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# Dynamic Subarray Architecture for Wideband Hybrid Precoding in Millimeter Wave MIMO Systems

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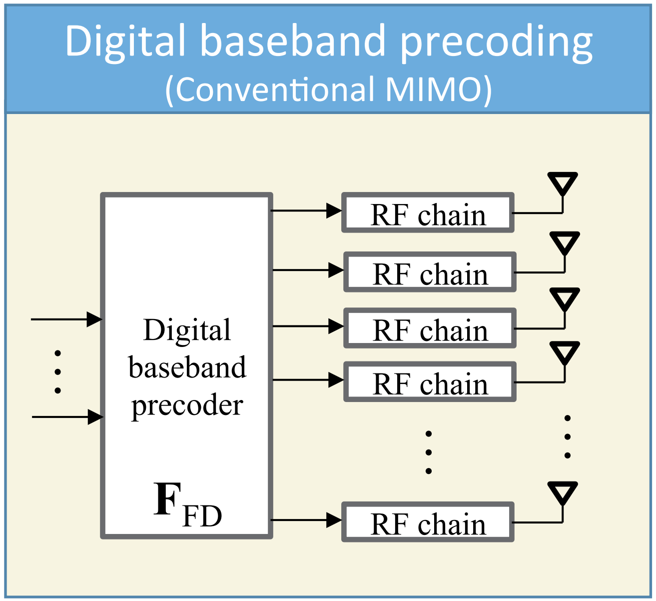
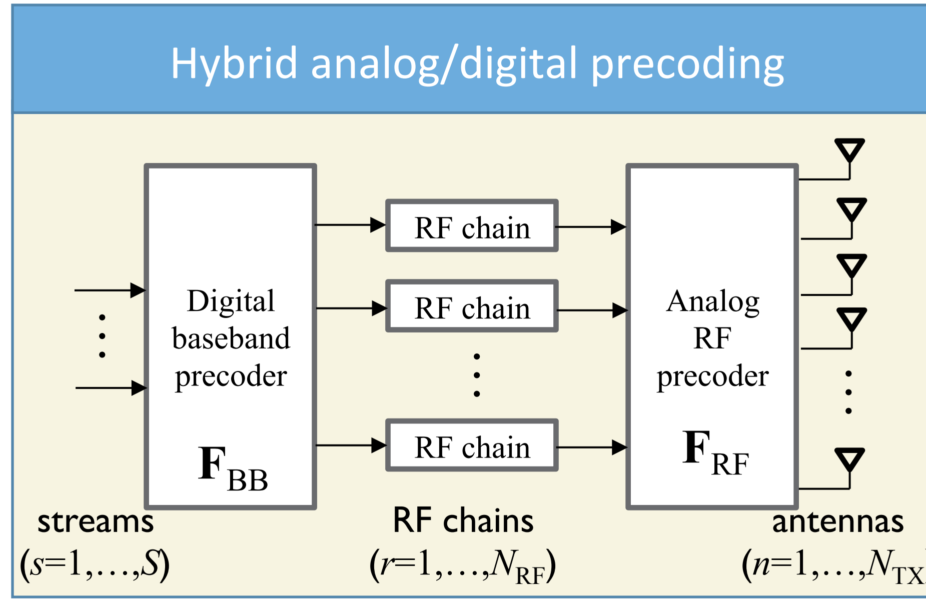
# Digital precoding vs. Hybrid precoding



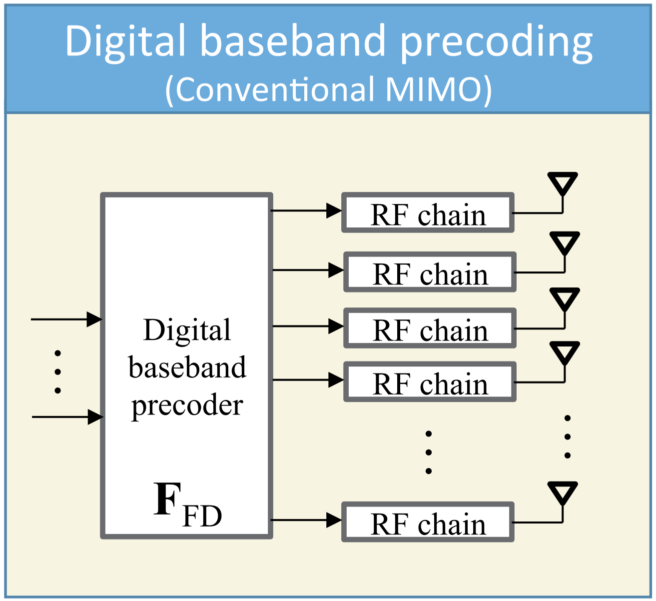



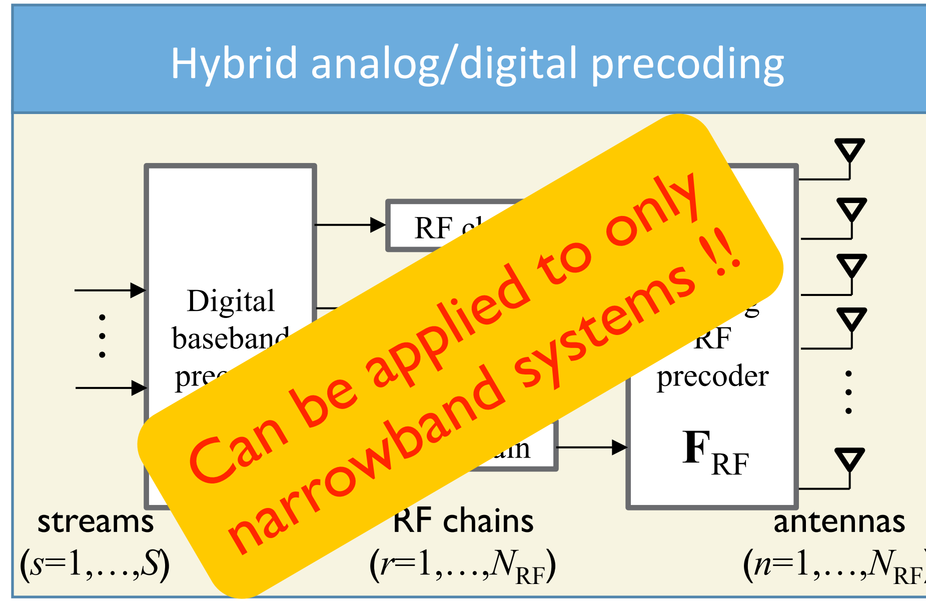

$$N_{\mathrm{RF}} < N_{\mathrm{TX}}$$

$$\mathbf{F}_{\mathrm{HB}} = \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}$$

[1] O. El Ayach et el., “Spatially sparse precoding in millimeter wave MIMO systems,” *IEEE Trans. on Wireless Comm.*, Mar. 2014.
[2] A. Alkhateeb et el, “MIMO precoding and combining solutions for millimeter-wave systems,” *IEEE Comm. Mag.*, Dec. 2014.
[3] W. Ni et el., “Hybrid block diagonalization for massive multiuser MIMO systems,” *IEEE Trans. on Comm.*, Jan 2016.

