Reduced Dimension Minimum BER PSK Precoding for Constrained Transmit Signals in Massive MIMO

Lee Swindlehurst

Center for Pervasive Communications and Computing, UC Irvine, USA

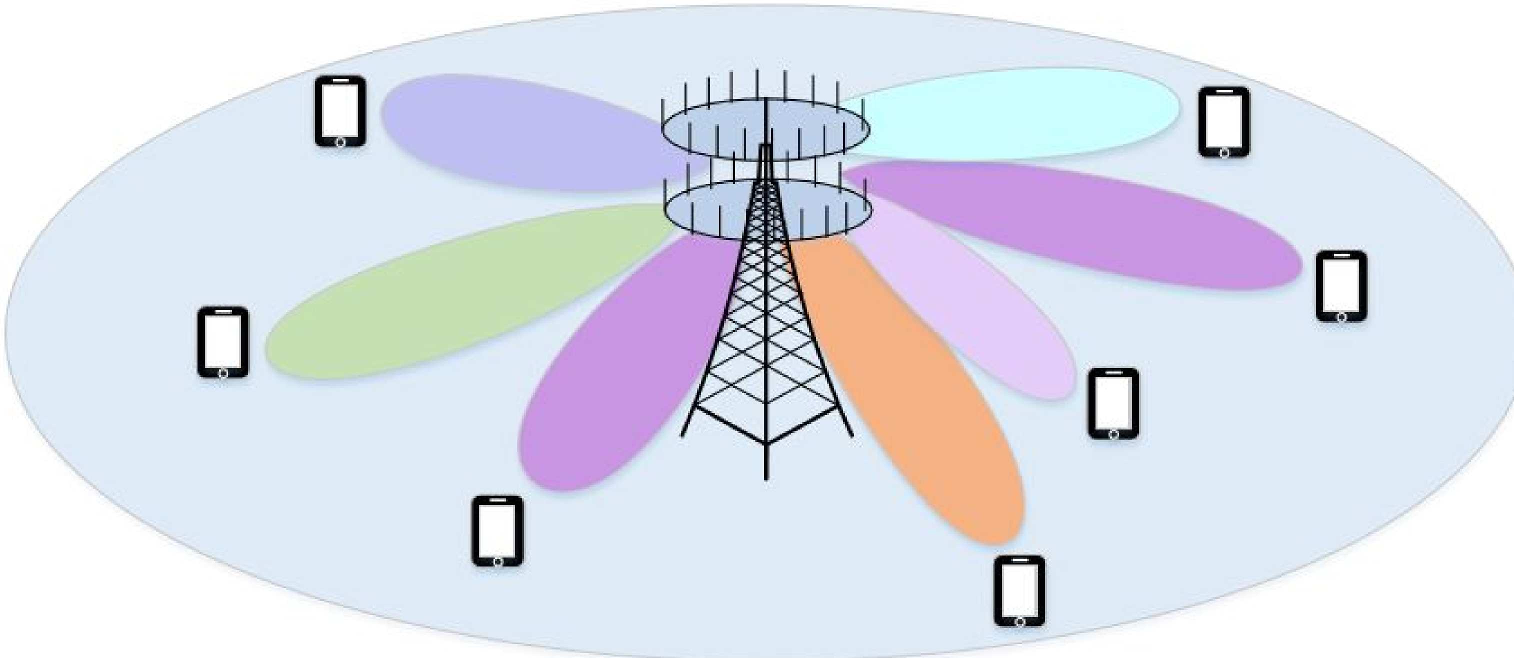
Hela Jedda

Dept. of Electrical & Computer Eng., Technical University of Munich, Germany

Inbar Fijalkow

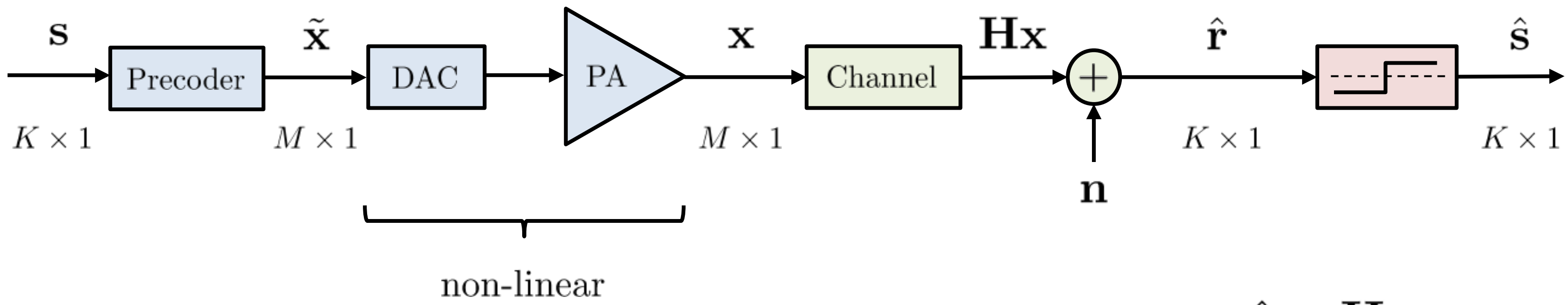
University of Cergy-Pointoise, ENSEA, France

Energy Efficiency in Massive MIMO



- Energy efficiency and hardware complexity are important issues for massive MISO/MIMO systems
- Use low fidelity hardware (e.g., one-bit ADCs/DACs) to minimize power consumption, low PAPR waveforms to (1) lower OOB interference and spectral regrowth and (2) allow PAs to operate with no back-off

Massive MISO Downlink



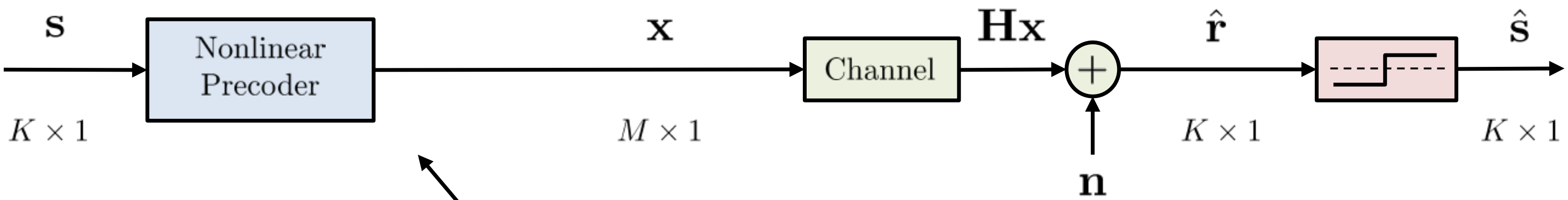
$$\hat{\mathbf{r}} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\mathbf{r} = \mathbf{H}\mathbf{x}$$

Model and Assumptions

- Known flat-fading channel
