DICTIONARY LEARNING IN THE ANALYSIS SPARSE REPRESENTATION WITH OPTIMIZATION ON STIEFEL MANIFOLD

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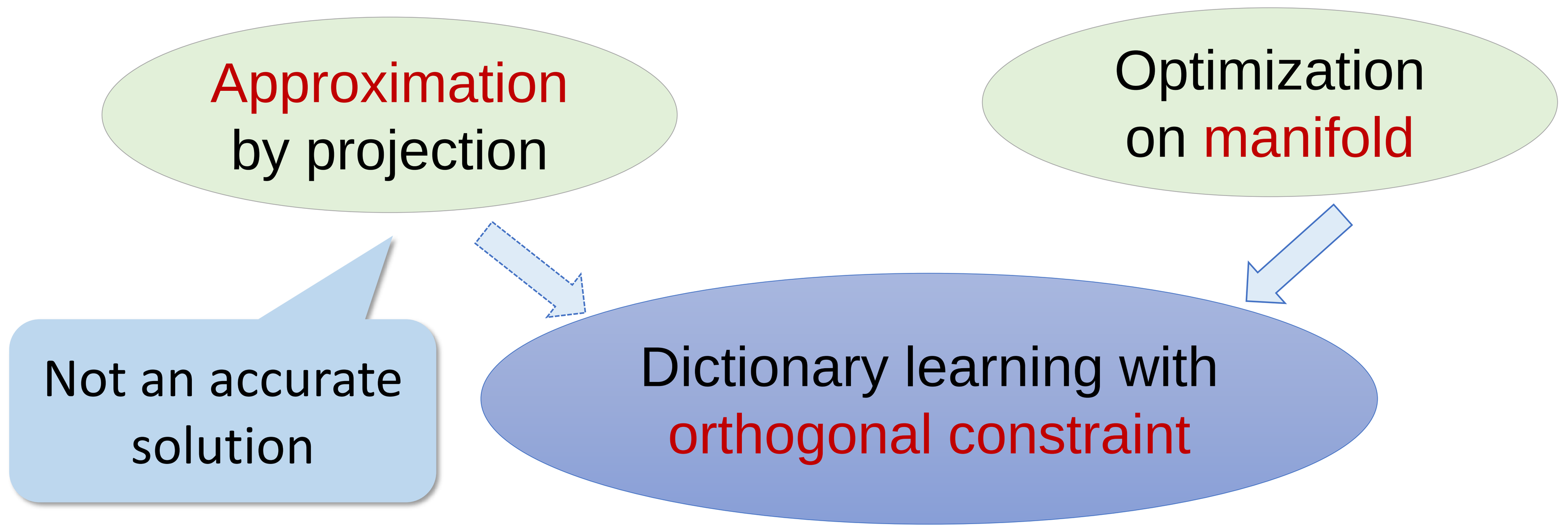
INTRODUCTION

Sparse representation has been proven to be a powerful tool for signals and images processing. This paper addresses sparse representation with the so-called analysis model. We pose the problem as to learn an analysis dictionary from signals using an optimization formulation with an orthogonal constraint.



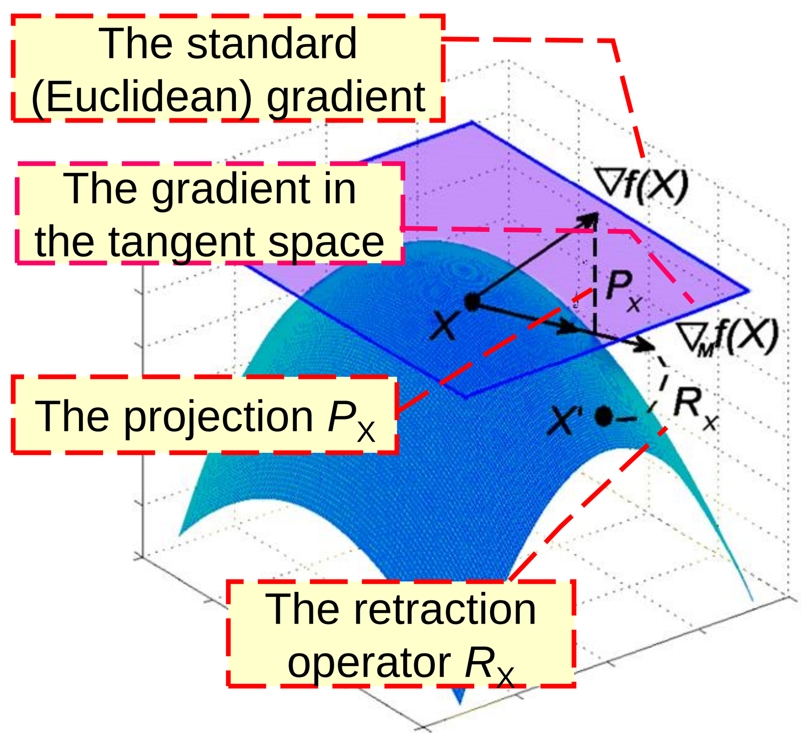
The optimization problem with constraints,

