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Conjugate gradient acceleration of non-linear smoothing filters

Iterated edge-preserving smoothing

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Outline:

- Bilateral (BF) filter
- Guided (GF) filter
- Preconditioned conjugate gradient iteration
- Numerical results

Announcement of numerical results:
For the test 1D signal of length 4730, the PCG acceleration provides the 9 times speedup for BF and 4 times speedup for GF filters.

Standard references for BF and GF filters

- C. Tomasi and R. Manduchi, Bilateral filtering for gray and color images, in *Proc. IEEE International Conference on Computer Vision*, Bombay, 1998, pp. 839–846.
- J. Sun, K. He and X. Tang, Guided image filtering, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 35(6):1397–1409, 2013.

Bilateral filter (BF)

A discrete function $x[j]$, $j \in \{1, 2, \ldots, N\}$, is an input signal for the bilateral filter. The output signal $y[i]$ is a weighted average of the signal values $x[j]$:

$$y[i] = \sum_j \frac{w_{ij}}{\sum_j w_{ij}} x[j].$$

Every index i has a spatial position p_i, and a spatial distance $\|p_i - p_j\|$ is determined for all pairs i and j.
The weights w_{ij} are defined by means of a guidance signal $g[i]$:

$$w_{ij} = \exp\left(-\frac{\|p_i - p_j\|^2}{2\sigma_d^2}\right) \exp\left(-\frac{(g[i] - g[j])^2}{2\sigma_r^2}\right),$$

where σ_d and σ_r are the filter parameters. When $g = x$, the bilateral filter is nonlinear and called self-guided. Numerical cost can be $O(N)$.

Iterated bilateral filter

- The nonnegative weights w_{ij} are the entries of the symmetric matrix W. Let us denote by D the diagonal matrix with the positive diagonal entries $d_i = \sum_j w_{ij}$. Then BF is the vector transform $y = D^{-1}Wx$. The spectrum of $D^{-1}W$ is real, and the eigenvalues corresponding to highly oscillatiing eigenvectors lie near 0.
  The symmetric nonnegative definite matrix $L = D - W \geq 0$ is called the Laplacian. The normalized Laplacian is $D^{-1}L = I - D^{-1}W$.
- The BF transform $y = D^{-1}Wx$ can be applied iteratively,
  1. changing the weights w_{ij} at each iteration using the result of the previous iteration as a guidance signal g, or
  2. using the fixed weights, calculated from the initial signal as a guidance signal, for all iterations.

  The former results in a nonlinear filter, the latter generates a linear filter, which may be faster to evaluate, since the BF weights are computed only once at the very beginning.

Guided filter (GF)

Algorithm 1 Guided Filter (GF)

Input: x, g, w, ϵ
Output: y

$$mean_g = f_{mean}(g, w)$$
$$mean_x = f_{mean}(x, w)$$
$$corr_g = f_{mean}(g. * g, w)$$
$$corr_{gx} = f_{mean}(g. * x, w)$$
$$var_g = corr_g - mean_g. * mean_g$$
$$cov_{gx} = corr_{gx} - mean_g. * mean_x$$
$$a = cov_{gx}./(var_g + \epsilon)$$
$$b = mean_x - a. * mean_g$$
$$mean_a = f_{mean}(a, w)$$
$$mean_b = f_{mean}(b, w)$$
$$y = mean_a. * g + mean_b$$

Guided filter (GF)

$f_{mean}(\cdot, w)$ is a mean filter with the window width w. The constant ϵ determines the smoothness degree: the larger ϵ the larger smoothing effect. The dot preceded operations $.*$ and $./$ denote the componentwise multiplication and division. Numerical complexity of the GF algorithm can be $O(N)$.

GF is $y = Wx$, where the entries of the symmetric matrix $W(g)$ are

$$W_{ij}(g) = \frac{1}{|\omega|^2} \sum_{k:\ (i,j)\in\omega_k} \left(1 + \frac{(g_i - \mu_k)(g_j - \mu_k)}{\sigma_k^2 + \epsilon}\right).$$

The windows ω_k of width w around all k have the number of pixels $|\omega|$. The values μ_k and σ_k^2 are the mean and variance of g over ω_k. Since $d_i = \sum_j w_{ij} = 1$, the graph Laplacian matrix is automatically normilized, i.e. $L = I - W$. The eigenvalues of $L(g)$ are real nonnegative with the low frequencies accumulated near 0 and high frequencies near 1. Similar to BF, the guided filter can be applied iteratively.

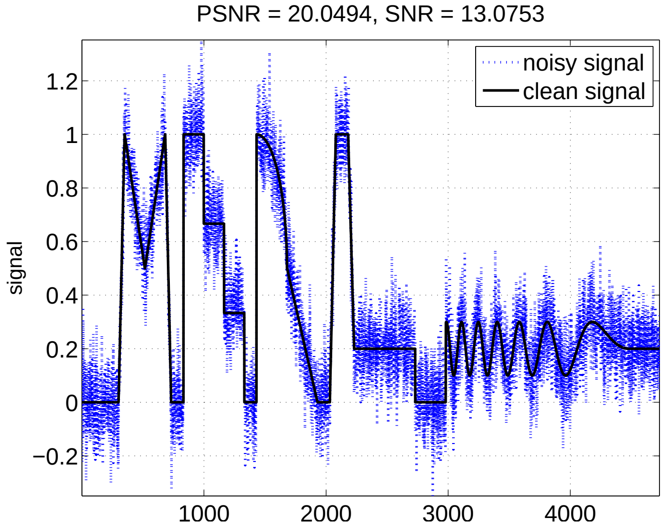
Preconditioned Conjugate Gradient acceleration of a smoothing filter

