Cell-free Massive MIMO Systems with Multi-antenna Users

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28 November 2018

Table of Contents

Cell-free Massive MIMO

M geographically distributed APs joinly serve all K users in the same time-frequency resource of the network.

- 3 phases: Uplink training phase, Uplink data transmission and Downlink data transmission.
- Work in TDD mode.
- Exploitation of the channel reciprocity, CSI only need to be estimated one times over the coherence time.

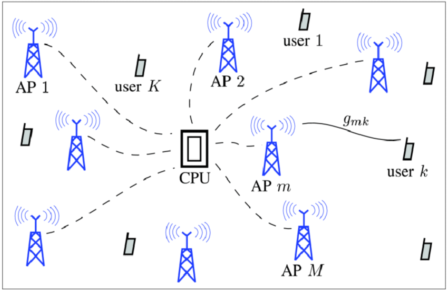

Figure 1: Cell-free Massive MIMO systems
(**Source:** http://ieeexplore.ieee.org/document/7827017/)

System Model

- Consider a cell-free massive MIMO system with M APs and K users, each AP has L antennas while each user has N antenas.
- The channel response matrix between the k-th user and the m-th AP

$$\mathbf{G}_{mk} = \beta_{mk}^{1/2}\mathbf{H}_{mk}, \tag{1}$$

where β_{mk} is large-scale fading, and $\mathbf{H}_{mk}$ is random matrix where each its element is i.i.d. $\mathcal{CN}(0,1)$.

- The received signal at the m-th AP is

$$\mathbf{Y}_m = \sum_{k=1}^{K} \sqrt{\tau\rho_p}\mathbf{G}_{mk}\Phi_k^H + \mathbf{W}_m \tag{2}$$

Uplink Channel Estimation

MMSE estimation of $\mathrm{vec}(\mathbf{G}_{mk})$ given $\mathrm{vec}(\mathbf{Y}_{mk})$ is

$$\mathrm{vec}(\hat{\mathbf{G}}_{mk}) = \sqrt{\tau\rho_p}\beta_{mk}\mathbf{I}_{LN}\left(\tau\rho_p\sum_{i=1}^{K}\tilde{\Phi}_{ik}\beta_{mi}\mathbf{I}_{LN}\tilde{\Phi}_{ik}^{H} + \mathbf{I}_{LN}\right)^{-1}\mathrm{vec}(\mathbf{Y}_{mk}). \tag{3}$$

The estimate of the channel matrix $\mathbf{G}_{mk}$ is given by

$$\hat{\mathbf{G}}_{mk} = \mathbf{Y}_{mk}\mathbf{A}_{mk}, \tag{4}$$

where

$$\mathbf{A}_{mk} \triangleq \sqrt{\tau\rho_p}\beta_{mk}\left(\tau\rho_p\sum_{i=1}^{K}\beta_{mi}\Phi_{ik}^{H}\Phi_{ik} + \mathbf{I}_N\right)^{-1}. \tag{5}$$

Downlink Data Transmission

The received signal at the k-th user is

$$\mathbf{r}_k = \sum_{m=1}^{M} \mathbf{G}_{mk}^H \mathbf{x}_m + \mathbf{n}_k, \tag{6}$$

where $\mathbf{x}_m$ is transmitted signal from the m-th AP and

$$\mathbf{x}_m = \sqrt{\rho_d} \sum_{k=1}^{K} \eta_{mk}^{1/2} \hat{\mathbf{G}}_{mk} \mathbf{q}_k, \tag{7}$$

Spectral Efficiency

An achievable downlink SE for the k-th user when using conjugate beamforming and MMSE-SIC receiver as

$$R_k = (1 - \tau/\tau_c) \log_2 \left| \mathbf{I}_N + \bar{\mathbf{H}}_k^H \bar{\Xi}_k \bar{\mathbf{H}}_k \right|, \quad (8)$$

where

$$\bar{\mathbf{H}}_k \triangleq \mathbb{E}\left\{ \sqrt{\rho_d} \sum_{m=1}^{M} \eta_{mk}^{1/2} \mathbf{G}_{mk}^H \hat{\mathbf{G}}_{mk} \right\}, \quad (9)$$

and

$$\bar{\Xi}_k \triangleq \left(\mathbb{E}\left\{ \rho_d \sum_{m=1}^{M} \sum_{n=1}^{M} \sum_{k'=1}^{K} \eta_{mk'}^{1/2} \eta_{nk'}^{1/2} \mathbf{G}_{mk}^H \hat{\mathbf{G}}_{mk'} \hat{\mathbf{G}}_{nk'}^H \mathbf{G}_{nk} \right\} - \bar{\mathbf{H}}_k \bar{\mathbf{H}}_k^H + \mathbf{I}_N \right)^{-1}. \quad (10)$$