$$\min_{\mathbf{X}\in M} f(\mathbf{X})$$

M can be any constraints

e.g. $\{\mathbf{X} \mid \|\mathbf{X}\|_F^2 \le 1\}, \{\mathbf{X} \mid \sum_{j=1}^{N} \mathbf{X}_{ij}^2 = 1\}$

In our situation: $M = \{\mathbf{X} \mid \mathbf{X}^T\mathbf{X} = \mathbf{I}\}$



PROPOSED METHOD

➢Problem formulation.

Given an observed signals set **Y**, which is a noisy version of a signal **X**. We formulate the sparse representation in analysis model with ℓ_1 norm as,

$$\min_{\mathbf{\Omega},\mathbf{X}} \|\mathbf{\Omega X}\|_1, \quad \text{s.t.} \quad \|\mathbf{Y}-\mathbf{X}\|_F^2 \le \sigma, \quad \mathbf{\Omega} \in L,$$

where L means the constraints on the dictionary $\mathbf{\Omega}$.

$$\min_{\mathbf{\Omega},\mathbf{X},\mathbf{Z}} \|\mathbf{Z}\|_1 + \lambda\|\mathbf{Y}-\mathbf{X}\|_F^2 + \beta\|\mathbf{\Omega X}-\mathbf{Z}\|_F^2,$$

$$\text{s.t.} \quad \mathbf{\Omega}^T\mathbf{\Omega} = \mathbf{I}; \quad \forall i \quad \|\omega_i\|_2 = c.$$

Use an alternative method.

The orthogonal constraint

The uniformly normalized constraint

Update $\mathbf{\Omega}$ $\mathbf{Z}$ $\mathbf{X}$

➢Dictionary update with manifold method.

We only consider the terms with dictionary $\mathbf{\Omega}$,

$$\min_{\mathbf{\Omega}} \|\mathbf{\Omega X}-\mathbf{Z}\|_F^2 \quad \text{s.t.} \quad \mathbf{\Omega}^T\mathbf{\Omega} = \mathbf{I}; \quad \forall i \quad \|\omega_i\|_2 = c.$$

Use nonmonotone line search with Barzilai-Borwein (BB) steps size to solve the problem with orthogonality constraint.

$$\mathbf{A} = (2\mathbf{\Omega XX}^T - 2\mathbf{ZX}^T)\mathbf{\Omega}^T - \mathbf{\Omega}(2\mathbf{\Omega XX}^T - 2\mathbf{ZX}^T),$$

$$\mathbf{\Omega}_{k+1} = (\mathbf{I}+\frac{\tau}{2}\mathbf{A})^{-1}(\mathbf{I}-\frac{\tau}{2}\mathbf{A})\mathbf{\Omega}_k. \qquad (14)$$

➢Algorithm 1 : Manifold based Analysis Dictionary Learning Algorithm (MADL).

1: Input: $\mathbf{X}$ and K_{max}.
2: Initialize $\mathbf{\Omega}_1$ as random matrices, $k=1$
3: While $k \le K_{max}$ do
4: Update $\mathbf{\Omega}_k$ as (14) with nonmonotone line search with BB steps,
5: $\mathbf{\Omega}_{k+1} = P_{UN}\{\mathbf{\Omega}_k\}$,
6: $\mathbf{Z}_k = \mathbf{\Omega}_{k+1}\mathbf{X}$,
7: $\mathbf{Z}_{k+1} = \mathbf{Z}_k - \eta\partial\|\mathbf{Z}\|_1 = \mathbf{Z}_k - \eta\,\text{sign}(\mathbf{Z})$,
8: $k = k+1$
9: end while,
10: Output: $\mathbf{\Omega} = \mathbf{\Omega}_{k+1}$.