# Digital precoding vs. Hybrid precoding




$N_{RF} < N_{TX}$

$$\mathbf{F}_{HB} = \mathbf{F}_{RF}\,\mathbf{F}_{BB}$$

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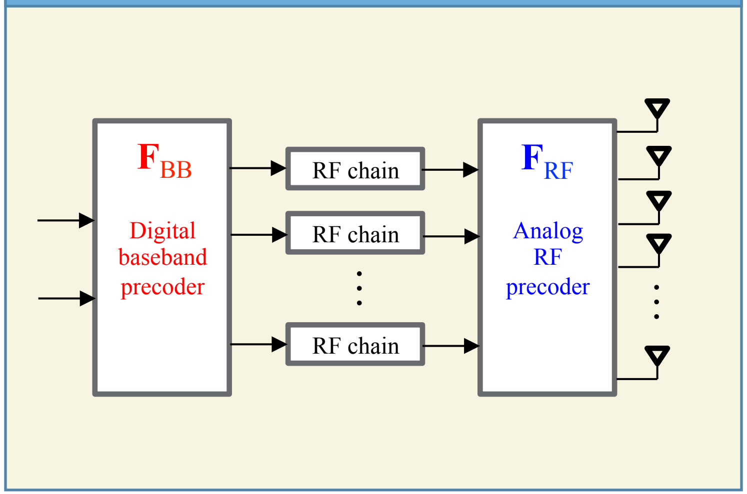
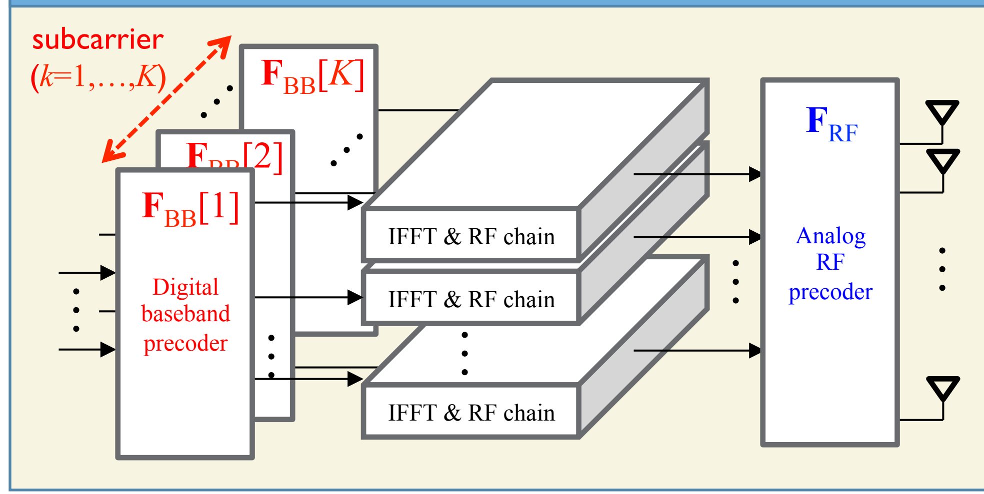
# Hybrid precoding: Narrowband vs. Wideband

## *Narrowband* hybrid precoding




$$\mathbf{F}_{\text{HB}} = \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}$$

## *Wideband* Hybrid precoding (MIMO-OFDM)

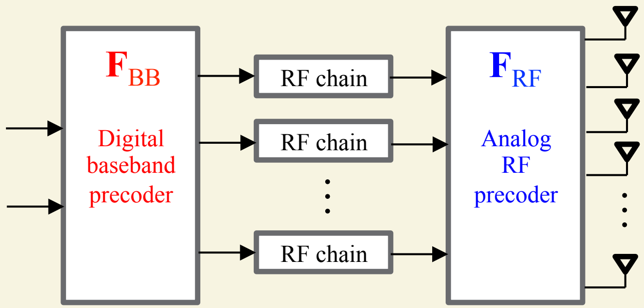



$$\mathbf{F}_{\text{HB}}[k] = \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}[k] \text{ for } k=1,\ldots,K$$

wideband & time-domain ($\mathbf{F}_{\text{RF}}$) per-subcarrier & frequency-domain ($\mathbf{F}_{\text{BB}}[k]$)

# Hybrid precoding: Narrowband vs. Wideband

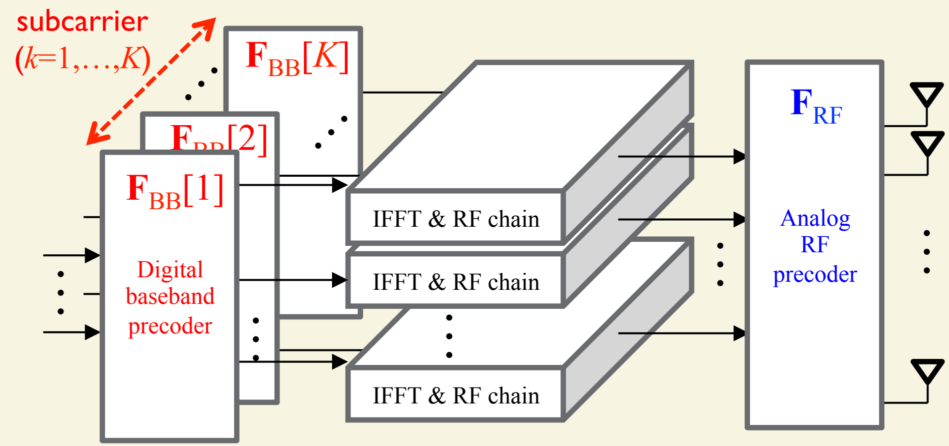
## *Narrowband* hybrid precoding




$$\mathbf{F}_{\mathrm{HB}} = \mathbf{F}_{\mathrm{RF}}\,\mathbf{F}_{\mathrm{BB}}$$

| Constraints on $\mathbf{F}_{\mathrm{RF}}$ | 1) $N_{\mathrm{RF}} < N_{\mathrm{TX}}$<br>2) Composed of phase shifters<br>$([\mathbf{F}_{\mathrm{RF}}]_{m,n} = e^{j\theta_{m,n}})$ |
|---|---|

## *Wideband* Hybrid precoding (MIMO-OFDM)




$$\mathbf{F}_{\mathrm{HB}}[k] = \mathbf{F}_{\mathrm{RF}}\,\mathbf{F}_{\mathrm{BB}}[k] \text{ for } k=1,\ldots,K$$

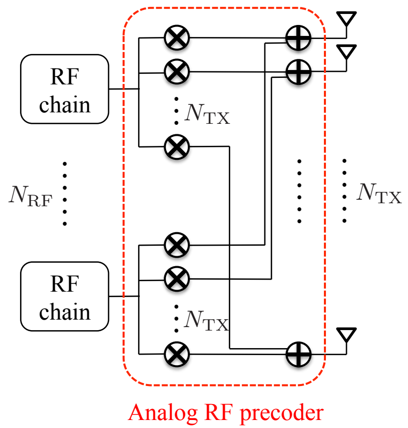
$\mathbf{F}_{\mathrm{RF}}$: wideband & time-domain

