Bayesian Learning based Millimeter-Wave Sparse Channel Estimation with Hybrid Antenna Arrays

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Motivation and Introduction

mmWave-Introduction:

- mmWvae one of the key ingredients for 5G
- Several GHz of spectrum (10 - 100 GHz) to provide Gbps data rates
- Leverage array gain

mmWave Channel Estimation:

- mmWave channel estimation is challenging
  - Large transmit and receive antenna arrays
  - Limited number of RF chains
- Beam training required: exhaustive search or bisection search
- Channel comprise of few dominant paths i.e. *sparse*
  - Compressive sensing and sparse signal recovery for CSI estimation

System and Channel model

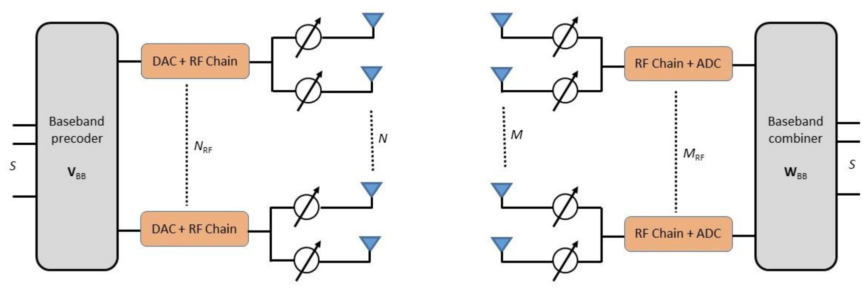


Figure: Hybrid Analog-digital Architecture.

mmWave Channel:

- The geometric channel model

$$\mathbf{H} = \sum_{\ell=1}^{L} \xi_\ell \mathbf{s}_R(\theta_\ell)\mathbf{s}_T^H(\phi_\ell)$$

Obtain: ξ_ℓ (*the complex channel gain*), θ_ℓ (*Angle of arrival*) and ϕ_ℓ (*Angle of departure*) for channel estimation

$$\mathbf{H} = \mathbf{S}(\Theta)\mathbf{H}^v\mathbf{S}(\Phi)^H$$

- Exploit sparsity in the angular domain
- $\mathbf{S}(\Theta)$ and $\mathbf{S}(\Phi)$ matrix of all resolvable directions Q ($Q \gg L$)
- $\mathbf{H}^v$ is (*jointly sparse*) consists of the complex channel gains

mmWave Channel Sensing:

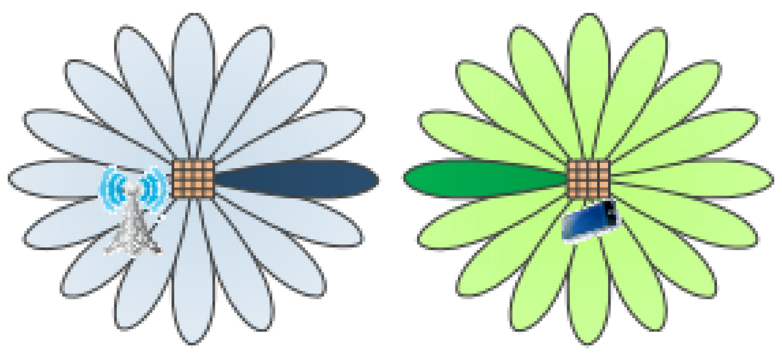


Figure: Exhaustive Search- a beam at a time[1]

- Stacking all D received vectors

$$\mathbf{y} = \left[\mathbf{z}_1^T, \ldots, \mathbf{z}_D^T\right]^T = \underbrace{\left(\mathbf{V}^T \mathrm{conj}(\mathbf{S}(\Phi)) \otimes \mathbf{W}^H\mathbf{S}(\Theta)\right)}_{\Psi}\mathbf{h} + \mathbf{n}$$

- $\mathbf{h} = \mathrm{vec}(\mathbf{H}^v) \in \mathbb{C}^{Q^2\times 1}$ is L sparse vector
- $\mathbf{y} \in \mathbb{C}^{D^2\times 1}$ is the received signal
- $\Psi \in \mathbb{C}^{D^2\times Q^2}$ is the sensing matrix

[1] Desai, Vip, et al. "Initial beamforming for mmWave communications. " Signals, Systems and Computers, on. IEEE 48th Asilomar Conference, 2014.