EXPERIMENTS AND DISCUSSION

➢Exact recovery of analysis operators.

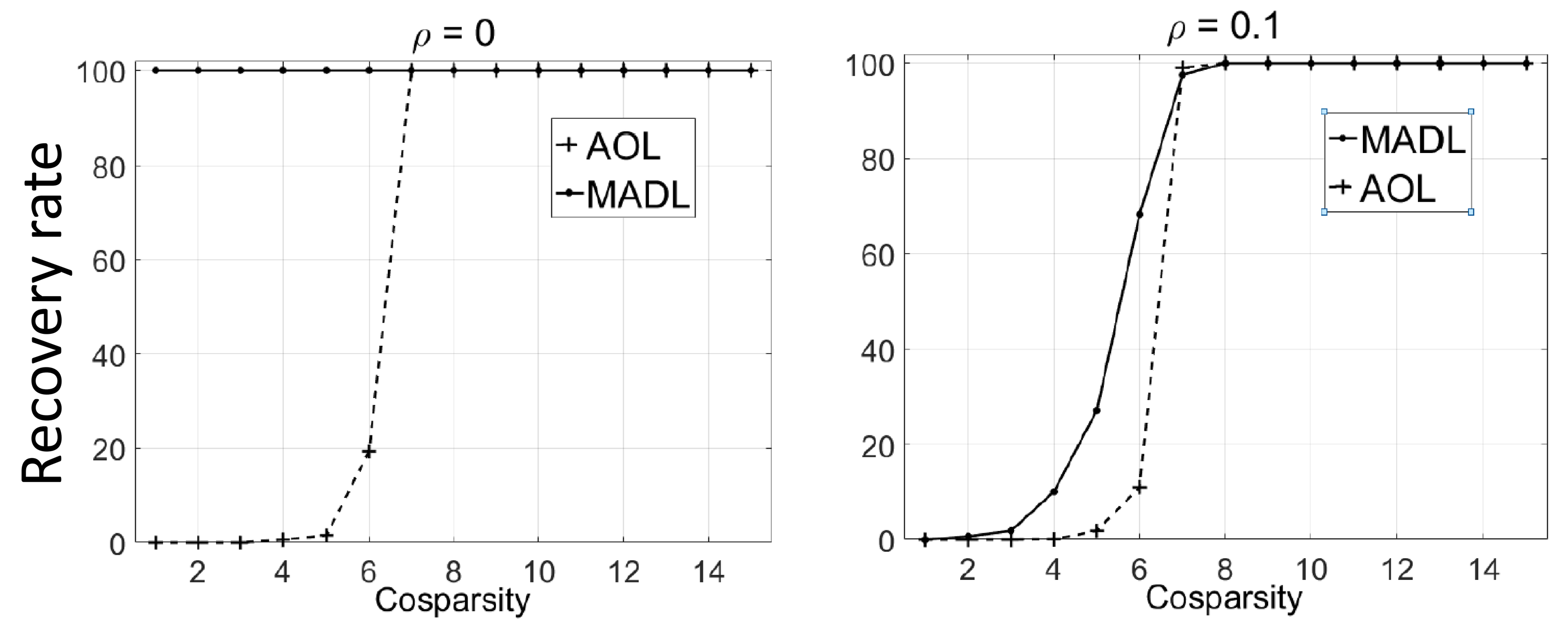
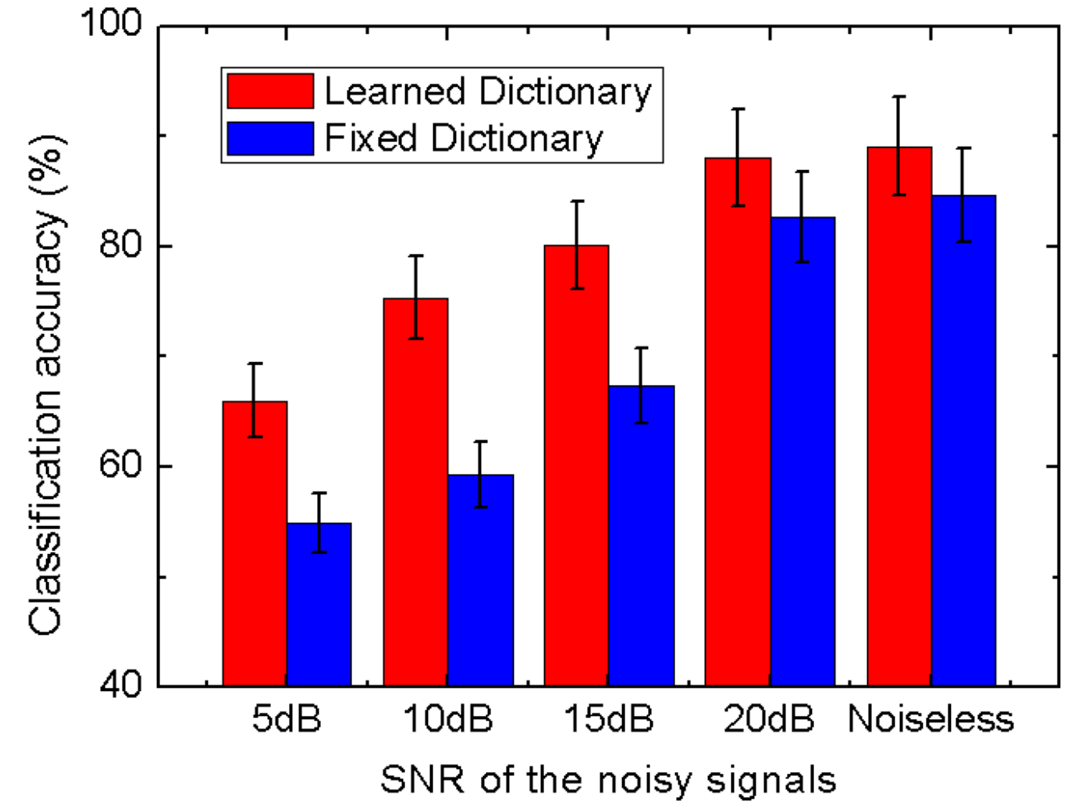


