Directional Maximum Likelihood Self-Estimation of the Path-Loss Exponent

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Objectives

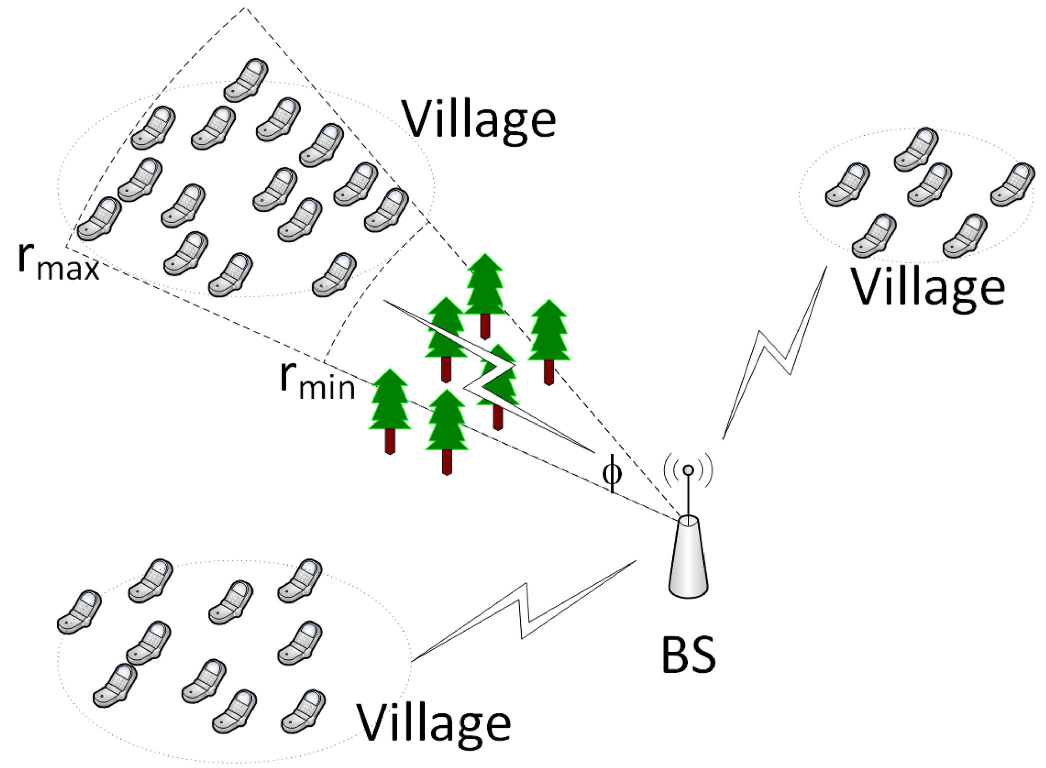
1. Estimation of PLE in case of clustered nodes.
2. Maximum likelihood (ML) solution, using the distribution of RSS.
3. Derive Cramér-Rao lower bound (CRLB).

Introduction

- The path-loss exponent (PLE) is very **crucial** for efficiently designing wireless communications and networking systems.
- Most existing methods for estimating the PLE:
  - require nodes with **known locations**;
  - assume **an omni-directional PLE**;
  - change some **configurations** of the receiver;
  - require some **information of the network**, e.g., the node density.
- Our previous work [1]:
  - two **(weighted) total least squares** solutions;
  - **Simple**, **pervasive**, **local**, **sole**, **collective**, **secure** and **directional** estimation for the PLE.
- However, the remaining problems:
  - can **NOT** cope with **clustered nodes**.
  - **NOT** the **ML** solution.
  - obtaining the **CRLB** is **NOT** possible.
- In $\mathbb{R}^m$, the RSS with only geometric path loss

$$P_r = Cr^{-\gamma},$$

  - C is the constant including the transmit power P_t
  - r is the nodal distance
  - γ is the PLE