Bayesian Channel Estimation

Sparse Bayesian Learning:

- Goal: to find MAP estimate (Bayes Rule)
- Two-layer hierarchical prior model
- Fast SBL algorithm to solve the Type-II ML

Setup:

- MAP function

$$p(\mathbf{h}, \alpha, \sigma^2|\mathbf{y}) = p(\mathbf{h}|\mathbf{y}, \alpha, \sigma^2)p(\alpha, \sigma^2|\mathbf{y})$$

- The first term in the RHS

$$p(\mathbf{h}|\mathbf{y}, \alpha, \sigma^2) = \mathcal{CN}\Big(\underbrace{\sigma^{-2}\Sigma\Psi^H\mathbf{y}}_{\mu}, \underbrace{(\sigma^{-2}\Psi^H\Psi + \mathbf{A})^{-1}}_{\Sigma}\Big) \quad (1)$$

- Estimate α from the data: Type-II ML

$$p(\mathbf{y}|\alpha) = \mathcal{CN}\Big(\mathbf{0}, \underbrace{\sigma^2\mathbf{I} + \Psi\mathbf{A}^{-1}\Psi^H}_{\mathbf{C}}\Big)$$

- SBL cost function

$$\mathcal{L}(\alpha) = \log|\mathbf{C}| + \mathbf{y}^H\mathbf{C}^{-1}\mathbf{y}$$

- Isolating the effect of each α_m

$$\mathcal{L}(\alpha) = \mathcal{L}(\alpha_{-\mathbf{m}}) + \underbrace{\log\alpha_m - \log(\alpha_m + s_m) + \frac{|q_m|^2}{\alpha_m + s_m}}_{\ell(\alpha_m)} \quad (2)$$

- α_m^* that minimizes $\ell(\alpha_m^*)$

$$\alpha_m = \begin{cases} \dfrac{s_m}{(|q_m|^2 - s_m)} & \text{if } |q_m|^2 > s_m, \\ \infty & \text{if } |q_m|^2 \le s_m \end{cases} \quad (3)$$

- $s_m = \psi_m^H\mathbf{C}_{-m}^{-1}\psi_m$ and $q_m = \psi_m^H\mathbf{C}_{-m}^{-1}\mathbf{y}$

- **Fast SBL algorithm**:
  1. initialize α by choosing an m, compute α_m using (3)
  2. Compute Σ and μ from (1) (which are scalers initially)
  3. Select any i-th column vector of Ψ, say ψ_i
  4. If $|q_i|^2 > s_i$ and $\alpha_i < \infty$, (ψ_i is in the model), update α_i using (3)
  5. If $|q_i|^2 > s_i$, and $\alpha_i = \infty$, **add** ψ_i to the model, update α_i using (3)
  6. If $|q_i|^2 \le s_i$, and $\alpha_i < \infty$, **delete** ψ_i from the model, set $\alpha_i = \infty$
  7. If converged terminate, otherwise go to 2

Non-Bayesian Channel Estimation

LASSO Regression:

- Problem: ℓ_2 norm minimization problem
- ADMM algorithm to avoid matrix inversion in each iteration
- SCA to solve the problem

Formulation:

- ℓ_2 norm minimization problem

$$\underset{\mathbf{h}}{\text{minimize}} \ \|\mathbf{y} - \Psi\mathbf{h}\|_2^2 + \lambda\sum_{m}^{M}\log(|h_m| + \epsilon)$$

- ADMM reformulation

$$\underset{\mathbf{h},\mathbf{z}}{\text{minimize}} \ \|\mathbf{y} - \Psi\mathbf{h}\|_2^2 + \lambda\textstyle\sum_m \log(|z_m| + \epsilon)$$
$$\text{subject to } \mathbf{h} = \mathbf{z}$$