Fig. 3. The recovery curves of analysis dictionary compared with AOL.

We compared our algorithm MADL with the state-of-the-art algorithm Analysis operator learning (AOL) and the results is represented in Fig.3. We can see our algorithm can reach to a better recovery ratio even when the cosparsity is lower compared with AOL.

➢Classification with Learned Analysis Dictionary.



We applied our proposed algorithm to the classification, which is conducted on the database of USPS handwritten digits.

➢Image Denoising with Learned Analysis Dictionary.

Table 2. PSNR (dB) and running time of denoising results with different algorithms

	PSNR(dB)			Time(s)
Given noisy	12.56	16.13	28.17	-
Global-KSVD	**13.31**	**16.92**	30.31	**13.79**
Adaptive-KSVD	13.12	16.71	**30.41**	148.62
Global-AOL	**13.23**	**16.51**	29.91	**1962.67**
Adaptive-AOL	13.23	**16.51**	**29.94**	1990.57
Adaptive-AKSVD	13.22	16.49	29.69	4532.19
Global-MADL	13.22	16.50	29.59	2682.80
Adaptive-MADL	13.22	16.50	29.72	2864.90

CONCLUSION

- ➢we introduce manifold constraint for the dictionary learning algorithm for analysis sparse representation, which is parallel to the synthesis model in its rationale and structure.
- ➢To efficiently solve the orthogonality constraint in analysis dictionary learning formulation, project to Stiefel Manifold is an efficient method to solve this optimization problem.
- ➢Numerical experiments on recovery of analysis dictionary show the effectiveness of the proposed algorithm. In addition, for realistic applications, the proposed algorithms show good performances in signal denoising and classification.
- ➢However, with a wealth of mathematical and computer tools already developed, much work remains to be done. In the future, we will apply the proposed algorithm to more applications such as inpainting and deblurring.