quad & \sum_{i=1}^{M}\sum_{k\in\mathcal{P}_i}\|\mathbf{y}_{k,i}-(\mathbf{O}_i\mathbf{x}_{k,i}+\mathbf{t}_i)\|^2 \\ \text{s.t.} \quad & \mathbf{y}_{k,i}=\mathbf{z}_k,\ \forall k\in\mathcal{P}_i,\ \ i\in[1:M]. \end{aligned} \tag{5}$$

Membership Graph





An example of a point set configuration, and the correspondence graph.

$$\begin{aligned} \min_{\mathbf{y}_{k,i}, \mathbf{z}_k, \mathbf{t}_i \in \mathbb{R}^n, \mathbf{O}_i \in \mathbb{O}(d)} \quad & \sum_{i=1}^{M} \sum_{k \in \mathcal{P}_i} \|\mathbf{y}_{k,i} - (\mathbf{O}_i \mathbf{x}_{k,i} + \mathbf{t}_i)\|^2 \\ \text{s.t.} \quad & \mathbf{y}_{k,i} = \mathbf{z}_k, \ \forall k \in \mathcal{P}_i, \ i \in [1:M]. \end{aligned} \tag{6}$$

Augmented Lagrangian and the ADMM Solver

- Problem:

$$\begin{aligned} \min_{\mathbf{y}_{k,i},\mathbf{z}_k,\mathbf{t}_i\in\mathbb{R}^n,\mathbf{O}_i\in\mathbb{O}(d)} \quad & \sum_{i=1}^{M}\sum_{k\in\mathcal{P}_i}\|\mathbf{y}_{k,i}-(\mathbf{O}_i\mathbf{x}_{k,i}+\mathbf{t}_i)\|^2 \\ \text{s.t.} \quad & \mathbf{y}_{k,i}=\mathbf{z}_k,\ \forall k\in\mathcal{P}_i,\ i\in[1:M]. \end{aligned} \tag{7}$$

- Augmented Lagrangian:

$$\mathcal{L}_\rho=\sum_{k\sim i}\left(\|\mathbf{y}_{k,i}-(\mathbf{O}_i\mathbf{x}_{k,i}+\mathbf{t}_i)\|^2+\lambda_{k,i}^\top(\mathbf{y}_{k,i}-\mathbf{z}_k)+\frac{\rho}{2}\|\mathbf{y}_{k,i}-\mathbf{z}_k\|^2\right). \tag{8}$$

- Alternating direction methods for multipliers (ADMM) solver:

$$\begin{aligned} &(\mathbf{Y}^{(t)},\mathbf{O}^{(t)},\mathbf{T}^{(t)})=\operatorname*{argmin}_{\mathbf{Y},\mathbf{O},\mathbf{T}}\ \mathcal{L}_\rho(\mathbf{Y},\mathbf{O},\mathbf{T},\mathbf{Z}^{(t-1)},\boldsymbol{\Lambda}^{(t-1)}), \\ &\mathbf{Z}^{(t)}=\operatorname*{argmin}_{\mathbf{Z}}\ \mathcal{L}_\rho(\mathbf{Y}^{(t)},\mathbf{O}^{(t)},\mathbf{T}^{(t)},\mathbf{Z},\boldsymbol{\Lambda}^{(t-1)}), \\ &\boldsymbol{\lambda}_{k,i}^{(t)}=\boldsymbol{\lambda}_{k,i}^{(t-1)}+\rho(\mathbf{y}_{k,i}^{(t)}-\mathbf{z}_k^{(t)}), \qquad (k\sim i). \end{aligned} \tag{9}$$

Update Y, O, T

$$\min_{\mathbf{y}_{k,i},\mathbf{t}_i,\mathbf{O}_i} \sum_{i=1}^{M} \sum_{k\in\mathcal{P}_i} \left(\|\mathbf{y}_{k,i} - (\mathbf{O}_i\mathbf{x}_{k,i} + \mathbf{t}_i)\|^2 + \frac{\rho}{2}\|\mathbf{y}_{k,i} - (\mathbf{z}_k - \lambda_{k,i}/\rho)\|^2 \right) \tag{10}$$

- For each point cloud:

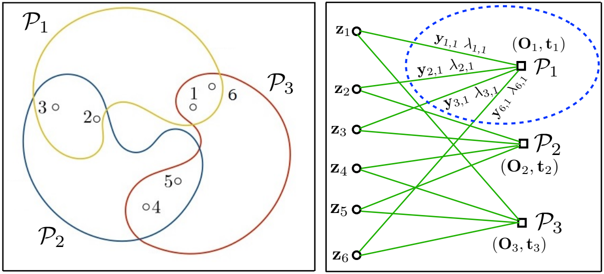
$$\min_{\mathbf{y}_{k,i},\mathbf{t}_i,\mathbf{O}_i} \sum_{k\in\mathcal{P}_i} \left(\|\mathbf{y}_{k,i} - (\mathbf{O}_i\mathbf{x}_{k,i} + \mathbf{t}_i)\|^2 + \frac{\rho}{2}\|\mathbf{y}_{k,i} - (\mathbf{z}_k - \lambda_{k,i}/\rho)\|^2 \right) \tag{11}$$

- First, minimizes over $\mathbf{y}_{k,i}$ and $\mathbf{t}_i$.
- Finally, solve the following:

$$\max_{\mathbf{O}_i\in\mathbb{O}(d)} \mathrm{Tr}(\mathbf{C}_i\mathbf{O}_i). \tag{12}$$

Update $\mathbf{Z}$

$$\min_{\mathbf{z}_k} \sum_{k \sim i} \|\mathbf{z}_k - (\mathbf{y}_{k,i}^{(t)} + \rho^{-1} \boldsymbol{\lambda}_{k,i}^{(t-1)})\|^2 \tag{13}$$



$$\mathbf{z}_k^{(t)} = \frac{1}{|\mathcal{N}_k|} \sum_{i \in \mathcal{N}_k} \left(\mathbf{y}_{k,i}^{(t)} + \rho^{-1} \boldsymbol{\lambda}_{k,i}^{(t-1)} \right) \tag{14}$$

Summary

- Computation is distributed over each point-cloud.
- Main computation per processor is an SVD: $\mathcal{O}(d^3)$.
- Solve the non-convex problem directly.