$\mathbf{F}_{\mathrm{BB}}[k]$: per-subcarrier & frequency-domain

| Constraints on $\mathbf{F}_{\mathrm{RF}}$ | 1) $N_{\mathrm{RF}} < N_{\mathrm{TX}}$<br>2) Composed of phase shifters<br>3) Common for all subcarriers |
|---|---|

# Hybrid structure – Fully-connected vs. Partially-connected

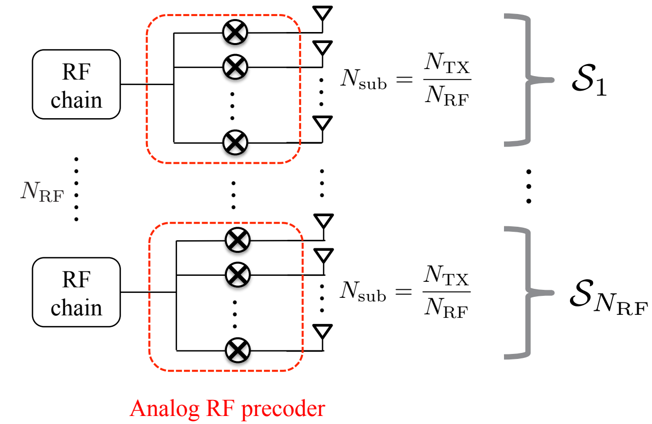
## Fully-connected structure




$$\mathbf{F}_{\mathrm{RF}} = \left[\mathbf{f}_{\mathrm{RF},1} \quad \mathbf{f}_{\mathrm{RF},2} \quad \cdots \quad \mathbf{f}_{\mathrm{RF},N_{\mathrm{RF}}}\right]$$

$N_{\mathrm{RF}}N_{\mathrm{TX}}$ phase shifters and $N_{\mathrm{TX}}$ adders are necessary.

## Partially-connected (subarray) structure




$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{f}_{\mathrm{RF},\mathcal{S}_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_{\mathrm{RF},\mathcal{S}_2} & \mathbf{0} & \vdots \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{f}_{\mathrm{RF},\mathcal{S}_{N_{\mathrm{RF}}}} \end{bmatrix}$$

$N_{\mathrm{TX}}$ phase shifters are necessary.

# WB hybrid precoding design - Problem formulation (1/2)

Goal: Maximize the sum of per-subcarrier mutual information

| Constraints on $\mathbf{F}_{\mathrm{RF}}$ | 1) $N_{\mathrm{RF}} < N_{\mathrm{TX}}$<br>2) Composed of phase shifters<br>3) Common for all subcarriers |
| --- | --- |

$$\arg \max_{\{\mathbf{F}_{\mathrm{RF}}, \mathbf{F}_{\mathrm{BB}}(1), \cdots, \mathbf{F}_{\mathrm{BB}}(K)\}} \sum_{k=1}^{K} \log\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{H}(k)\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}(k)\mathbf{F}_{\mathrm{BB}}^*(k)\mathbf{F}_{\mathrm{RF}}^*\mathbf{H}^*(k)\right)$$

$$\text{s.t.} \sum_{k=1}^{K} ||\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}(k)||_{\mathrm{F}}^2 \le P_{\mathrm{tot}} \quad \text{(Eq. 1)}$$

Using

$$\mathbf{H}_{\mathrm{eff}}(k) = \mathbf{H}(k)\mathbf{F}_{\mathrm{RF}}(\mathbf{F}_{\mathrm{RF}}^*\mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}}$$

$$\hat{\mathbf{F}}_{\mathrm{BB}}(k) = (\mathbf{F}_{\mathrm{RF}}^*\mathbf{F}_{\mathrm{RF}})^{\frac{1}{2}}\,\mathbf{F}_{\mathrm{BB}}(k)$$

⇓ equivalent

$$\arg \max_{\{\mathbf{F}_{\mathrm{RF}}, \hat{\mathbf{F}}_{\mathrm{BB}}(1), \cdots, \hat{\mathbf{F}}_{\mathrm{BB}}(K)\}} \sum_{k=1}^{K} \log\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{H}_{\mathrm{eff}}(k)\hat{\mathbf{F}}_{\mathrm{BB}}(k)\hat{\mathbf{F}}_{\mathrm{BB}}^*(k)\mathbf{H}_{\mathrm{eff}}^*(k)\right)$$

$$\text{s.t.} \sum_{k=1}^{K} ||\hat{\mathbf{F}}_{\mathrm{BB}}(k)||_{\mathrm{F}}^2 \le P_{\mathrm{tot}} \quad \text{(Eq. 2)}$$

If $\mathbf{F}_{\mathrm{RF}}$ is given, the optimum solution of $\left\{\hat{\mathbf{F}}_{\mathrm{BB}}(1), \cdots, \hat{\mathbf{F}}_{\mathrm{BB}}(K)\right\}$ can be found by using a conventional SVD scheme with respect to the effective channel at each subcarrier.

# WB hybrid precoding design - Problem formulation (2/2)

Once the optimum $\mathbf{F}_{\mathrm{BB}}(k)$ for each $k$ is determined given $\mathbf{F}_{\mathrm{RF}}$, the optimization problem is only with respect to $\mathbf{F}_{\mathrm{RF}}$.

equivalent

$$\arg\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K}\sum_{s=1}^{S} \log\left(1+\frac{\lambda_s^2\left(\mathbf{H}\left(k\right)\mathbf{F}_{\mathrm{RF}}\left(\mathbf{F}_{\mathrm{RF}}^{*}\mathbf{F}_{\mathrm{RF}}\right)^{-\frac{1}{2}}\right)p_{s,k}}{N_0}\right) \quad \text{(Eq. 3)}$$

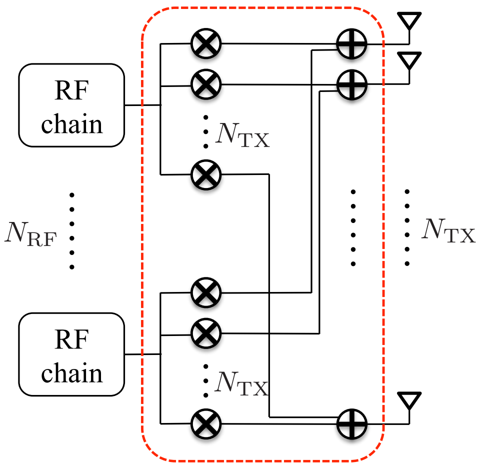
$p_{s,k}$: water-filling power control

The above problem is a non-convex optimization problem.
Instead, let us maximize the sum of squared singular values of effective channels ( = maximize the sum of effective SNR per subcarrier & stream)

approximated (relaxation)

$$\arg\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K}\sum_{s=1}^{S} \lambda_s^2\left(\mathbf{H}\left(k\right)\mathbf{F}_{\mathrm{RF}}\left(\mathbf{F}_{\mathrm{RF}}^{*}\mathbf{F}_{\mathrm{RF}}\right)^{-\frac{1}{2}}\right) \quad \text{(Eq. 4)}$$