Spectral Efficiency

Lemma 1:
Let $\mathbf{B} = \mathbf{Y}^H\mathbf{X}$, where $\mathbf{X}$, $\mathbf{Y}$ are $M \times N$ random matrix which its elements are assumed to be i.i.d. $\mathcal{CN}(0,1)$ and $\mathbf{C}$ is $N \times N$ matrix. Then

$$\mathbb{E}\left\{\mathbf{B}^H\mathbf{C}\mathbf{B}\right\} = M\,\mathrm{tr}(\mathbf{C})\mathbf{I}_N. \tag{11}$$

Spectral Efficiency

Apply Lemma 1, the achievable downlink SE for the k-th user can be represented in closed-form as

$$R_k = (1 - \tau/\tau_c) \log_2 \left| \mathbf{I}_N + \bar{\mathbf{H}}_k^H \left(\mathcal{S} + \mathbf{I}_N\right)^{-1} \bar{\mathbf{H}}_k \right|, \quad (12)$$

where

$$\bar{\mathbf{H}}_k = L\sqrt{\tau \rho_d \rho_p} \sum_{m=1}^{M} \eta_{mk}^{1/2} \beta_{mk} \mathbf{A}_{mk}, \quad (13)$$

Spectral Efficiency

and

$$
\begin{aligned}
\mathcal{S} = {} & L^2\tau\rho_p\rho_d \sum_{m=1}^{M}\sum_{n\neq m}^{M}\sum_{k'\neq k}^{K} \eta_{mk'}^{1/2}\eta_{nk'}^{1/2}\beta_{mk}\beta_{nk}\Phi_{kk'}\times \\
& \times \mathbf{A}_{mk'}\mathbf{A}_{nk'}^H\Phi_{kk'}^H - L^2\tau\rho_p\rho_d \sum_{m=1}^{M}\eta_{mk}\beta_{mk}^2\mathbf{A}_{mk}\mathbf{A}_{mk}^H \\
& + L^2\tau\rho_p\rho_d \sum_{m=1}^{M}\sum_{k'=1}^{K}\eta_{mk'}\beta_{mk}^2 \,\mathrm{tr}(\Phi_{kk'}\mathbf{A}_{mk'}\mathbf{A}_{mk'}^H\Phi_{kk'}^H)\mathbf{I}_N \\
& + L\tau\rho_p\rho_d \sum_{m=1}^{M}\sum_{k'=1}^{K}\sum_{i=1}^{K}\eta_{mk'}\beta_{mk}\beta_{mi}\,\mathrm{tr}(\Phi_{ik'}\mathbf{A}_{mk'}\mathbf{A}_{mk'}^H\Phi_{ik'}^H)\mathbf{I}_N \\
& + L\rho_d \sum_{m=1}^{M}\sum_{k'=1}^{K}\beta_{mk}\eta_{mk'}\,\mathrm{tr}(\mathbf{A}_{mk'}\mathbf{A}_{mk'}^H)\mathbf{I}_N. \qquad (14)
\end{aligned}
$$

Numerical Results

Numerical results are conducted by using orthogonal pilot sequences and per-user net throughput, which defined as $T_k = BR_k$.

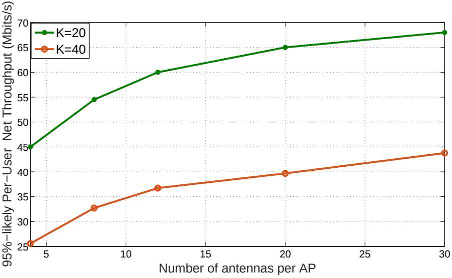

Figure 2: 95%-likely per-user downlink net throughput versus the number of antennas per AP with $N = 2$.

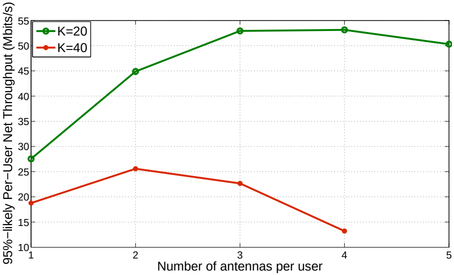

Figure 3: 95%-likely per-user downlink net throughput versus the number of antennas per user with $L = 4$.

Conclusion

- We analyse the performance of cell-free massive MIMO with multiple antennas at both APs and users.
- The closed-form expression of downlink SE is derived.
- Effect of the number of antennas at APs and users on SE is analyzed and exploited through the use of max-min fairness power control.

Thank you for your attention!