- Symbol-rate model, ignoring spectral regrowth, distortion due to non-linearities
- Possible transmit signal non-linearities/constraints:
  - one-bit DACs: $x_{i,n} = \sqrt{\frac{\rho}{2}}(\pm 1 \pm j)$
  - PA saturation: $|x_{i,n}| \leq \sqrt{\rho}$
  - constant modulus: $|x_{i,n}| = \sqrt{\rho}$

Massive MISO Downlink



non-linear precoder implements transmit constraints so that $\tilde{\mathbf{x}} = \mathbf{x}$

$$\hat{\mathbf{r}} = \mathbf{Hx} + \mathbf{n}$$

$$\mathbf{r} = \mathbf{Hx}$$

- Possible transmit signal non-linearities/constraints:
  - one-bit DACs: $x_{i,n} = \sqrt{\frac{\rho}{2}}(\pm 1 \pm j)$
  - PA saturation: $|x_{i,n}| \leq \sqrt{\rho}$
  - constant modulus: $|x_{i,n}| = \sqrt{\rho}$

Linear Precoding Performance

Non-linear effects applied to linear precoder output: $\mathbf{x} = \mathcal{Q}(\mathbf{Ps})$

$$\mathbf{r} = \mathbf{H}\mathcal{Q}(\mathbf{Ps})$$

Example: One-bit ADCs, Zero-Forcing (ZF) Precoder, $\mathbf{P} = \mathbf{H}^H(\mathbf{HH}^H)^{-1}$

Assume $M \gg K \gg 1$, resulting asymptotic SER [1]:

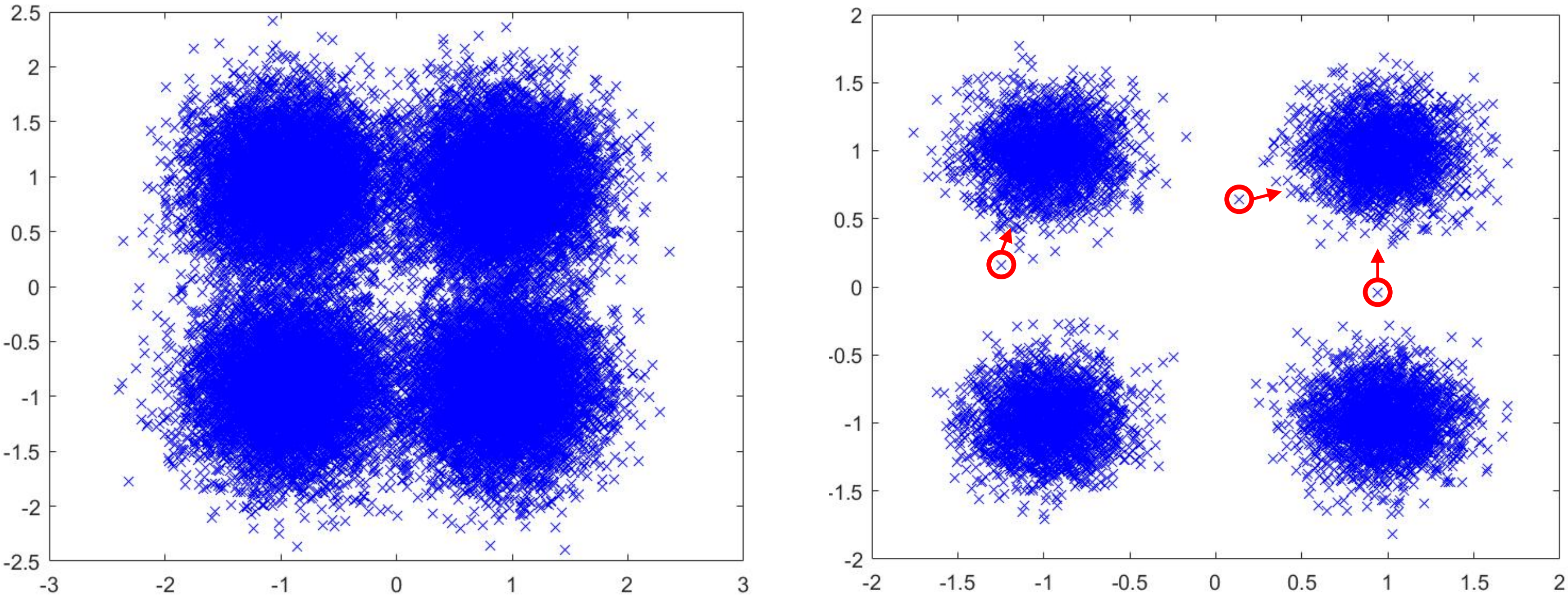
$$P_e = 2Q\left(\sqrt{\frac{\frac{4\sigma^2(M-K)^2}{MK\pi}}{\frac{2\sigma^2}{M}\left(1-\frac{2}{\pi}\right)(M-K)+\sigma_n^2}}\right)$$

High SNR error floor:

$$P_e \longrightarrow 2Q\left(\sqrt{\frac{\frac{2}{\pi}}{1-\frac{2}{\pi}}\left(\frac{M}{K}-1\right)}\right)$$

[1] A. Saxena, I. Fijalkow, A. Swindlehurst; Analysis of One-Bit Quantized Precoding for the Multiuser Massive MIMO Downlink, *IEEE Trans. SP*, Sept. 2017.

Linear Precoding Requires Large *M/K*



$$\frac{M}{K} = \frac{128}{32} = 4$$

$$\frac{M}{K} = \frac{128}{10} = 12.8$$

Since problem symbols are known beforehand, can adjust/perturb $\mathbf{x}$ to push them away from decision boundaries

Non-Linear MSE Precoding

Directly minimize (noise-free) MSE at receivers:

$$\mathbf{x} = \arg\min_{\substack{\beta \in \mathbb{R} \\ \mathbf{x} \in \mathcal{X}}} \|\mathbf{s} - \beta \mathbf{H}\mathbf{x}\|$$

- Probhibitively complex, especially for a massive antenna array (for one-bit DACs, d^M possible $\mathbf{x}$ for constellations of size d)
- Approximate solution for one-bit DACs based on ℓ_∞ relaxation proposed in [2] (SQUID)
- Approximate solutions also proposed for constant modulus and PA saturated signals
- MSE criterion does not directly minimize decoding error probability at users.