# Hybrid precoding solution – Fully-connected case




Analog RF precoder

**Optimization problem**

$$\mathbf{F}_{\mathrm{RF}}^{\star} = \arg\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K}\sum_{s=1}^{S} \lambda_s^2\left(\mathbf{H}[k]\mathbf{F}_{\mathrm{RF}}(\mathbf{F}_{\mathrm{RF}}^{*}\mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}}\right)$$

**Solution**

$$\mathbf{F}_{\mathrm{RF}}^{\star} = [\mathbf{V}_{\mathrm{R}}]_{1:N_{\mathrm{RF}}}\,\mathbf{A}$$

where $\mathbf{A}$ is an arbitrary invertible matrix and

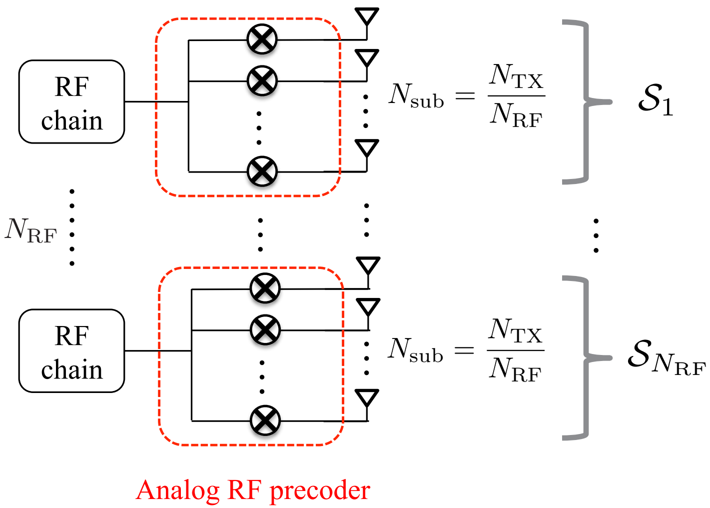
$$\mathbf{R} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{H}^{*}[k]\mathbf{H}[k] = \mathbf{V}_{\mathrm{R}}\boldsymbol{\Lambda}_{\mathrm{R}}\mathbf{V}_{\mathrm{R}}^{*}$$

**Solution w/ phase shifter constraint**

$$\hat{\mathbf{F}}_{\mathrm{RF}}^{\star} = \arg\min_{\mathbf{X},|[\mathbf{X}]_{m,n}|=1} \|\mathbf{X}-\mathbf{F}_{\mathrm{RF}}^{\star}\|_{\mathrm{F}}^{2} = \measuredangle\mathbf{F}_{\mathrm{RF}}^{\star}$$

where $\measuredangle\mathbf{F}_{\mathrm{RF}}^{\star}$ is a matrix with $[\hat{\mathbf{F}}_{\mathrm{RF}}^{\star}]_{m,n} = e^{j\measuredangle([\mathbf{F}_{\mathrm{RF}}^{\star}]_{m,n})}$

# Hybrid precoding solution – Partially-connected case




Subarray partitioning

$$\begin{aligned}
\mathcal{S}_1 &= \{1, \cdots, N_{\mathrm{sub}}\} \\
\mathcal{S}_2 &= \{N_{\mathrm{sub}}+1, \cdots, 2N_{\mathrm{sub}}\} \\
&\vdots \\
\mathcal{S}_{N_{\mathrm{RF}}} &= \{(N_{\mathrm{RF}}-1)N_{\mathrm{sub}}+1, \cdots, N_{\mathrm{RF}}N_{\mathrm{sub}}\}
\end{aligned}$$

**Optimization problem**

$$\mathbf{F}_{\mathrm{RF}}^{\star} = \arg\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K}\sum_{s=1}^{S} \lambda_s^2\left(\mathbf{H}[k]\mathbf{F}_{\mathrm{RF}}(\mathbf{F}_{\mathrm{RF}}^{*}\mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}}\right)$$

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{f}_{\mathrm{RF},\mathcal{S}_1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{f}_{\mathrm{RF},\mathcal{S}_{N_{\mathrm{RF}}}} \end{bmatrix}$$

**Solution**

$$\mathbf{f}_{\mathrm{RF},\mathcal{S}_r}^{\star} = \alpha_r \mathbf{v}_{\mathbf{R}_{\mathcal{S}_r},1}, \quad \text{for} \quad r = 1, \cdots, N_{\mathrm{RF}}$$

where $\alpha_r$ is an arbitrary nonzero complex number, $\mathbf{v}_{\mathbf{R}_{\mathcal{S}_r},1}$ is the dominant eigenvector of $\mathbf{R}_{\mathcal{S}_r}$, and

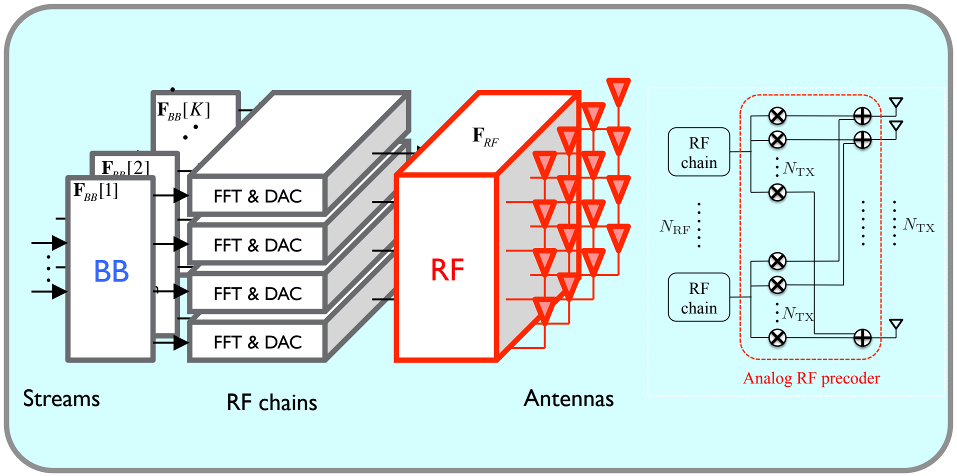
$$\mathbf{H}[k] = \left[\mathbf{H}_{\mathcal{S}_1}[k] \quad \mathbf{H}_{\mathcal{S}_2}[k] \quad \cdots \quad \mathbf{H}_{\mathcal{S}_{N_{\mathrm{RF}}}}[k]\right]$$