Performance Metric

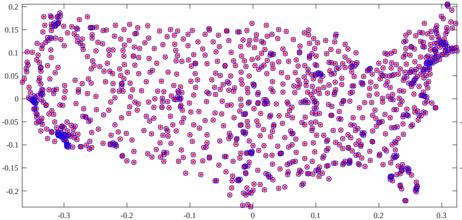
- Performance metric: Average Normalized Error[2] (ANE).

$$\texttt{ANE} = \left\{ \frac{\sum_{i=1}^{N} \|\hat{\mathbf{x}}_i - \bar{\mathbf{x}}_i\|^2}{\sum_{i=1}^{N} \|\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_c\|^2} \right\}^{1/2},$$

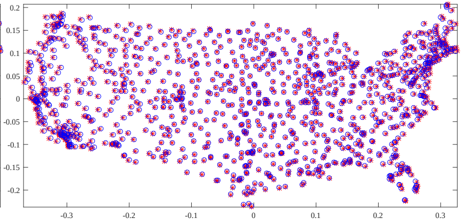
where,
$\hat{\mathbf{x}}_i$: estimated sensor position after the alignment,
$\mathbf{x}_i$: actual position of the sensor,
$\bar{\mathbf{x}}_c$: the centroid of the original sensor positions

$$\bar{\mathbf{x}}_c = \frac{1}{N} \sum_{i=1}^{N} \bar{\mathbf{x}}_i.$$

[2] M. Cucuringu, Y. Lipman, and A. Singer, "*Sensor network localization by eigenvector synchronization over the Euclidean group*," ACM Trans. on Sensor Networks, vol. 8, no. 3, pp. 19-42, 2012.

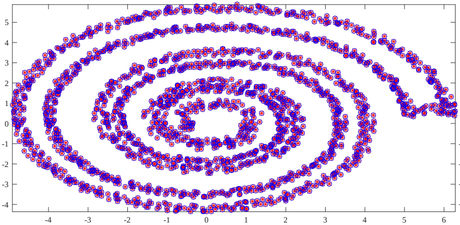


(d) $\eta = 0$, ANE = 9.5e-13.

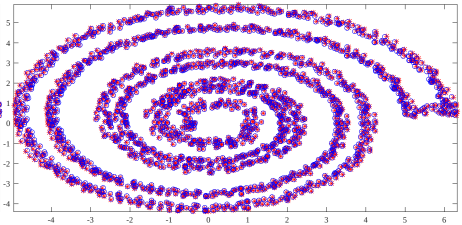


(e) $\eta = 0.006$, ANE = 9.1e-3.

Localization of the US cities dataset consisting of 1101 points. The sensing radius used for both (a) and (b) is $r = 0.06$, which is about 9% of the diameter of the dataset (0.704). The original and estimated locations are marked using blue circles and red stars.

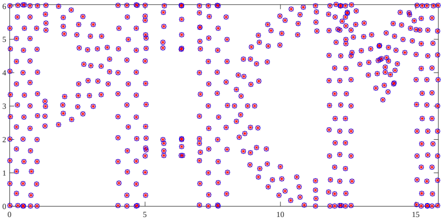


(f) $\eta = 0$, ANE $=$ 4.6e-11.

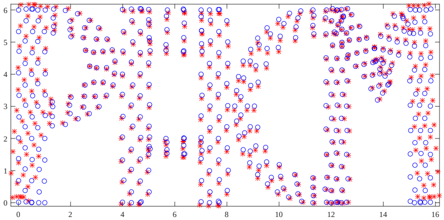


(g) $\eta = 0.01$, ANE $=$ 7.6e-3.

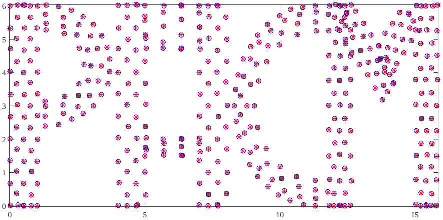
Localization of the spiral dataset consisting of 2259. The sensing radius used for both (a) and (b) is $r = 1$, which is about 9% of the diameter of the dataset (11.2). The original and estimated locations are marked using blue circles and red stars.



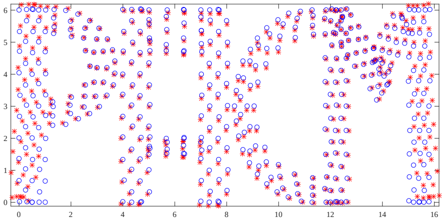
(h) Proposed (ANE = 4.4e-12).



(i) SNLSDP (ANE = 2.9e-2)



(j) Proposed (ANE = 2.6e-3).



(k) SNLSDP (ANE = 3e-2)

Localization of PACM dataset consisting of 495 points. The top and bottom rows correspond to $\eta = 0$ and $\eta = 0.03$. The original and estimated locations are marked using blue circles and red stars.

[3]

Questions?

[3] P.C. planwallpaper