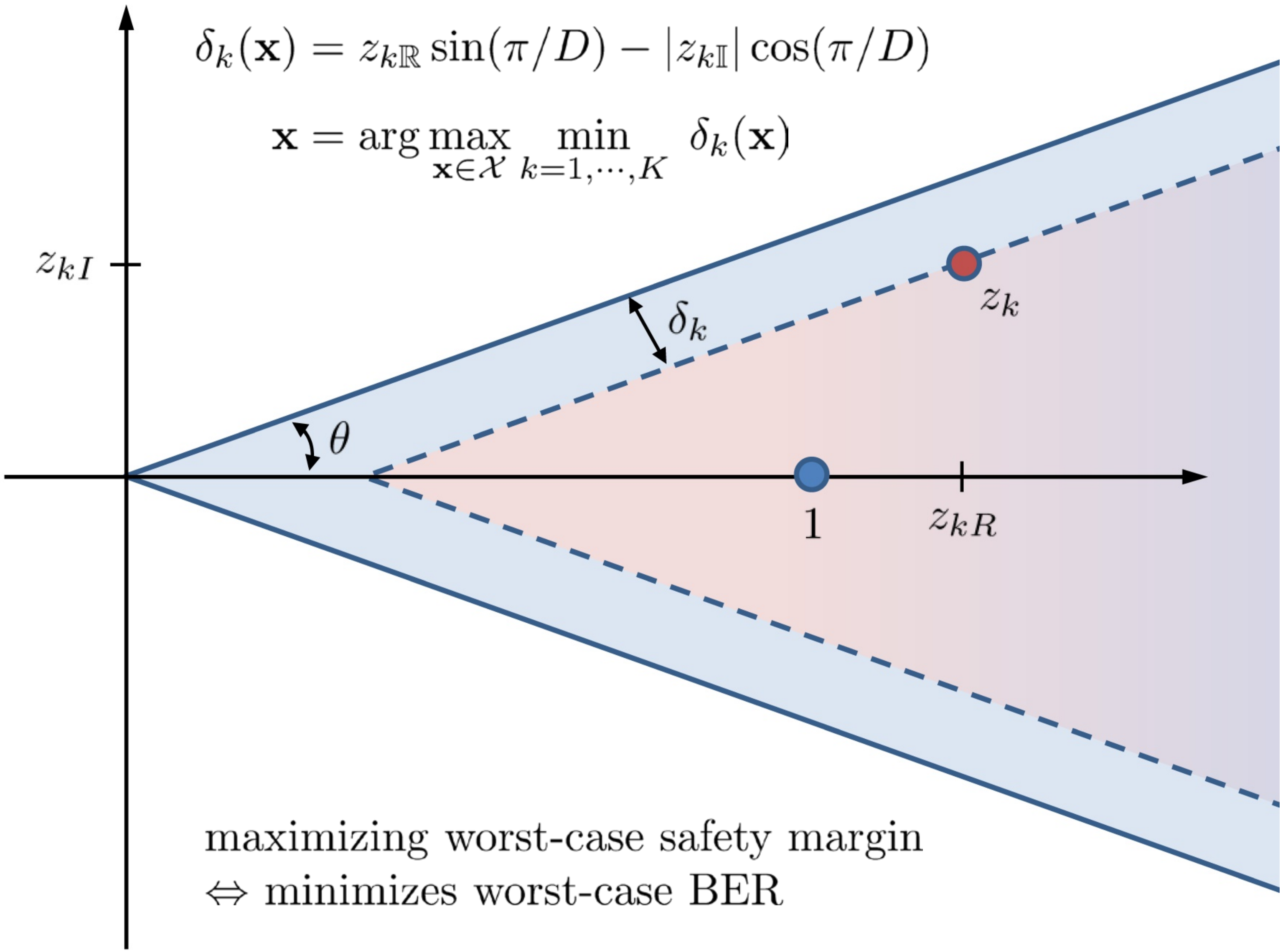
[2] S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer; Quantized Precoding for Massive MU-MIMO, *IEEE Trans. Commun.*, Nov. 2017.