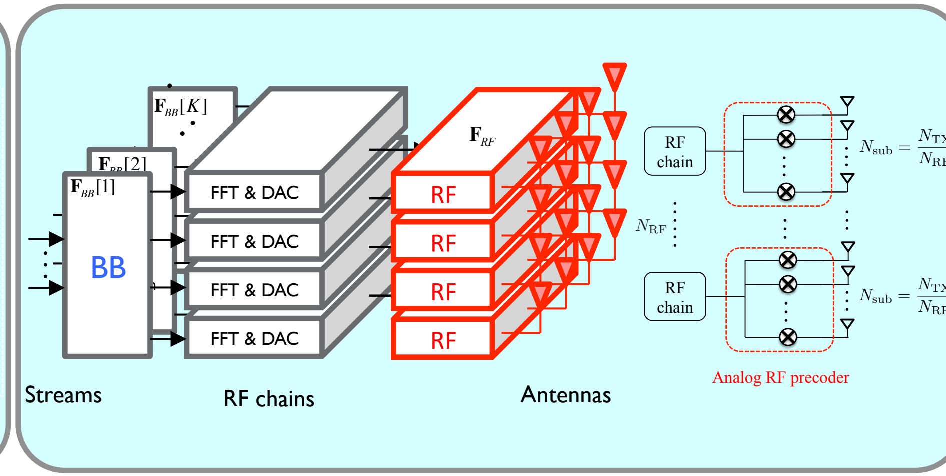
$$\mathbf{R}_{\mathcal{S}_r} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{H}_{\mathcal{S}_r}^{*}[k]\mathbf{H}_{\mathcal{S}_r}[k], \quad \text{for} \quad r = 1, \cdots, N_{\mathrm{RF}}$$

Solution w/ phase shifter constraint → the same as the fully-connected case

# Comparison between fully-connected vs. partially-connected

| Fully-connected structure | Partially-connected (subarray) structure |
| --- | --- |



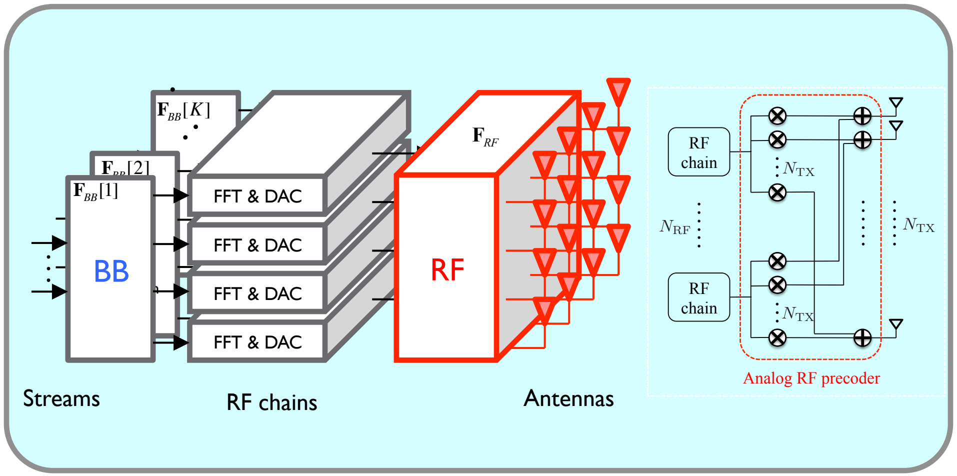

$$\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K} \sum_{s=1}^{S} \lambda_s^2 \left( \mathbf{H}[k]\, \mathbf{F}_{\mathrm{RF}} (\mathbf{F}_{\mathrm{RF}}^* \mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}} \right) = K \sum_{r=1}^{N_{\mathrm{RF}}} \lambda_r (\mathbf{R})$$

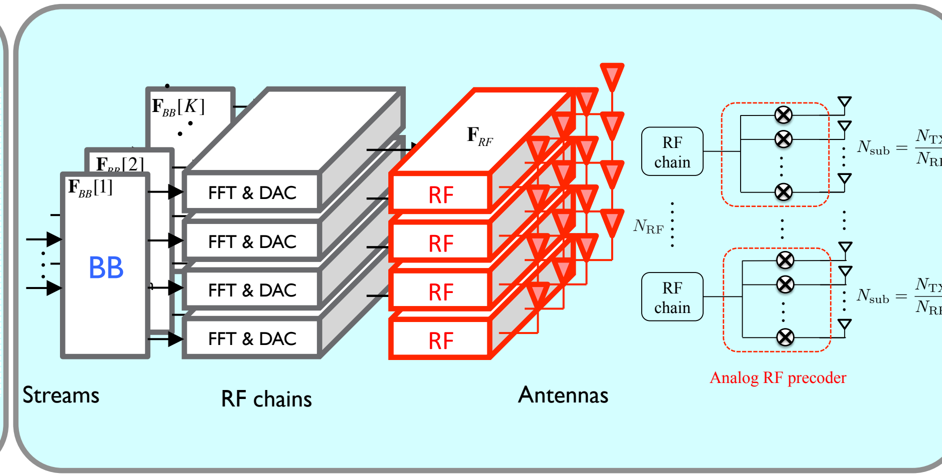
$$\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K} \sum_{s=1}^{S} \lambda_s^2 \left( \mathbf{H}[k]\, \mathbf{F}_{\mathrm{RF}} (\mathbf{F}_{\mathrm{RF}}^* \mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}} \right) = K \sum_{r=1}^{N_{\mathrm{RF}}} \lambda_1 (\mathbf{R}_{\mathcal{S}_r})$$

$\lambda_i(\mathbf{A})$ : $i$-th largest singular value of $\mathbf{A}$

# Comparison between fully-connected vs. partially-connected

| Fully-connected structure | Partially-connected (subarray) structure |
| --- | --- |




$$\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K} \sum_{s=1}^{S} \lambda_s^2 \left( \mathbf{H}[k] \mathbf{F}_{\mathrm{RF}} (\mathbf{F}_{\mathrm{RF}}^* \mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}} \right) = K \sum_{r=1}^{N_{\mathrm{RF}}} \lambda_r(\mathbf{R})$$

$$\max_{\mathbf{F}_{\mathrm{RF}}} \sum_{k=1}^{K} \sum_{s=1}^{S} \lambda_s^2 \left( \mathbf{H}[k] \mathbf{F}_{\mathrm{RF}} (\mathbf{F}_{\mathrm{RF}}^* \mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}} \right) = K \sum_{r=1}^{N_{\mathrm{RF}}} \lambda_1(\mathbf{R}_{\mathcal{S}_r})$$

$\lambda_s(\mathbf{A})$ : $s$-th largest singular value of $\mathbf{A}$

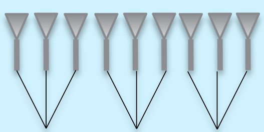
Depends on $\mathbf{R} \left( = \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}^*[k] \mathbf{H}[k] \right)$

Depends on $\mathbf{R}$ *and partitioning* $(\mathcal{S}_1, \ldots, \mathcal{S}_{N_{\mathrm{RF}}})$

# Which is the best sub-array structure (best partitioning) ?

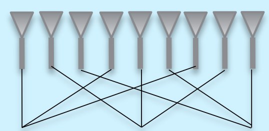
## Uniform Linear Array (ULA) case

### Adjacent type




$\mathcal{S}_1 = \{1,2,3\}$
$\mathcal{S}_2 = \{4,5,6\}$
$\mathcal{S}_3 = \{7,8,9\}$

