Iterative Beam Alignment Algorithms for TDD MIMO Systems

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Overview

Part 1: Introduction

- Problem Statement & Background
- System Model
- Prior work

Part 2: Proposed Beam Alignment Algorithms

- Sequential Least Squares (SLS) Power Method
- Summed Power Method
- Least-squares initialized Summed Power Method (LISP method)

Part 3: Simulation Results

- I.I.D Rayleigh fading model
- Sparse mmWave model

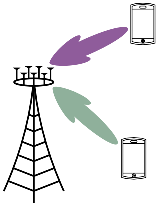
Part 4: Conclusion

- Further research
- Acknowledgments

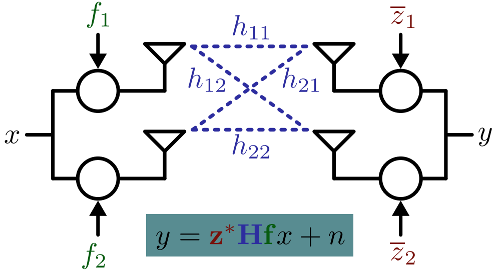
Part 1: Introduction

Introduction

- 5G technologies rely on **beamforming gains** to realize data rate requirements
  - **Millimeter-wave (mmWave)**: Compensation for increased path and penetration loss in 25-100 GHz band
  - **Massive MIMO**: Multi-user beamforming in sub-6 GHz bands



- However: **Optimal beamforming weights depend on the channel matrix**



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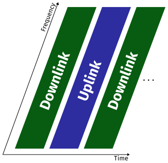
Introduction

- With sufficiently small arrays,
  a) Use sounding sequences and feedback for each antenna
  b) Directly compute optimal beamformers (i.e. singular vectors of channel matrix)
- Problem: **Sounding approach is impractical with many antennas**
- Solution: **Beam-based sounding**
  - Users always transmit on beams
  - Acquire beamformers using a TDD **beam alignment phase**



- **Need for practical approaches to TDD-based beam alignment** (i.e. with additive noise, mmWave channel models)
  - Beamsweeping (codebook-based)
  - Greedy → *ping-pong* framework

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System Model

Ping-pong beam alignment framework divides each channel use k into two time slots

Slot 1 (*ping*)

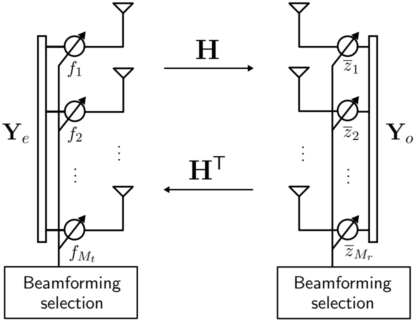
Node 1 sounds beam $\mathbf{f}[k]$ as

$$\mathbf{y}_o[k] = \sqrt{\rho_o}\,\mathbf{H}\mathbf{f}[k] + \mathbf{n}_o[k]$$

Slot 2 (*pong*)

Node 2 sounds beam $\mathbf{z}[k]$ as

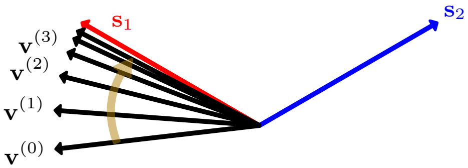
$$\mathbf{y}_e[k] = \sqrt{\rho_e}\,\mathbf{H}^{\mathsf{T}}\overline{\mathbf{z}}[k] + \mathbf{n}_e[k]$$



Notation: $\mathbf{H}$ — $M_r \times M_t$ channel matrix, ρ_e, ρ_o — beam alignment SNR, $\mathbf{n}_e[k], \mathbf{n}_o[k]$ — complex additive white Gaussian noise

Power Method

- Propose new beam alignment algorithms based on **power method**



- Works well for the noiseless case
- **Convergence can slow down dramatically under additive noise**

Power Method (one-way)

Given: Diagonalizable $\mathbf{A} \in \mathbb{C}^{n \times n}$ and unit 2-norm $\mathbf{v}^{(0)}$
for $k = 1, 2, \ldots$ **do**
  $\mathbf{v}^{(k)} = \mathbf{A}\mathbf{v}^{(k-1)} / \|\mathbf{A}\mathbf{v}^{(k-1)}\|_2$
end for