Pareto distribution for RSS

RSS follows a truncated *Pareto* distribution Type I

$$\mathbb{P}(P_r|m,\gamma,P_{r,min},P_{r,max}) = \begin{cases} \frac{m}{\gamma}\frac{P_{r,min}^{m/\gamma}P_r^{-m/\gamma-1}}{1-(P_{r,min}/P_{r,max})^{m/\gamma}}, & P_r \in [P_{r,min}, P_{r,max}], \\ 0, & \text{otherwise}, \end{cases}$$

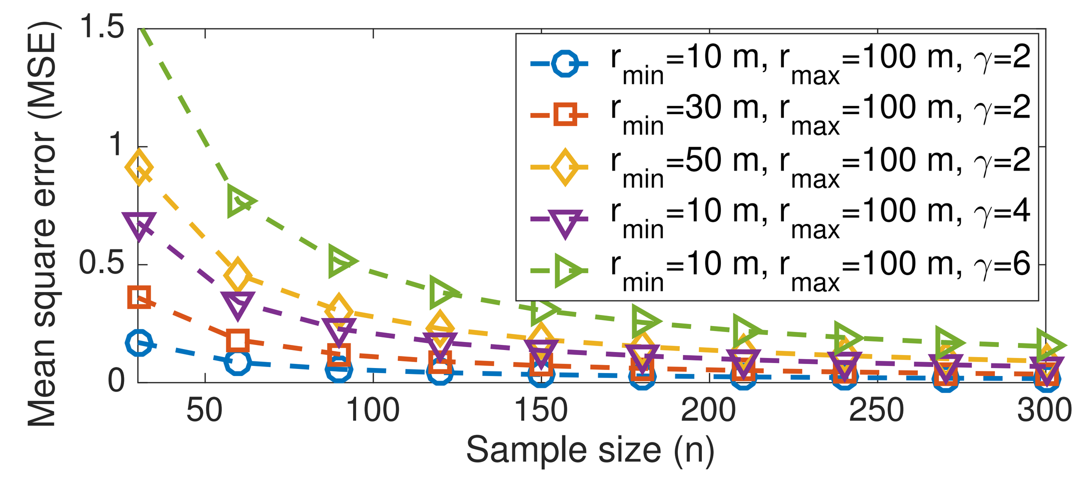
- $P_{r,min} \triangleq Cr_{max}^{-\gamma}$
- $P_{r,max} \triangleq \min\{Cr_{min}^{-\gamma}, P_t\}$

Accordingly, the CRLB is $CRLB(\gamma) = \frac{1}{\mathcal{I}(\gamma)}$.

$$\mathcal{I}(\gamma) = -\frac{n}{\gamma^2} - \frac{2mnln(P_{r,min})}{\gamma^3} + \frac{2n[(\gamma+mln(P_{r,max}))(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} - (\gamma+mln(P_{r,min}))]}{\gamma^3 \left((\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} - 1\right)} + \frac{nm(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} ln(\frac{P_{r,min}}{P_{r,max}})[2\gamma(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}} - 2\gamma - mln(\frac{P_{r,min}}{P_{r,max}})]}{(1-(\frac{P_{r,min}}{P_{r,max}})^{\frac{m}{\gamma}})^2\gamma^4}$$

CRLB

In $\mathbb{R}^2$, $P_t = 1\,Watt$.



Important Result

1. The RSS based on only a geometric path-loss is first found to follow a truncated *Pareto* distribution.
2. The CRLB is introduced and decreases with more samples, a small PLE and a close node cluster.
3. Two ML self-estimators of the PLE are derived and yield a good performance close to the CRLB.

ML solutions

- $P_{r,min}$ and $P_{r,max}$ are known: solve

$$\frac{n\gamma}{m} - \sum_{i=1}^{n}(ln\frac{P_i}{P_{r,min}}) + \frac{n(\frac{P_{r,min}}{P_{r,max}})^{m/\gamma}ln(\frac{P_{r,min}}{P_{r,max}})}{1-(\frac{P_{r,min}}{P_{r,max}})^{m/\gamma}} = 0.$$

