Detection Diversity of Spatio-Temporal Data using Pitman's Efficiency for low SNR Regimes

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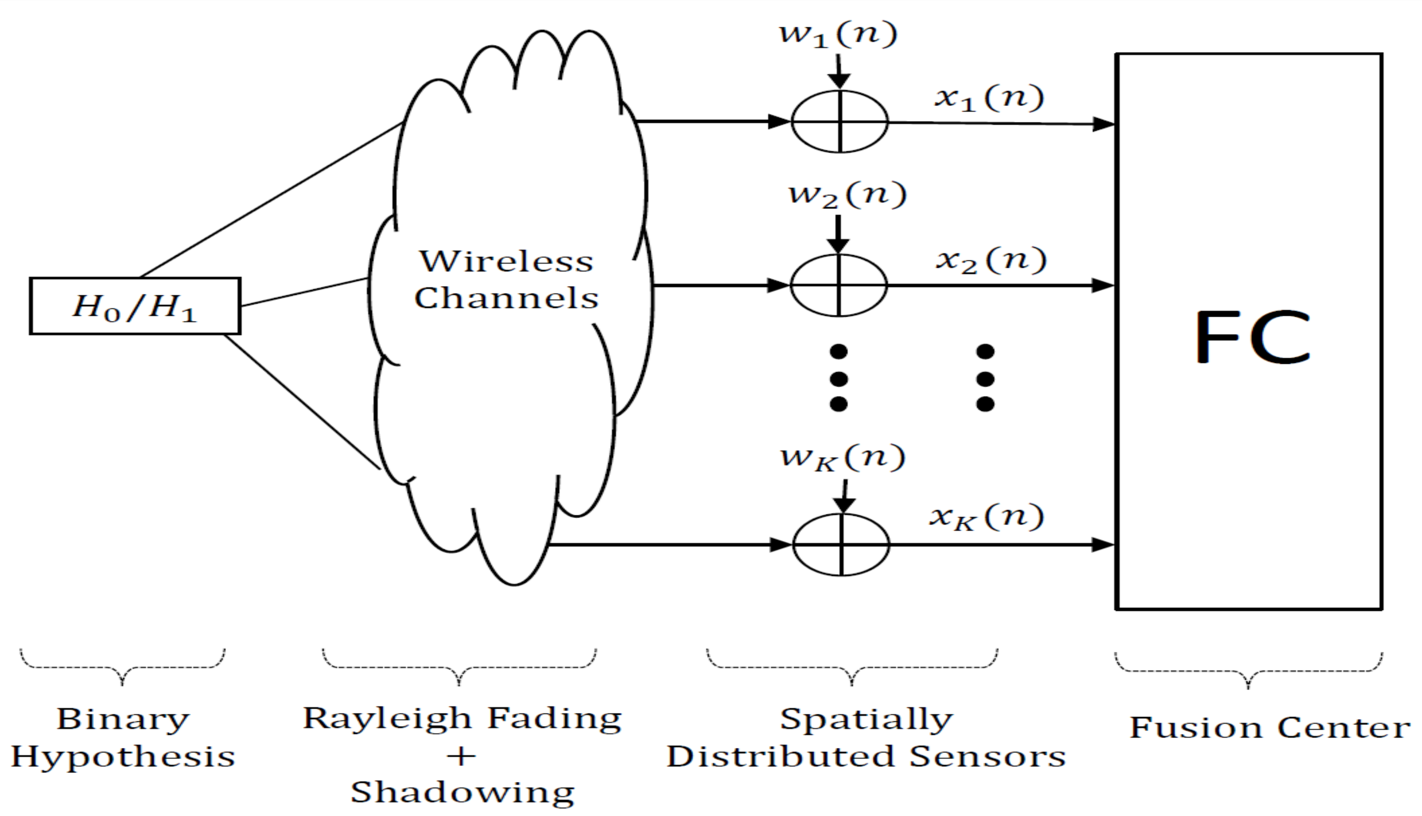
Motivation

- **Wireless Sensor Network (WSN), Distributed detection systems:**
  - Spatially distributed sensors observe a process over wireless channels
  - Forward either quantized or unquantized data to a fusion center (FC)
  - FC processes data received from local sensors to make a decision
- **Channels:** Fading, shadowing and path-loss adversely affect the performance
- **Diversity is inherent:**
  - Random nature of wireless channels
  - Multiple sensors taking multiple observations over these channels
- **Operating signal to noise ratio (SNR) of WSNs are typically very low**
- **Most diversity measures are defined for asymptotically high SNRs [1]**

System Model



Goal

- **Goal:** To define a new notion of detection diversity which captures system performance at low SNRs
- **Idea:** Use a definition based on Pitman's efficiency

Pitman's Efficiency

- ***Pitman's Efficiency***: *Let T_0 and T_1 be two test statistics that satisfy*

$$\sqrt{n}\frac{T_i-\mu_i(a_n)}{\sigma_i(a_n)} \to \mathcal{N}(0,1),$$

in distribution under H_i, for $i = 0,1$, $a_n \to 0$ while the two error probabilities are kept constant. Assume $\mu_i(a)$ is differentiable with $\mu_i'(0) > 0$ and σ_i continuous at 0. Then Pitman's efficiency for T_1 w.r.t T_0 is

$$\lim_{a_n\to 0}\frac{N_1}{N_0} = \left(\frac{\mu_1'(0)/\sigma_1(0)}{\mu_0'(0)/\sigma_0(0)}\right)^2$$

where, N_i is the number of samples needed for T_i for $i = 0,1$ to achieve the specified probability of error.