Alternative for PSK Symbols: Minimum BER Precoding

- MSE penalizes received signals far from constellation point, even though they may be reliably decoded
- Use performance metric that maximizes the distance or "safety margin" δ of the received signal from the decision boundary
- Also referred to as exploiting "constructive interference" in work by Masouros, Ottersten, etc.
- Here we focus on PSK signals, although formulations are available for QAM as well
- To define the metric, rotate symbols to 1:

$$\mathbf{z} = \mathrm{diag}(s)^H \mathbf{H}\mathbf{x} = \tilde{\mathbf{H}}\mathbf{x}$$

Safety Margin for D-PSK Signals



Examples from Prior Work

- CVX-CIO [3], constant modulus $\mathbf{x}$, relaxed convex optimization

$$\mathbf{x} = \arg \max_{|x_i| \leq c} \min_{k=1,\cdots,K} \delta_k(\mathbf{x})$$

[3] P. V. Amadori and C. Masouros, "Constant envelope precoding by interference exploitation in phase shift keying-modulated multiuser transmission," *IEEE Trans. Wireless Commun.*, Jan 2017.

- Direct perturbation method [4] for one-bit DACs, greedy perturbation of $\hat{\mathbf{x}} = \mathcal{Q}(\mathbf{Ps})$ to increase $\delta(\mathbf{x})$

[4] A. Swindlehurst, A. Saxena, A. Mezghani, and I. Fijalkow, "Minimum probability-of-error perturbation precoding for the one-bit massive MIMO downlink," in *Proc. IEEE ICASSP*, March 2017.

- Linear programming method [5] for one-bit DACs with relaxed box constraint

$$\mathbf{x} = \arg \max_{\substack{|x_{i\mathbb{R}}| \leq c \\ |x_{i\mathbb{I}}| \leq c}} \min_{k=1,\cdots,K} \delta_k(\mathbf{x})$$

[5] H. Jedda, A. Mezghani, J. Nossek, and A. Swindlehurst, "Massive MIMO downlink 1-bit precoding with linear programming for PSK signaling," in *Proc. IEEE SPAWC*, July 2017.

Motivation for Reduced Dimension Algorithm

- Previous algorithms require (1) a relaxation of the non-linear transmit constraint, and (2) an iterative search over a constrained vector $\mathbf{x}$ of dimension M, which for massive MIMO is very large:

$$\mathbf{x}(p+1) = \mathbf{x}(p) + \boldsymbol{\epsilon}(p)$$

- Idea: (1) Maintain the constraint at each iteration, and (2) reduce complexity by searching in the K-dimensional symbol space:

$$\mathbf{x}(p+1) = \mathcal{Q}\left(\mathbf{P}(\mathbf{s}(p) + \boldsymbol{\epsilon}(p))\right)$$

- In the proposed approach, we used gradient descent to update the perturbation $\boldsymbol{\epsilon}(p)$:

$$\boldsymbol{\epsilon}(p+1) = \boldsymbol{\epsilon}(p) + \mu \tilde{\nabla}^*_{\boldsymbol{\epsilon}} \delta(p)$$

where the gradient $\tilde{\nabla}^*_{\boldsymbol{\epsilon}} \delta(p)$ is obtained using a continuous approximation of the non-linearity, or ignoring it altogether.

Reduced Dimension Algorithm

1. Given $\mathbf{s}, \tilde{\mathbf{H}}, \mathbf{P}$, number of iterations N_p, and stepsize μ, set $p = 1$ and $\boldsymbol{\epsilon}(1) = 0$.