Power Method (two-way)

Given: $\mathbf{H} \in \mathbb{C}^{n \times n}$ and unit 2-norm $\mathbf{x}^{(0)}$
for $k = 1, 2, \ldots$ **do**
  $\mathbf{y}^{(k)} = \mathbf{H}\mathbf{x}^{(k-1)}$
  $\mathbf{w}^{(k)} = \mathbf{y}^{(k)} / \|\mathbf{y}^{(k)}\|_2$
  $\mathbf{z}^{(k)} = \mathbf{H}^{\mathsf{T}}\overline{\mathbf{w}}^{(k)}$
  $\mathbf{x}^{(k)} = \overline{\mathbf{z}}^{(k)} / \|\mathbf{z}^{(k)}\|_2$
end for

Prior work

- Precoding for sparse mmWave channels[1]
- Hybrid beamforming[2]
- Beamspace MIMO for mmWave systems[3]
- Alternative eigenvalue iterations (i.e. Arnoldi iteration[4])

[1] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath Jr., "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, Mar. 2014.

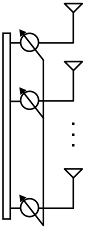
[2] F. Sohrabi and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE Journ. Sel. Topics in Sig. Proc.*, vol. 10, no. 3, pp. 501–513, Apr. 2016.

[3] J. Brady, N. Behdad, and A. M. Sayeed, "Beamspace MIMO for millimeter-wave communications: System architecture, modeling, analysis and measurements," *IEEE Trans. Ant. Propag.*, vol. 61, no. 7, pp. 3814–3827, Jul. 2013.

[4] H. Ghauch, T. Kim, M. Skoglund, and M. Bengtsson, "Subspace Estimation and Decomposition in large millimeter-wave MIMO systems," *IEEE Journ. Sel. Topics in Sig. Proc.*, vol. 10, no. 3, pp. 528–542, Apr. 2016.

Part 2: Proposed Beam Alignment Algorithms

Sequential Least Squares (SLS) Power Method



Compute least-squares channel estimate $\widehat{\mathbf{H}}_e$

↓

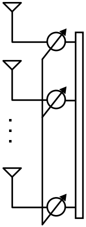
Compute beamforming weights $\mathbf{f}[k]$

→ Transmit →

Compute least-squares channel estimate $\widehat{\mathbf{H}}_o$

↓

Compute beamforming weights $\mathbf{z}[k]$



← Transmit

⋮

Main Ideas

- Construct a least-squares (LS) estimate of the channel matrix **using the sounding beams**
- Compute **greedy estimates** of the singular vectors

Beamforming weights

$$\mathbf{f}[k] = \frac{\widehat{\mathbf{H}}^*_{e,k}\mathbf{z}[k-1]}{\left\|\widehat{\mathbf{H}}^*_{e,k}\mathbf{z}[k-1]\right\|_2}$$

$$\mathbf{z}[k] = \frac{\widehat{\mathbf{H}}_{o,k}\mathbf{f}[k]}{\left\|\widehat{\mathbf{H}}_{o,k}\mathbf{f}[k]\right\|_2}$$

Sequential Least Squares (SLS) Power Method

- Batch LS estimator:

$$\widehat{\mathbf{H}}_{o,k} = \frac{\mathbf{Y}_{o,k}\left(\mathbf{F}_k\right)^{\dagger}}{\sqrt{\rho_o}}$$

- Requires full-rank observation matrix $\mathbf{Y}_{o,k}$
- Instead, construct estimates sequentially:

$$\widehat{\mathbf{H}}_{o,k} = f\left(\widehat{\mathbf{H}}_{o,k-1},\ \mathbf{y}_o[k],\ \mathbf{f}[k]\right)$$