Test Statistics

- *Consider a **binary hypothesis test** as*

$$H_0: x_k(n) = w_k(n),$$
$$H_1: x_k(n) = h_k(n)a + w_k(n),$$

  - *a is deterministic known quantity*
  - *$h_k(n)$ is the channel gain for k^{th} channel at n^{th} time instant*
  - *$w_k(n)$ is i.i.d. across n and independent across k*
  - $\mathbb{E}[w_k(n)] = 0$ *and* $\mathbb{E}[w_k(n)^2] = 1$

- ***Test Statistic:***
  - *For $n = 1,2,\dots,N_1$ and $k = 1,2,\dots,K$*

$$T_L = \frac{1}{N_1}\sum_{k=1}^{K}\frac{h_k}{\sigma_k^2}\sum_{n=1}^{N_1}x_k(n)$$

- ***Benchmark System**: Consider a single sensor system as,*

$$H_0: x(n) = w(n)$$
$$H_1: x(n) = a + w(n)$$

  - *$\{w(n)\}$ is i.i.d. Gaussian noise*
  - $\mathbb{E}[w(n)] = 0$ *and* $\mathbb{E}[w(n)^2] = 1$

- ***Test Statistic:***
  - *For $n = 1,2,\dots,N_0$*

$$T_{BM} = \frac{1}{N_0}\sum_{n=1}^{N_0}x(n)$$

A New Measure of Detection Diversity

- ***Detection Diversity:** We define the detection diversity for any distributed system as the Pitman's efficiency of the system with respect to the above benchmark system which is the ratio $\frac{N_0}{N_1}$, where N_1 are the number of observations needed by the system of interest to achieve the same performance as the benchmark system.*

Diversity Measure for Fading Channels

- The diversity measure of the distributed detection system against the benchmark system for:
  - **Deterministic Channels:**

$$D_L = \lim_{a\to 0}\frac{N_0}{N_1} = \sum_{k=1}^{K}\frac{h_k^2}{\sigma_k^2}$$

  - **Log normal Shadowing + Rayleigh fading:**

$$\mathbb{E}[D_L] = \sum_{k=1}^{K}\frac{2\lambda_k^2 e^{\left(\frac{\mu_k^{\xi}}{\xi}+\frac{(\sigma_k^{\xi})^2}{2\xi^2}\right)}}{\sigma_k^2}$$