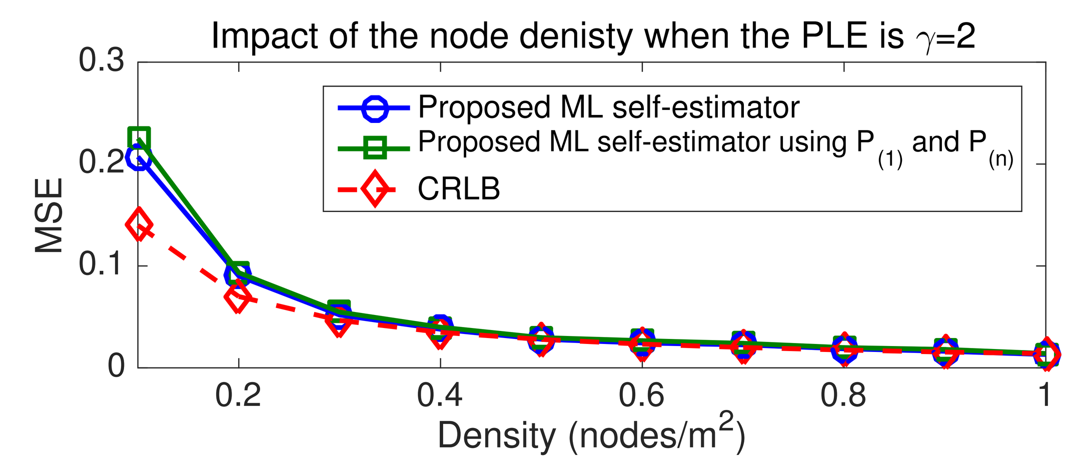
- $P_{r,min}$ and $P_{r,max}$ are unknown: rank the RSSs as $P_{(1)} < \cdots < P_{(n)}$ and solve

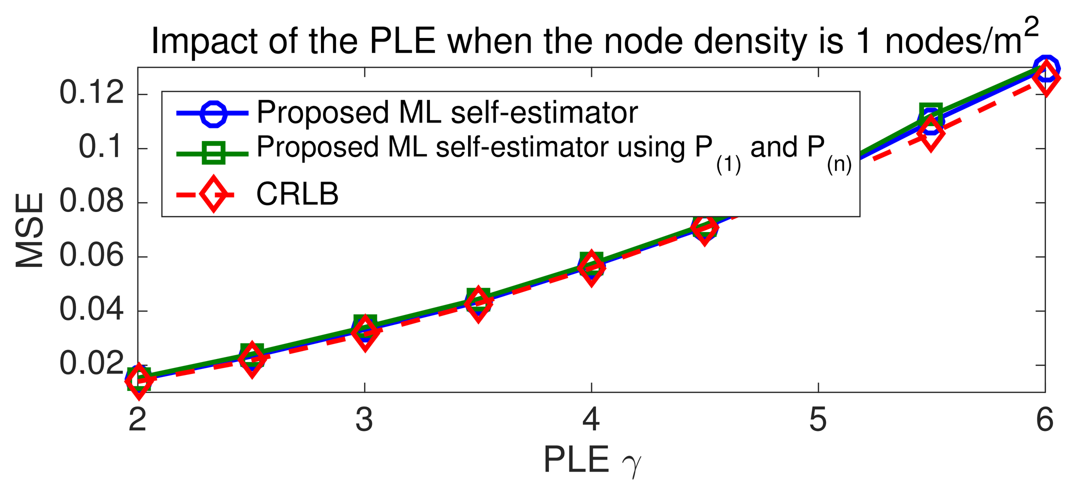
$$\frac{n\gamma}{m} - \sum_{i=1}^{n}(ln\frac{P_i}{P_{(1)}}) + \frac{n(\frac{P_{(1)}}{P_{(n)}})^{m/\gamma}ln(\frac{P_{(1)}}{P_{(n)}})}{1-(\frac{P_{(1)}}{P_{(n)}})^{m/\gamma}} = 0.$$

- Solve by a simple bisection method.
- Shadowing effect, if considered, has an insignificant influence.