Computational complexity

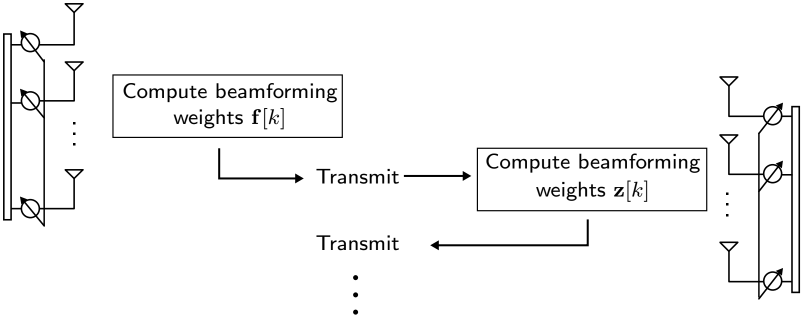
	Computational Count	**Feedback**
Sequential Least-Squares	$k_{\text{max}} \cdot \mathcal{O}(M^3)$	$k_{\text{max}} \cdot \mathcal{O}(M)$

k_{max} = Number of beam alignment ping-pong slots

$M = \max(M_t, M_r)$

- **SLS Power Method performs very well at high costs** (feedback and computational overhead)

Summed Power Method



Main Ideas

- Derive beamforming weights as a function of the **running sum of received observations**
- **Average over potentially noisy estimates** during beam alignment

Beamforming weights

$$\begin{aligned}\mathbf{z}[k+1] &= \beta_k \left[\mathbf{y}_o[k] + \mathbf{y}_o[k-1] + \cdots + \mathbf{y}_o[0]\right] \\ &= \beta_k\, \mathbf{s}_o[k] \\ \mathbf{f}[k+1] &= \alpha_k \left[\overline{\mathbf{y}}_e[k] + \overline{\mathbf{y}}_e[k-1] + \cdots + \overline{\mathbf{y}}_e[0]\right] \\ &= \alpha_k\, \mathbf{s}_e[k]\end{aligned}$$

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Summed Power Method

- Normalize using ℓ_2-norm

$$\alpha_k = \frac{1}{\|\mathbf{s}_e[k]\|_2}, \quad \beta_k = \frac{1}{\|\mathbf{s}_o[k]\|_2}$$

- Repeated conjugation and retransmission like in the simple power iteration
- Averaging observations reduces the effects of additive noise
- Little overhead
- No feedback necessary

Computational complexity

	Computational Count	**Feedback**
Sequential Least-Squares	$k_{\text{max}} \cdot \mathcal{O}(M^3)$	$k_{\text{max}} \cdot \mathcal{O}(M)$
Summed Power	$k_{\text{max}} \cdot \mathcal{O}(M)$	-

$k_{\text{max}} =$ Number of beam alignment ping-pong slots
$M = \max(M_t, M_r)$

How to combine the positive properties of both techniques?

What are the tradeoffs?

Least-squares initialized Summed Power Method (LISP method)[5]

- **Idea: "prime" the beamformer estimates up to period k_{switch} with the SLS method, then switch to the Summed Power Method**

	Computational Count	Feedback
Sequential Least-Squares	$k_{\text{max}} \cdot \mathcal{O}(M^3)$	$k_{\text{max}} \cdot \mathcal{O}(M)$
Summed Power	$k_{\text{max}} \cdot \mathcal{O}(M)$	-
LISP	$k_{\text{switch}} \cdot \mathcal{O}(M^3) + (k_{\text{max}} - k_{\text{switch}}) \cdot \mathcal{O}(M)$	$k_{\text{switch}} \cdot \mathcal{O}(M)$

[5] D. Ogbe, D. J. Love, and V. Raghavan, "Noisy Beam Alignment Techniques for Reciprocal MIMO Channels," *ArXiv:1609.03601 [cs.IT]*, Nov. 2016. [Online]. Available: http://arxiv.org/abs/1609.03601.

Part 3: Simulation Results

	Computational Count	Feedback
Sequential Least-Squares	$k_{\text{max}} \cdot \mathcal{O}(M^3)$	$k_{\text{max}} \cdot \mathcal{O}(M)$
Summed Power	$k_{\text{max}} \cdot \mathcal{O}(M)$	-
LISP	$k_{\text{switch}} \cdot \mathcal{O}(M^3) + (k_{\text{max}} - k_{\text{switch}}) \cdot \mathcal{O}(M)$	$k_{\text{switch}} \cdot \mathcal{O}(M)$
BIMA[6]	$k_{\text{max}} \cdot \mathcal{O}(M)$	-
BSM[7]	$k_{\text{max}} \cdot \mathcal{O}(M)$	-