- Augmented Lagrangian

$$L_\rho(\mathbf{h}, \mathbf{z}, \mathbf{u}) = \lambda\textstyle\sum_i \log(|z_i| + \epsilon) + \|\mathbf{y} - \Psi\mathbf{h}\|_2^2 + \rho\|\mathbf{z} - \mathbf{h} + \mathbf{u}\|_2^2 - \rho\|\mathbf{u}\|_2^2.$$

- **ADMM algorithm**:
  1. initialize $\mathbf{z}^k$, $\mathbf{u}^k$ and $k = 0$
  2. Update $\mathbf{h}^{k+1} = \left(\Psi^H\Psi + \rho\mathbf{I}\right)^{-1}\left[\Psi^H\mathbf{y} + \rho\left(\mathbf{z}^k + \mathbf{u}^k\right)\right]$
  3. Update $\mathbf{z}^{k+1} = \mathbf{h}^{k+1} - \mathbf{u}^k - 2\lambda/\rho\left(\frac{\hat{\mathbf{z}}_m}{(|\hat{\mathbf{z}}|_m^2 + \epsilon)}\right)$
  4. Update $\mathbf{u}^{k+1} = \mathbf{u}^k + \mathbf{z}^{k+1} - \mathbf{h}^{k+1}$
  5. Set $k = k + 1$
  6. If converged terminate, otherwise go to 2

Numerical Examples

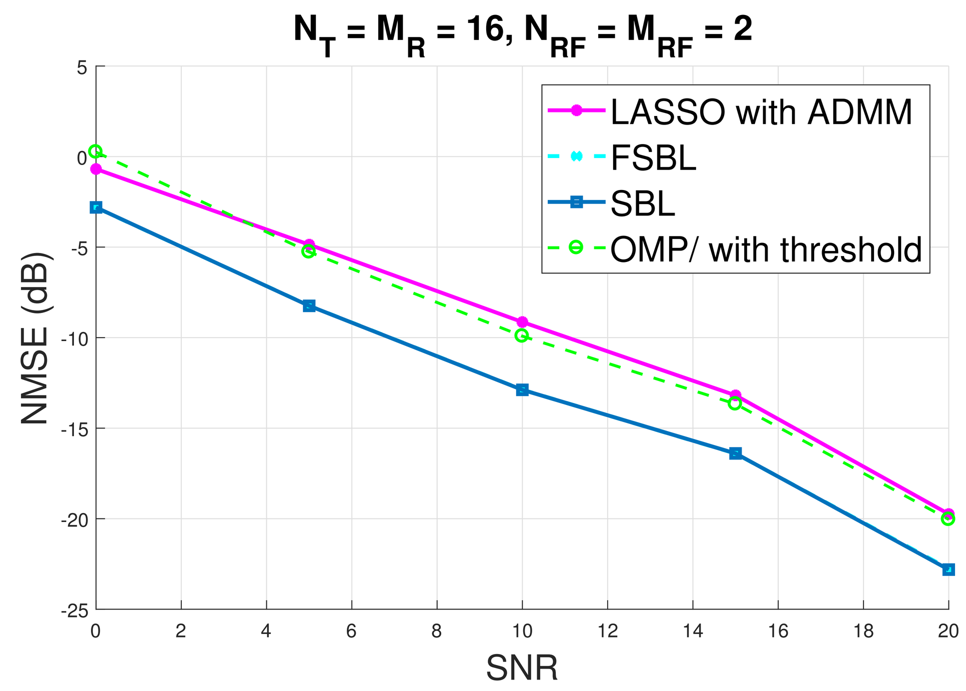
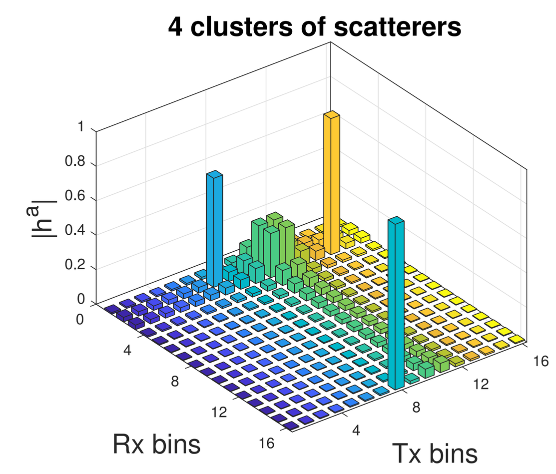
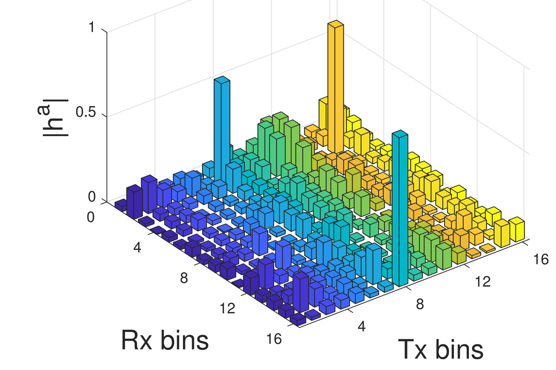