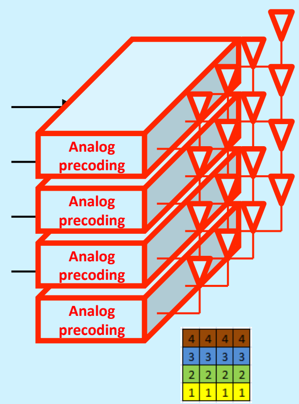
### Interlaced type




$\mathcal{S}_1 = \{1,4,7\}$
$\mathcal{S}_2 = \{2,5,8\}$
$\mathcal{S}_3 = \{3,6,9\}$

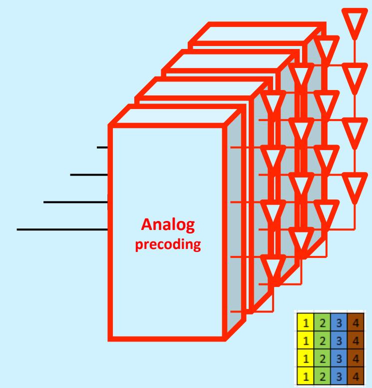
## Uniform Planar Array (UPA) case

### Horizontal type



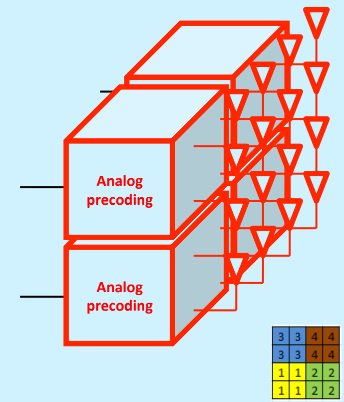

$\mathcal{S}_1 = \{1,2,3,4\}$
$\mathcal{S}_2 = \{5,6,7,8\}$
$\mathcal{S}_3 = \{9,10,11,12\}$
$\mathcal{S}_4 = \{13,14,15,16\}$

### Vertical type




$\mathcal{S}_1 = \{1,5,9,13\}$
$\mathcal{S}_2 = \{2,6,10,14\}$
$\mathcal{S}_3 = \{3,7,11,15\}$
$\mathcal{S}_4 = \{4,8,12,16\}$

### Squared type




$\mathcal{S}_1 = \{1,2,5,6\}$
$\mathcal{S}_2 = \{3,4,7,8\}$
$\mathcal{S}_3 = \{9,10,13,14\}$
$\mathcal{S}_4 = \{11,12,15,16\}$

The best subarray structure depends on the **R** matrix.
➔ We propose a ***dynamic subarray structure*** that adapts to the **R** matrix.

# Dynamic subarray - Problem formulation

**What to do**

Partition a set of $N_{\mathrm{TX}}$ antennas into $N_{\mathrm{RF}}$ subsets
- *cardinality of each subset : variable* (non-empty)

**Goal (optimum criterion)**

$$\{\mathcal{S}_r^\star\}_{r=1}^{N_{\mathrm{RF}}} = \arg \max_{\mathcal{S}_1,\ldots,\mathcal{S}_{N_{\mathrm{RF}}}} \sum_{r=1}^{N_{\mathrm{RF}}} \lambda_1\left(\mathbf{R}_{\mathcal{S}_r}\right)$$

$$\text{s.t.} \bigcup_{r=1}^{N_{\mathrm{RF}}} \mathcal{S}_r = \{1,\cdots,N_{\mathrm{TX}}\},\ \mathcal{S}_i \cap \mathcal{S}_j = \emptyset \text{ for } i \neq j,\ \ |\mathcal{S}_r| > 0\ \ \forall r$$

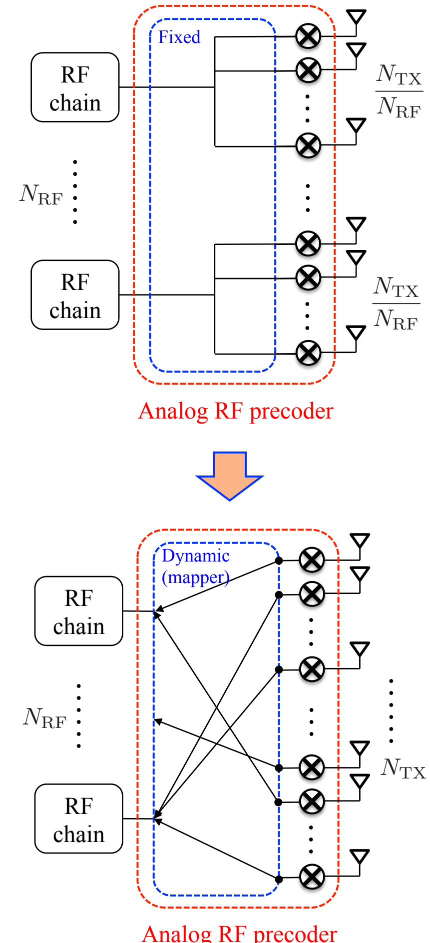
**Optimal solution**

* Optimum solution: Exhaustive search
- # of total cases: Stirling number of the second kind

$$\frac{1}{(N_{\mathrm{RF}})!}\sum_{k=0}^{N_{\mathrm{RF}}}(-1)^{N_{\mathrm{RF}}-k}\binom{N_{\mathrm{RF}}}{k}k^{N_{\mathrm{TX}}}$$

Ex) $\{N_{\mathrm{TX}}=16, N_{\mathrm{RF}}=4\} \rightarrow 1.7\times10^{8}$

1) Needs to check too many cases
2) Needs to calculate the singular value at every case



# Approximation of the largest singular value

**Definition of the approximate largest singular value**

$$\hat{\lambda}_1(\mathbf{R}_{\mathcal{S}}) \triangleq \frac{1}{|\mathcal{S}|}\sum_{i=1}^{|\mathcal{S}|}\sum_{j=1}^{|\mathcal{S}|}|[\mathbf{R}_{\mathcal{S}}]_{i,j}| = \frac{1}{|\mathcal{S}|}\sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{S}}|[\mathbf{R}]_{i,j}|$$

**Property of the approximate largest singular value**

$$\lambda_{1,\mathrm{LB}}(\mathbf{R}_{\mathcal{S}}) \le \hat{\lambda}_1(\mathbf{R}_{\mathcal{S}}) \le \lambda_{1,\mathrm{UB}}(\mathbf{R}_{\mathcal{S}})$$

* Lower & upper bound on the largest singular value (exact value)

$$\lambda_{1,\mathrm{LB}}(\mathbf{R}_{\mathcal{S}}) \le \lambda_1(\mathbf{R}_{\mathcal{S}}) \le \lambda_{1,\mathrm{UB}}(\mathbf{R}_{\mathcal{S}})$$