Metrics:

- Effective channel gain $|\mathbf{z}^*[k]\mathbf{H}\mathbf{f}[k]|^2$
- Chordal distance to dom. sing. vector $\phi_k = \cos^{-1}\left(|\mathbf{f}_{\text{opt}}^*\mathbf{f}[k]|\right)$

[6] T. Dahl, N. Christophersen, and D. Gesbert, "Blind MIMO eigenmode transmission based on the algebraic power method," *IEEE Trans. Sig. Proc.*, vol. 52, no. 9, pp. 2424–2431, Sep. 2004, ISSN: 1053-587X. DOI: 10.1109/TSP.2004.832000.

[7] S. Gazor and K. AlSuhaili, "Communications over the best singular mode of a reciprocal MIMO channel," *IEEE Trans. Commun.*, vol. 58, no. 7, pp. 1993–2001, Jul. 2010, ISSN: 0090-6778. DOI: 10.1109/TCOMM.2010.07.090297.

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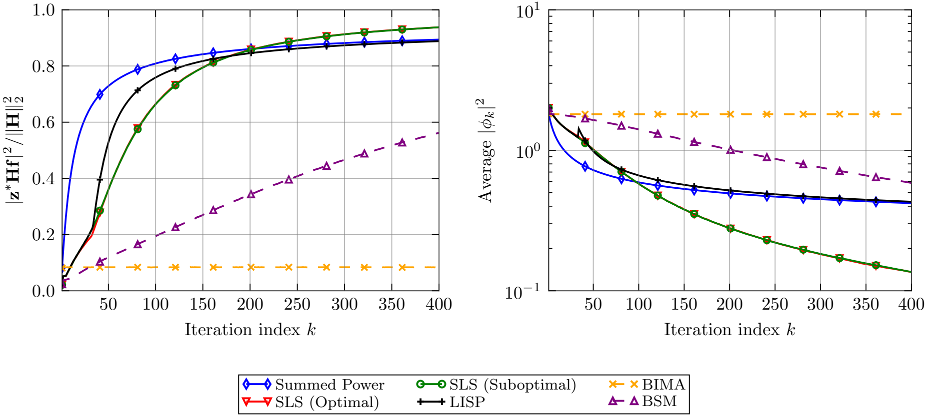
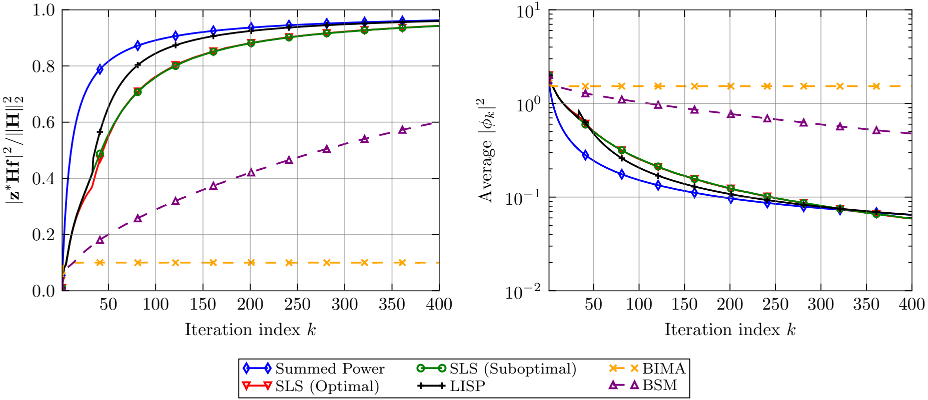
IID Rayleigh fading model



Parameters:

$\rho_e = \rho_o = -10$ dB, $M_r = 4, M_t = 32,\ k_{\text{switch}} = \max(M_r, M_t)$

Sparse mmWave model



Parameters:

$\rho_e = \rho_o = -10$ dB, $M_r = 4, M_t = 32$,

$k_{\text{switch}} = \max(M_r, M_t)$

Channel model:

$\lambda/2$-spaced ULAs, $f_c = 28$ GHz,

$K = 3$ dominant clusters, one path/cluster

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Future research

- Analytical framework for convergence analysis as function of SNR, antenna dimensions, etc.
- Impact of noisy feedback for SLS method
- Time-varying channels
- Application to hybrid beamforming systems
- Applications to machine learning, principal component analysis-type problems

Acknowledgments

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Backup

—Begin Backup Slides—

Sequential Least Squares (SLS) Power Method

Update Equations

- With the sequential algorithm, node 2 computes its estimate according to the update equation

$$\widehat{\mathbf{H}}_{o,k} = \widehat{\mathbf{H}}_{o,k-1} + \left(\frac{\mathbf{y}_o[k]}{\sqrt{\rho_o}} - \widehat{\mathbf{H}}_{o,k-1}\mathbf{f}[k] \right) \mathbf{K}_{o,k} \tag{1}$$

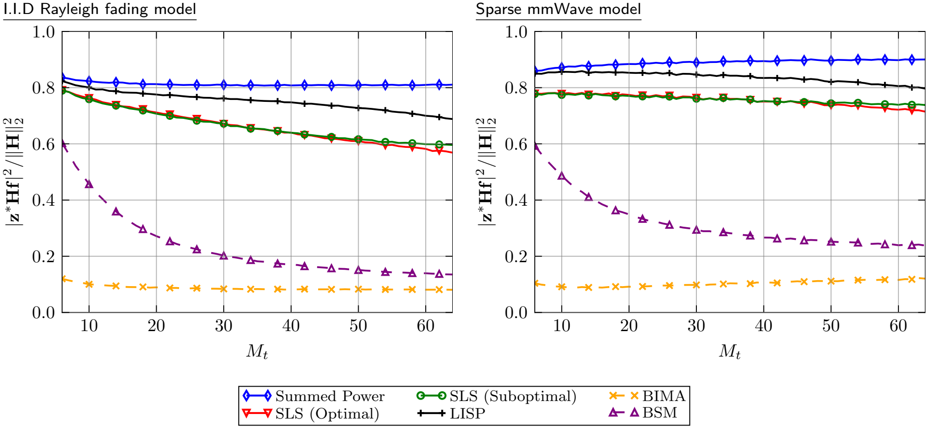
where

$$\mathbf{K}_{o,k} = \frac{\mathbf{f}^*[k]\mathbf{C}_{o,k-1}}{1 + \mathbf{f}^*[k]\mathbf{C}_{o,k-1}\mathbf{f}[k]} \tag{2}$$

and

$$\mathbf{C}_{o,k} = \mathbf{C}_{o,k-1}\left(\mathbf{I} - \mathbf{f}[k]\mathbf{K}_{o,k}\right) \tag{3}$$

Impact of Antenna Dimensions



Parameters:

$\rho_e = \rho_o = -10$ dB, $M_r = 4$, $M_t \in \{6, 8, \ldots, 64\}$, $k_{\text{switch}} = \max(M_r, M_t)$,

100 ping-pong slots

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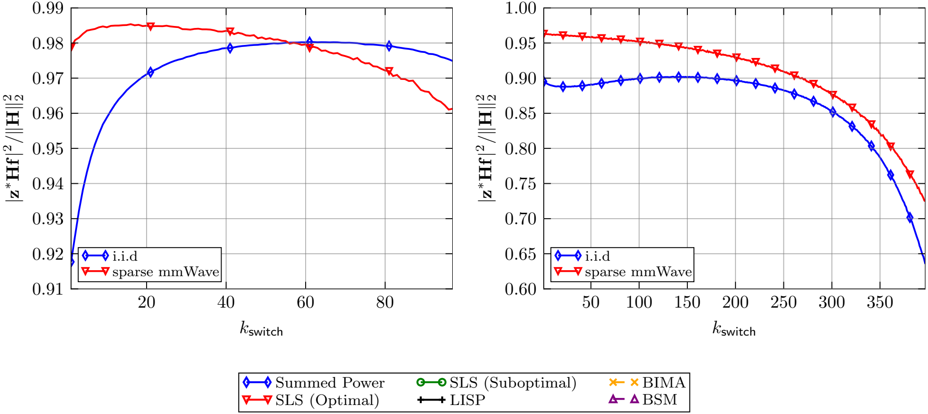
Impact of k_{switch}