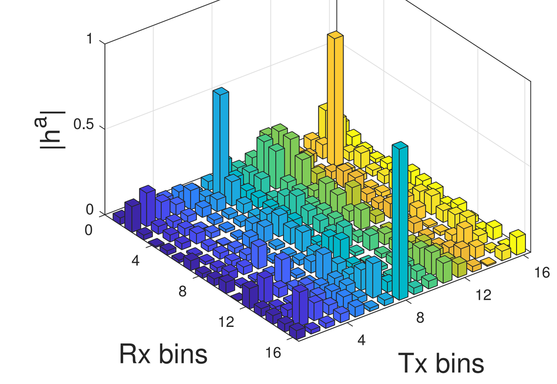
Figure: NMSE vs SNR plot



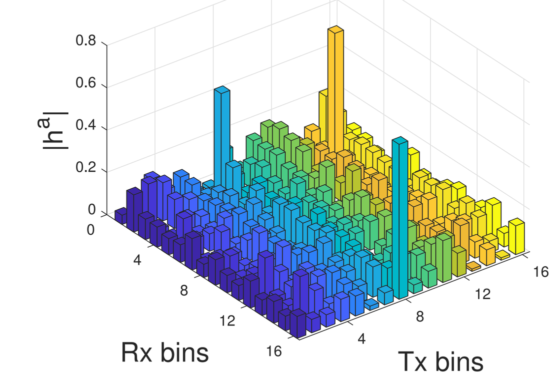
(a) original channel



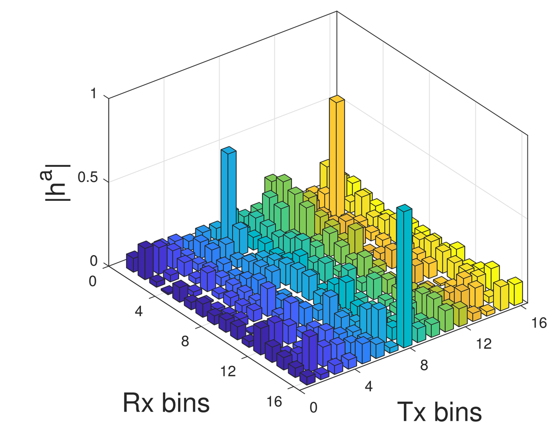
(b) FSBL



(c) SBL



(d) LASSO



(e) OMP

Figure: Angle domain representation of the channel at SNR = 3 dB.

Conclusion:

- Two channel estimation algorithms have been proposed: Bayesian (FSBL) and non-Bayesian (LASSO with ADMM)
- Future works:
  - Grid-less approach
  - Wideband Channel (Frequency Selective)
  - Tracking the AoDs, AoAs and path gains in a mobile scenario