  - **Only Rayleigh fading:**

$$\mathbb{E}[D_L] = \sum_{k=1}^{K}\frac{2\lambda_k^2}{\sigma_k^2}$$

Diversity Using Daher-Adve's Definition

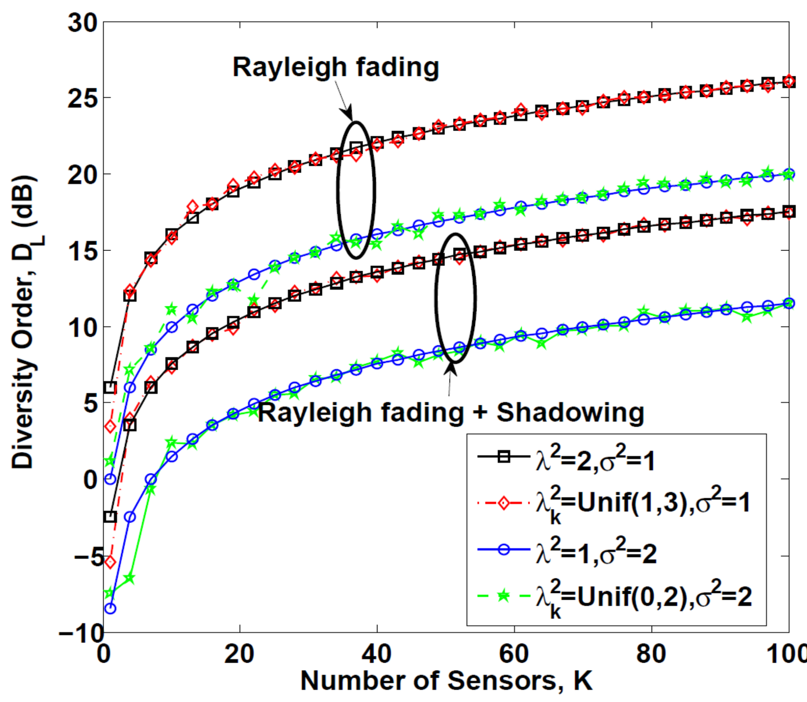
- ***Daher-Adve's definition [2]:***

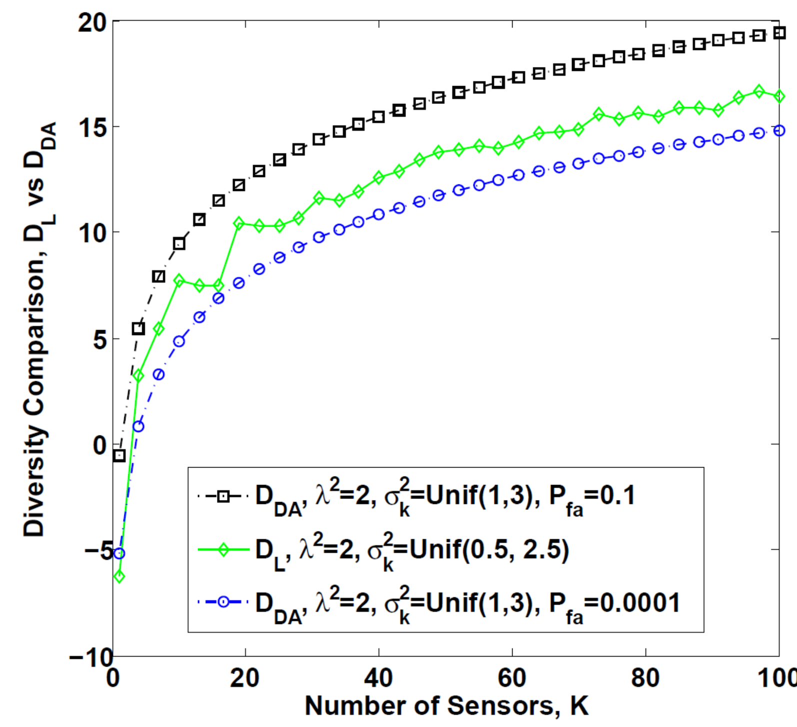
$$D_{DA} = \left.\frac{dP_d}{dSNR}\right|_{P_d=0.5}$$

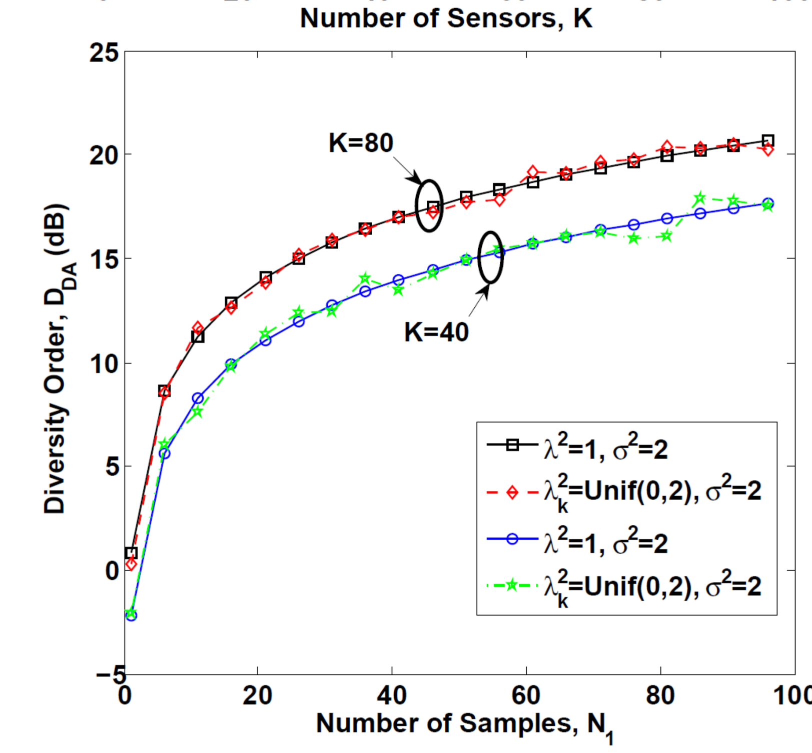
where, P_d is the probability of detection and SNR is the signal to noise ratio under H_1, defined as

$$SNR = \frac{1}{K}\sum_{k=1}^{K}\frac{a^2}{\sigma_k^2}$$

Simulation Results







- **Discussion:**
  - Higher channel gains improve diversity
  - Higher noise variances deteriorate diversity
  - Effect of heterogeneous channels is captured
  - Independent of the probabilities of error

Summary

- **We proposed a new measure of detection diversity for heterogeneous WSNs using Pitman's efficiency**
- **Definition naturally covers the low SNR regimes**
- **We showed the effect of fading and shadowing on our diversity measure**
- **We compare our definition to the definition of [2]:**
  - **Our definition captures spatial diversity better than the definition of [2]**
  - **It is independent of the probabilities of error**
- **Future work:**
  - **Extend the notion of detection diversity to the case of time varying channels**
  - **When the observations are dependent in space and/or time**

References

[1] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," IEEE Trans. Inf. Theory, vol. 49, no. 5, pp. 1073–1095, May 2003.

[2] R. Daher and R. Adve, "Notion of diversity order in distributed radar networks," IEEE Trans. Aerosp. Electron. Syst., vol. 46, no. 2, pp. 818–831, Apr. 2010.

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