$\rho_e = \rho_o = 0$ dB and $k_{\max} = 100$

$\rho_e = \rho_o = -10$ dB and $k_{\max} = 400$



Parameters:

$M_r = 4, \ M_t = 32$

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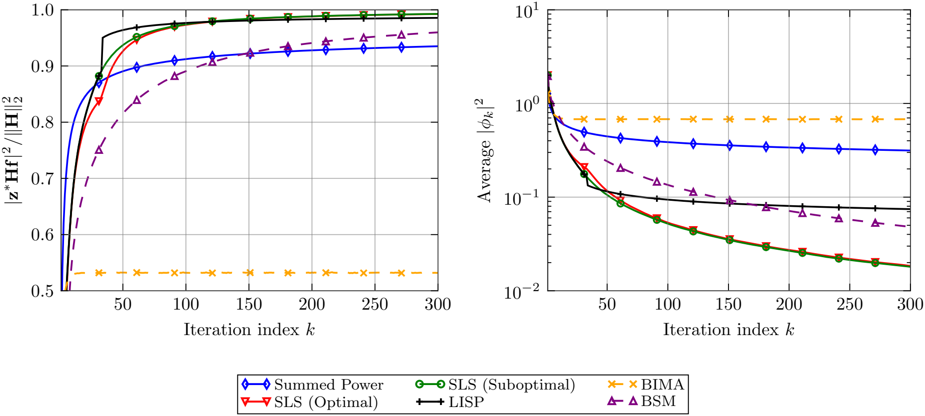
IID Rayleigh fading model



Parameters:

$\rho_e = \rho_o = 0$ dB, $M_r = 4, M_t = 32$, $k_{\text{switch}} = \max(M_r, M_t)$

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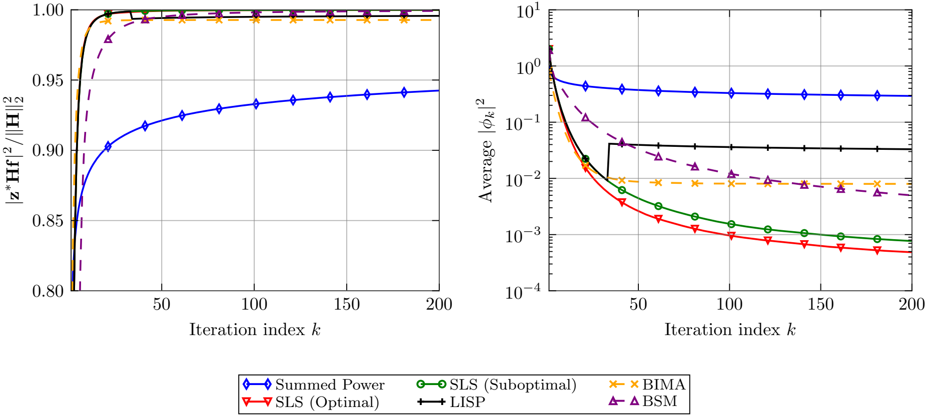
IID Rayleigh fading model

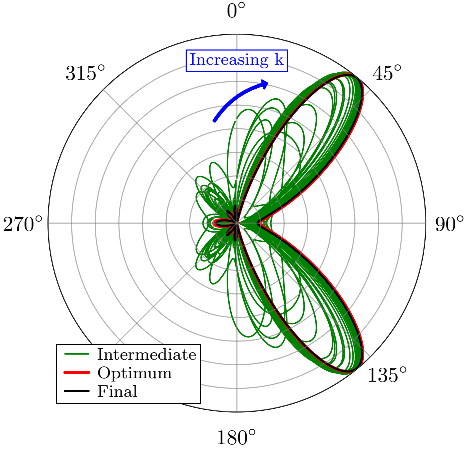


Parameters:

$\rho_e = \rho_o = 20$ dB, $M_r = 4, M_t = 32$, $k_{\text{switch}} = \max(M_r, M_t)$

Beam Pattern evolution

Beam pattern of $\mathbf{f}[k]$ vs. beam pattern of $\mathbf{f}_{\text{opt}}$



Parameters:

$\rho_e = \rho_o = -10$ dB, $M_r = 4, M_t = 32$, 200 ping-pong slots