Node Cluster

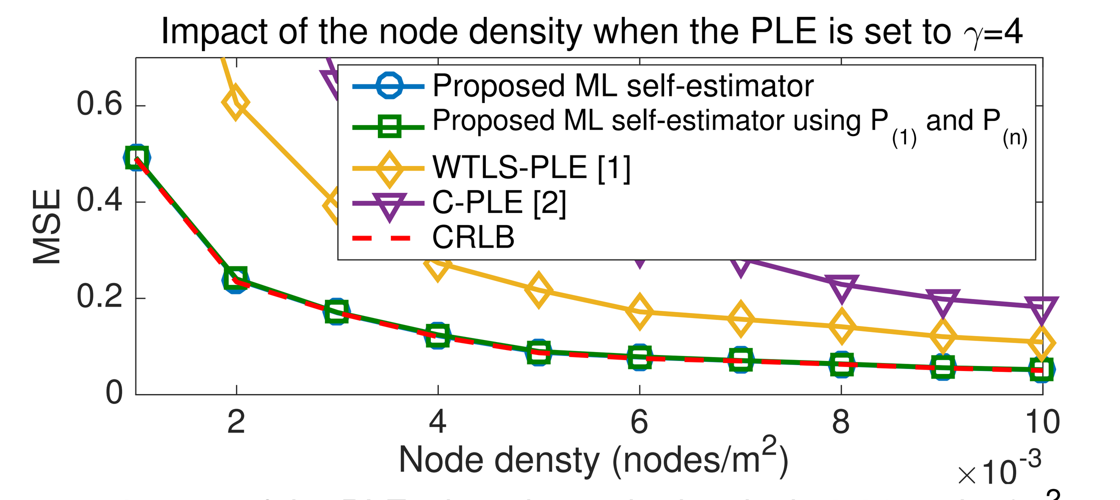
In $\mathbb{R}^2$, $r_{min} = 50\ m$, $r_{max} = 100\ m$, $\phi = \pi/6$ and $P_t = 1\,Watt$

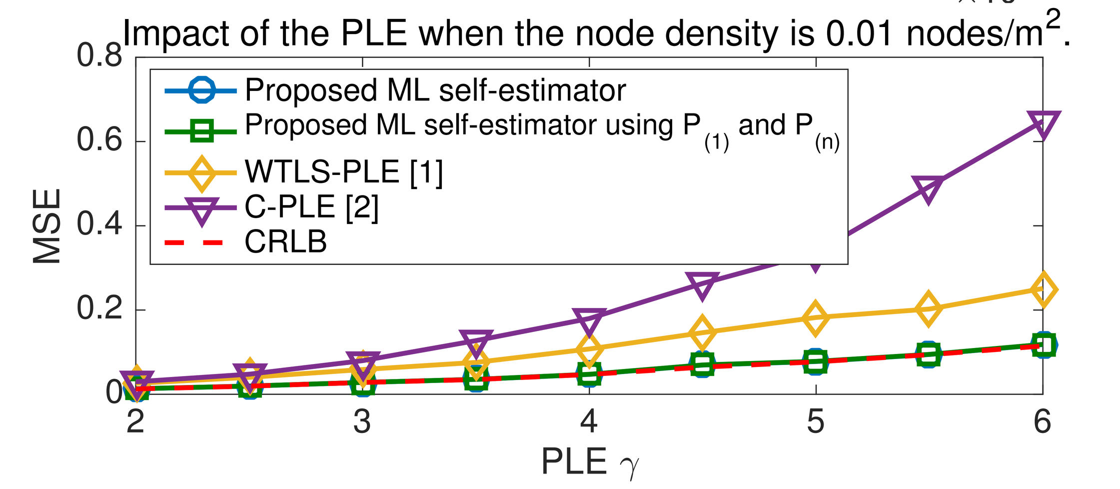




Homogeneous random networks

In $\mathbb{R}^2$, $r_{max} = 100\ m$ and $P_t = 1\,Watt$.





References

[1] Yongchang Hu and G. Leus. Self-estimation of path-loss exponent in wireless networks and applications. *IEEE Transactions on Vehicular Technology*, 64(11):5091–5102, Nov 2015.

[2] Sunil Srinivasa and Martin Haenggi. Path loss exponent estimation in large wireless networks. In *Information Theory and Applications Workshop, 2009*, pages 124–129. IEEE, 2009.

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