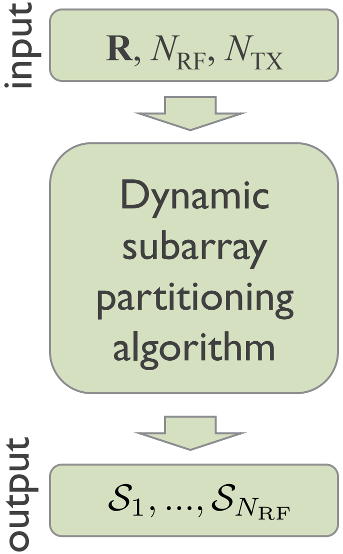
$$\lambda_{1,\mathrm{LB}}(\mathbf{R}_{\mathcal{S}}) = m + \frac{s}{(|\mathcal{S}|-1)^{\frac{1}{2}}}$$

$$\lambda_{1,\mathrm{UB}}(\mathbf{R}_{\mathcal{S}}) = m + s(|\mathcal{S}|-1)^{\frac{1}{2}}$$

$$m = \frac{\mathrm{Tr}(\mathbf{R}_{\mathcal{S}})}{|\mathcal{S}|}, \quad s = \left(\frac{\mathrm{Tr}(\mathbf{R}_{\mathcal{S}}^2)}{|\mathcal{S}|} - m^2\right)^{\frac{1}{2}}$$

[4] H. Wolkowicz and G. P. Styan, "Bounds for eigenvalues using traces," *Linear Algebra and Its Applications*, pp. 471-506, 1980

# Dynamic subarray partitioning algorithm (proposed)




**Algorithm 1** Dynamic subarray partitioning

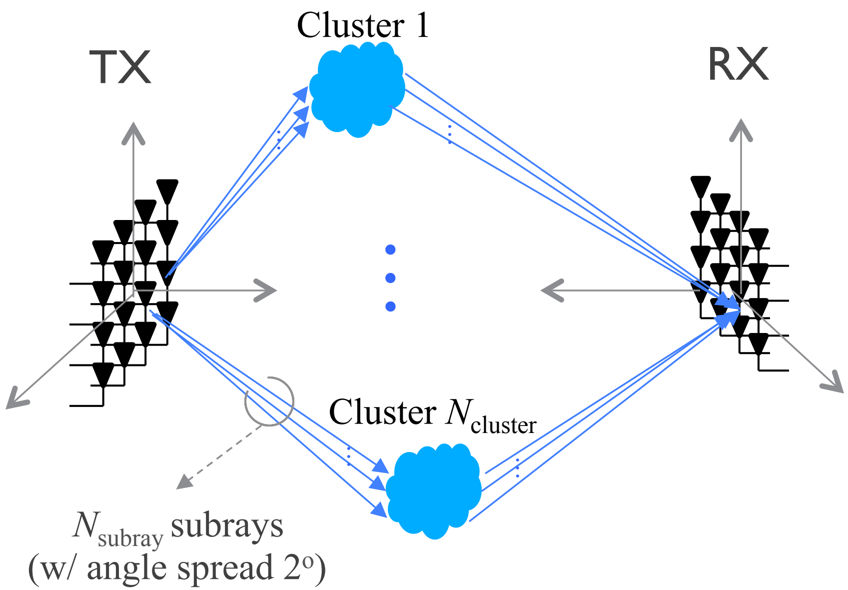
Input: $\mathbf{R}, N_{\mathrm{RF}}, N_{\mathrm{TX}}$
$\mathcal{S}_0 = \{1, \ldots, N_{\mathrm{RF}}\}, n = 0$
Sort $|[\mathbf{R}]_{i,j}|$ for $1 \leq i < j \leq N_{\mathrm{TX}}$ in descending order $\left(|[\mathbf{R}]_{i_1,j_1}| \geq \cdots \geq |[\mathbf{R}]_{i_K,j_K}|, \quad K = \frac{N_{\mathrm{TX}}(N_{\mathrm{TX}}-1)}{2}, \right)$ → Complexity: $O(N_{TX}^2 \log N_{TX})$
**for** $k = 1 : K$ **do**
  **if** $i_k, j_k \in \mathcal{S}_0$ **then**
    **if** $n < N_{\mathrm{RF}}$ **then**
      $n \leftarrow n + 1, \mathcal{S}_n \leftarrow \{i_k, j_k\}, \mathcal{S}_0 \leftarrow \mathcal{S}_0 \setminus \{i_k, j_k\}$
    **else**
      $\hat{r} = \underset{r \in \{1,\ldots,N_{\mathrm{RF}}\}}{\arg\max} \left(f\left(\mathcal{S}_r \cup \{i_k, j_k\}, n, r\right) - f\left(\mathcal{S}_r, n, r\right)\right)$ → Complexity: $O(N_{TX}^2 N_{RF})$
      $\mathcal{S}_{\hat{r}} \leftarrow \mathcal{S}_{\hat{r}} \cup \{i_k, j_k\}, \mathcal{S}_0 \leftarrow \mathcal{S}_0 \setminus \{i_k, j_k\}$
    **end if**
  **else if** $i_k \in \mathcal{S}_m, \quad j_k \in \mathcal{S}_l$ for $m \leq n, l \leq n, m \neq l$ **then** → Complexity: $O(N_{TX}^2 N_{RF})$
    $\nu = f\left(\mathcal{S}_m, n, m\right) + f\left(\mathcal{S}_l, n, l\right)$
    $\mu_j = f\left(\mathcal{S}_m \cup \{j_k\}, n, m\right) + f\left(\mathcal{S}_l \setminus \{j_k\}, n, l\right)$
    $\mu_i = f\left(\mathcal{S}_m \setminus \{i_k\}, n, m\right) + f\left(\mathcal{S}_l \cup \{i_k\}, n, l\right)$
    **if** $\mu_j > \mu_i, \mu_j > \nu$, and $m \neq 0$ **then**
      $\mathcal{S}_m \leftarrow \mathcal{S}_m \cup \{j_k\}, \mathcal{S}_l \leftarrow \mathcal{S}_l \setminus \{j_k\}$
    **else if** $\mu_i > \mu_j, \mu_i > \nu$, and $l \neq 0$ **then**
      $\mathcal{S}_m \leftarrow \mathcal{S}_m \setminus \{i_k\}, \mathcal{S}_l \leftarrow \mathcal{S}_l \cup \{i_k\}$
    **end if**
  **end if**
**end for**
Output: $\mathcal{S}_1, \cdots, \mathcal{S}_{N_{\mathrm{RF}}}$

Total complexity: $O(\,N_{TX}^2 \max(N_{RF}, \log N_{TX})\,)$

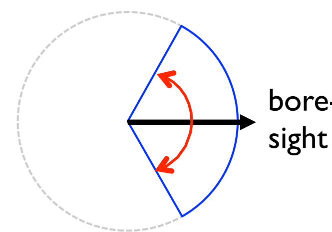
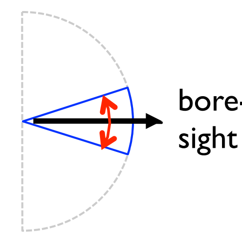
➔ Polynomial time !