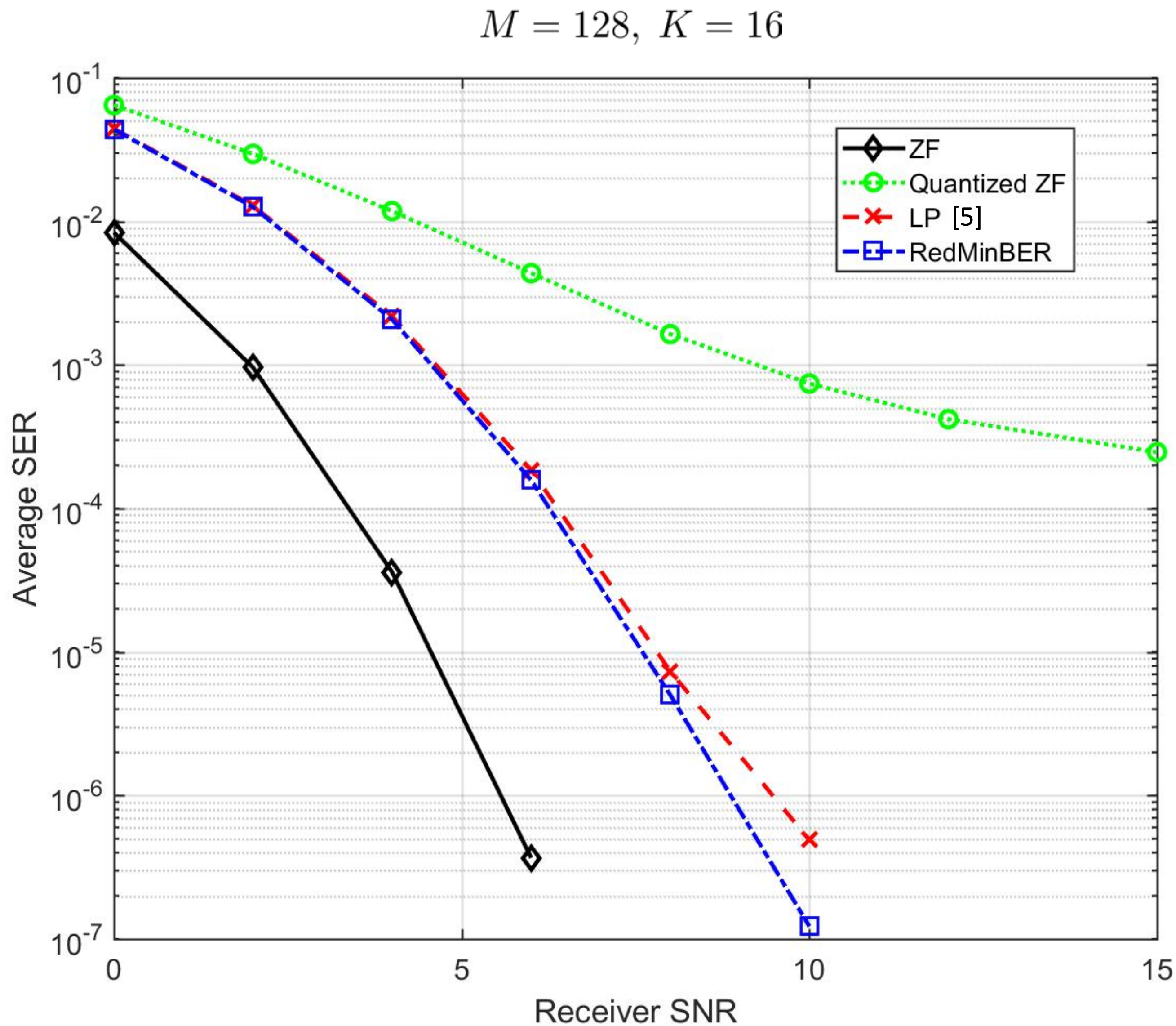
2. Calculate $\mathbf{z} = \tilde{\mathbf{H}}\mathcal{Q}(\mathbf{P}\mathbf{s})$ and $\delta(1)$.