Algorithm PCG($k_{\max}$) with $l_{\max}$ restarts

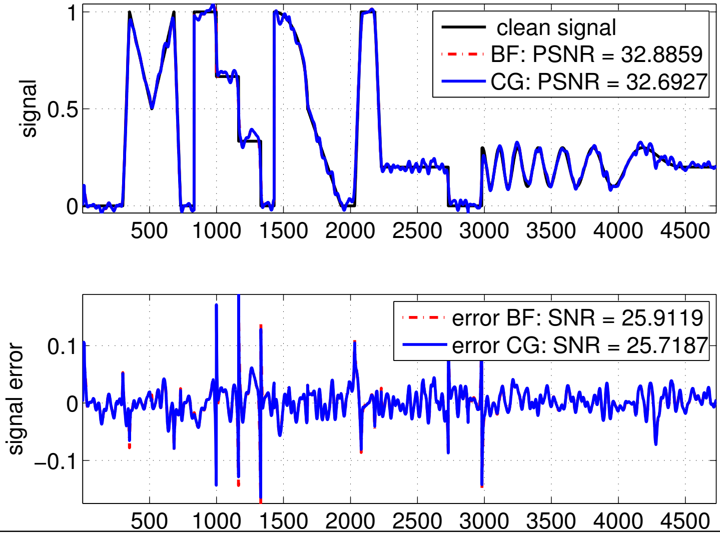
Input: x_0, $k_{\max}$, l_{max}
Output: x

$x = x_0$
for $l = 1, \ldots, l_{\max}$ **do**
  $r = W(x)x - D(x)x$
  for $k = 1, \ldots, k_{\max} - 1$ **do**
    $s = D^{-1}(x)r$; $\gamma = s^T r$
    if $k = 1$ **then** $p = s$ **else** $\beta = \gamma/\gamma_{old}$; $p = s + \beta p$ **endif**
    $q = D(x)p - W(x)p$; $\alpha = \gamma/(p^T q)$
    $x = x + \alpha p$; $r = r - \alpha q$; $\gamma_{old} = \gamma$
  endfor
endfor

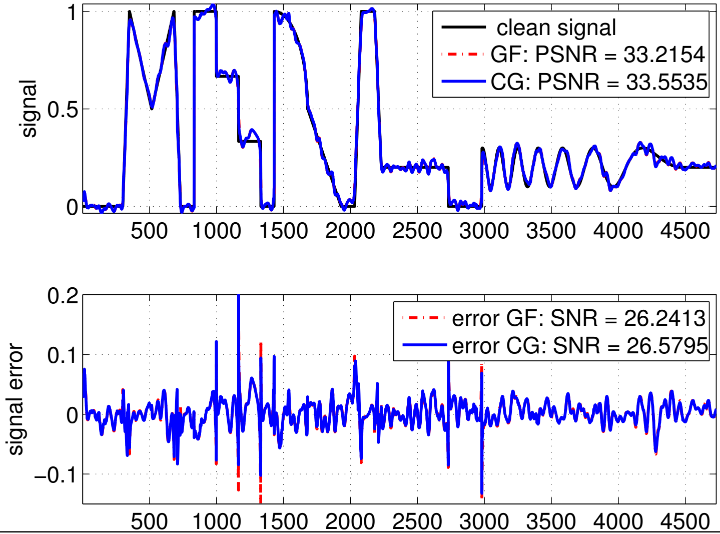
```
noisy = clean + randn(size(clean))*0.1
```



Bilateral filter: 390 iter. vs. $7 \times PCG(6)$



Guided filter: 66 iterations vs. $5 \times PCG(3)$



Krylov subspace acceleration of iterated smoothing filters

Conclusion:

- We present smoothing filters in the guided form.
- We can apply the preconditioned conjugate gradient method, with the zero right-hand side and restarts, to iterated smoothing filters in the self-guided form.
- The PCG acceleration may give dramatical speedup without quality degradation.

Numerical results in 1D:

For the test 1D signal of length 4730, the PCG acceleration provides the 9 times speedup for BF and 4 times speedup for GF.

Future work:

- Evaluation for 2D imaging.
- Total Variation Filter.

Related publications by the authors

- D. Tian, A. Knyazev, H. Mansour, and A. Vetro, Chebyshev and conjugate gradient filters for graph image denoising, in Proc. IEEE International Conf. Multimedia Expo Workshops (ICMEW), Chengdu, 2014, pp. 1–6.
- Andrew Knyazev and Alexander Malyshev. Accelerated graph-based spectral polynomial filters, in Proc. IEEE International Workshop on Machine Learning for Signal Processing, September 17-20, Boston, USA, 2015, arXiv:1509.02468.
- Andrew Knyazev and Alexander Malyshev. Conjugate Gradient Acceleration of Non-Linear Smoothing Filters, in Proc. IEEE Global Conf. on Signal and Information Processing, December 14-16, Orlando, Florida, USA, 2015, arXiv:1509.01514.