$$f\left(\mathcal{S}, n, r\right) \triangleq \begin{cases} 0, & \text{if } |\mathcal{S}| = 0 \text{ or } \{n = N_{\mathrm{RF}} \text{ and } r = 0\} \\ \frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} |[\mathbf{R}]_{i,j}|, & \text{otherwise.} \end{cases}$$

# Channel model for simulations




* Distribution of AoAs and AoDs : Uniform in a confined range

| Azimuth domain | Elevation domain |
| --- | --- |
|   range of cluster angles $[-\Delta_{AZ}, \Delta_{AZ}]$ |   range of cluster angles $[-\Delta_{EL}, \Delta_{EL}]$ |

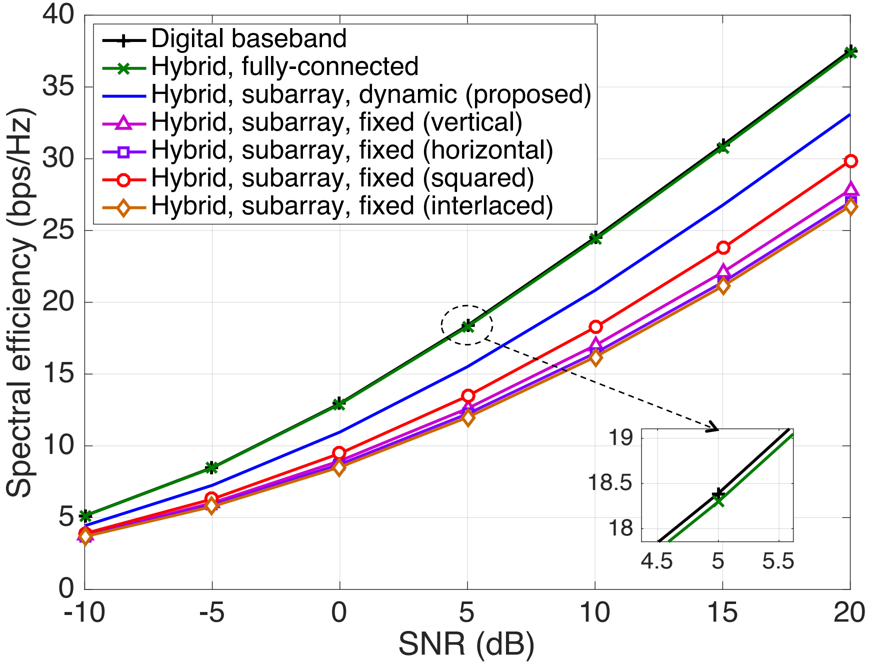
$$\mathbf{H}[k] = \sum_{c=1}^{N_{\text{cluster}}} \sum_{r=1}^{N_{\text{subray}}} \alpha_{c,r}\omega_{\tau_{c,r}}[k]\mathbf{a}_{\mathrm{R}}(\phi_{\mathrm{R},c,r}, \theta_{\mathrm{R},c,r})\mathbf{a}^*_{\mathrm{T}}(\phi_{\mathrm{T},c,r}, \theta_{\mathrm{T},c,r}) \qquad \omega_{\tau_p}[k] = \sum_{d=0}^{D-1} p(dT_{\mathrm{s}} - \tau_p)e^{-\frac{j2\pi kd}{K}}$$

$$\mathbf{a}(\phi_p, \theta_p) = \left[\begin{matrix} 1 & e^{j\frac{2\pi d}{\lambda}\sin(\phi_p)} & \dots & e^{j\frac{2\pi d}{\lambda}(N_{\mathrm{ANT}}-1)\sin(\phi_p)} \end{matrix}\right]^T$$ for ULA (uniform linear array)

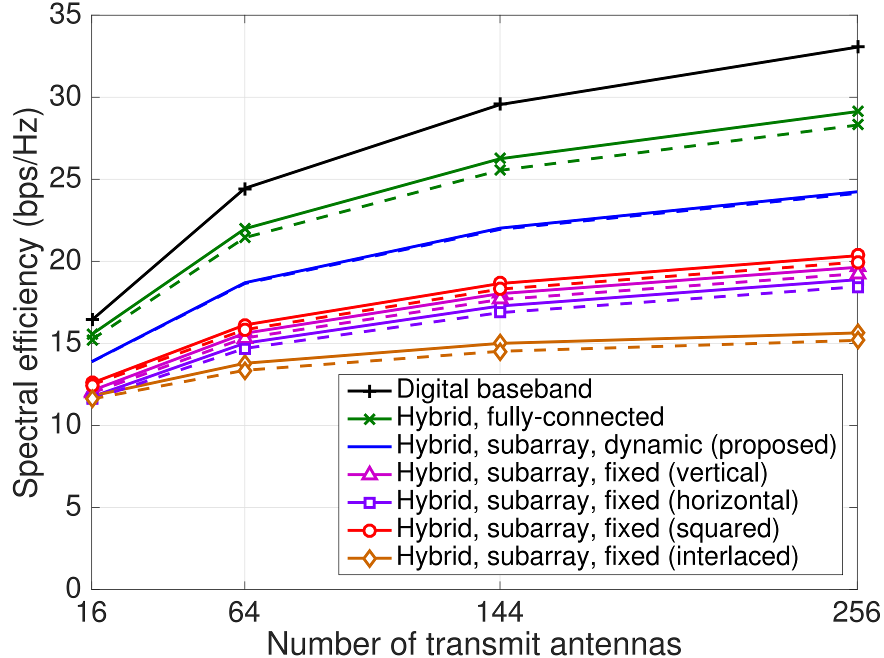
$$\mathbf{a}(\phi_p, \theta_p) = \left[\begin{matrix} 1 & \dots & e^{j\frac{2\pi d}{\lambda}(m\sin(\phi_p)\sin(\theta_p)+n\cos(\theta_p))} & \dots & e^{j\frac{2\pi d}{\lambda}((N_{\mathrm{W}}-1)\sin(\phi_p)\sin(\theta_p)+(N_{\mathrm{H}}-1)\cos(\theta_p))} \end{matrix}\right]^T$$ for UPA (uniform planar array)

# Simulation results

- TX: 64 antennas (UPA), 8 RF chains
- RX: 4 antennas (UPA), 4 RF chains
- CH: 8 clusters 10 subrays ($\Delta_{AZ}$=180°, $\Delta_{EL}$=90°)




- TX: $N_{\mathrm{TX}}$ antennas (UPA), 4 RF chains
- RX: 4 antennas (UPA), 4 RF chains
- CH: 8 clusters 10 subrays ($\Delta_{AZ}$=180°, $\Delta_{EL}$=90°), SNR 10 dB




- solid: w/o phase shifter constraint ( $\mathbf{F}_{\mathrm{RF}}^{\star}$ )
- dashed: w/ phase shifter constraint ( $\measuredangle\mathbf{F}_{\mathrm{RF}}^{\star}$ )

# Conclusions

We derived closed-form solutions for wideband hybrid precoding.

Fully connected structure

Partially-connected structure
(Subarray structure)

We proposed a dynamic subarray structure
based on spatial channel covariance.

# Thank you !