3. Set $\mathbf{s}_{opt} = \mathbf{s}$ and $\delta_{opt} = \delta(1)$.

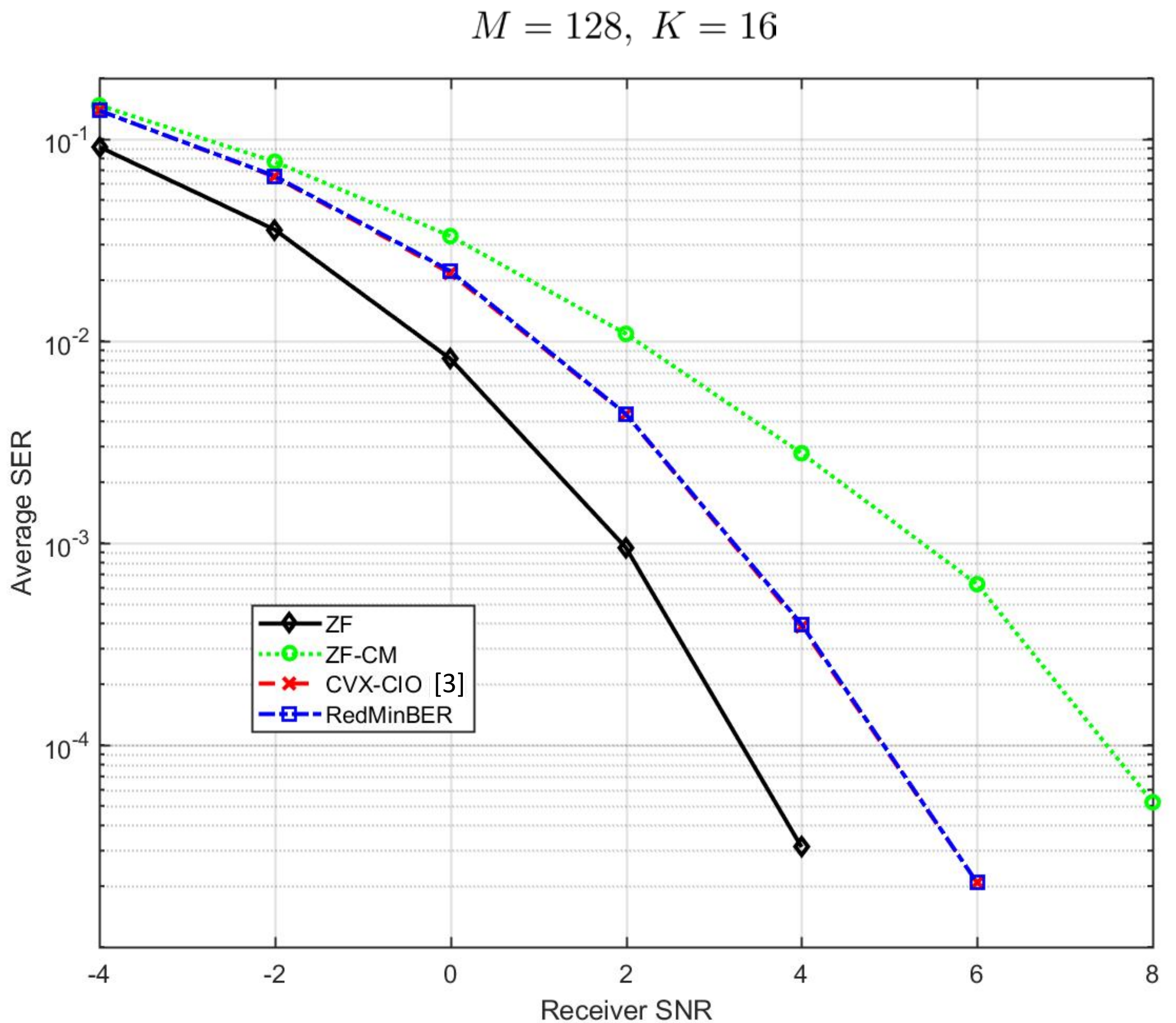
4. For $p = 1$ to N_p, do
   (a) Find $\boldsymbol{\epsilon}(p+1) = \boldsymbol{\epsilon}(p) + \mu\tilde{\nabla}^*_{\boldsymbol{\epsilon}}\delta(p)$.
   (b) Calculate $\mathbf{z} = \tilde{\mathbf{H}}\mathcal{Q}(\mathbf{P}(\mathbf{s} + \boldsymbol{\epsilon}(p+1)))$ and $\delta(p+1)$.
   (c) If $\delta(p+1) > \delta_{opt}$, set $\delta_{opt} = \delta(p+1)$ and $\mathbf{s}_{opt} = \mathbf{s} + \boldsymbol{\epsilon}(p+1)$.

5. Output solution $\mathbf{s}_{opt}$.

Example 1: One-Bit DACs



Example 2: Constant Modulus Signals



Conclusions

- Energy efficiency / hardware complexity important issues for massive MISO/MIMO
- Use low fidelity hardware (e.g., one-bit ADCs/DACs) to minimize power consumption, low-PAPR waveforms to lower OOB interference and spectral regrowth, eliminate PA back-off
- Linear precoding: low complexity, but good performance with transmit constraints/non-linearities requires large M/K
- Non-linear precoding: significantly better performance, but high complexity (M-dimensional search)
- Proposed algorithm: perturb linearly precoded symbols, requires search in only K-dimensional space
- Achieves performance similar to more complex methods