# **DISTRIBUTED LINEAR BLIND SOURCE SEPARATION OVER WIRELESS SENSOR NETWORKS** WITH ARBITRARY CONNECTIVITY PATTERNS



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# INTRODUCTION

- Communication is performed by transmitting signals through a medium
- Signals can be mixed in the transport medium before they are received
- The operation of separating source signals without prior information about the sources is referred to as blind source separation
- Wireless sensor networks (WSN) form a natural platform for effective, low cost BSS
  - facilitate a good coverage of an area
  - low deployment cost
- In the context of sensor networks, research on blind source separation can be divided into:
  - centralized approaches
  - de-centralized approaches

# PROBLEM

- In centralized approaches, the recordings of all sensors (microphones) are transmitted to a centralized processor. Drawbacks:
  - lack of scalability
  - high power consumption
  - the need for hardware that can transmit over long distances
- Drawbacks of the existing de-centralized approaches include:
  - the need for a full connectivity pattern over the graph of the network
  - high computational load

# CONTRIBUTIONS

- Developing a distributed BSS algorithm with the following features:
  - applicable to any connected graph with any connectivity patterns
  - low computational load
  - low power consumption

**ADAPTIVE LINEAR BSS** 

• Linear generative model:

$$b_{N \times 1}(t) = A_{N \times M} s_{M \times 1}(t) + n_{N \times 1}(t)$$
 (1)

• Likelihood function:

$$\mathbf{p}(b|A) = \int \mathbf{p}(b|s, A) \,\mathbf{p}(s) ds \qquad (2$$

• Natural gradient learning rule:

$$\Delta A \propto A A^T \frac{\partial}{\partial A} \log \mathbf{p}(b|A) \tag{3}$$

or 
$$A(t+1) = A(t) - \mu A(t)F[y(t)]$$

where  $F[y(t)] = I - \psi[y(t)]y(t)^T$ ,  $y(t) = A^{-1}(t)b(t)$  and

$$\psi[y] = [\psi_1[y_1], \dots, \psi_M[y_M]]^T$$

$$\psi_i[y_i] = -\frac{d}{dy_i} \ln p(y_i)$$
(4)

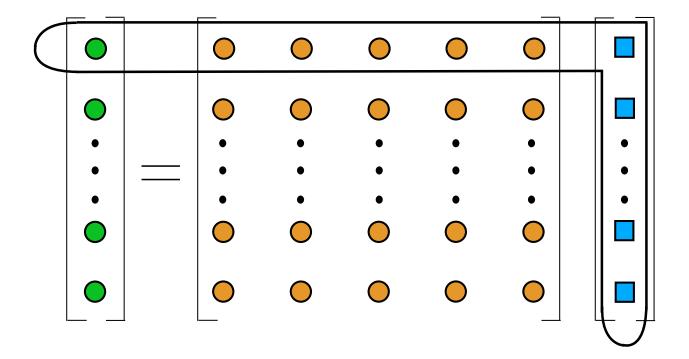
• A more numerically stable algorithm:

$$F[y(t)] = \Lambda(t) - \psi[y(t)]y(t)^T$$
(5)

where  $\Lambda(t) = \text{diag}[\text{diag}[\psi[y(t)]y(t)^T]]$ 

## **SPLIT OBJECTIVE FUNCTION**

- Finding  $y(t) = A^+(t)b(t)$  is equivalent to finding y that minimizes  $f(y) = \frac{1}{2} ||A(t)y - b(t)||_2^2$ Splitting the objective function on a row by row basis as  $f_i(y^{\{i\}}) = \frac{1}{2} \|A_i(t)y^{\{i\}} - b_i(t)\|_2^2$
- Visualization of b(t) = A(t)y(t) and data splitting:

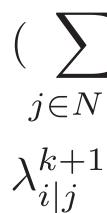


• Preserving the above splitting structure the update  $A(t+1) = A(t) - \mu A(t)F[y(t)]$  can be carried out in parallel on a row by row basis.

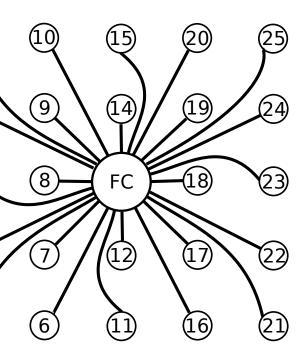
$${}_{\{i,k-i)}$$

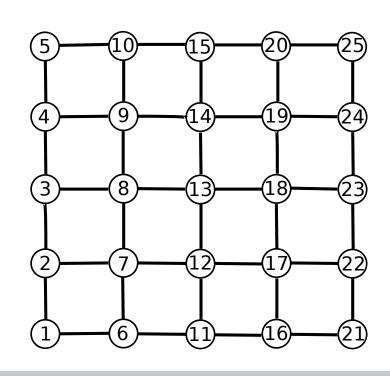
$$z^{k}$$

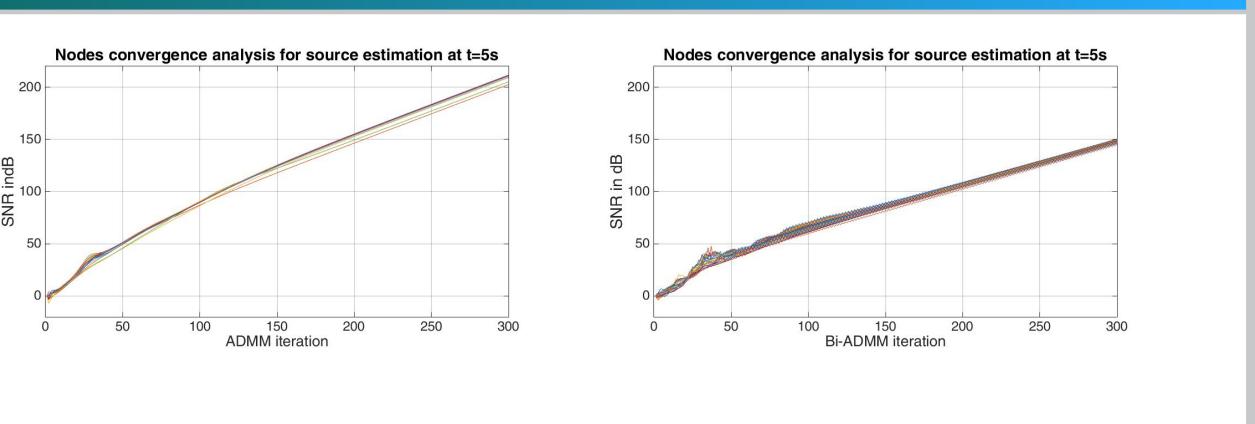
$$u_i^n$$



## **GRAPHICAL MODELS AND CONSENSUS ACHIEVEMENTS**







## **STRIBUTED PROCESSING**

- Distributed processing of source estimation:
- Fusion-center approach

$$\min_{y} \sum_{i=1}^{N} f_i(y^{\{i\}}) \quad \text{s.t.} \quad y^{\{i\}} - z = 0 \quad (6)$$

ADMM solution:

$$\begin{aligned} & +1 \} = \arg\min_{y^{\{i\}}} (f_i(y^{\{i\}}) + \frac{\rho}{2} || y^{\{i\}} - z^k + u_i^k ||_2^2) \\ & z^{k+1} = \frac{1}{N} \sum_{i=1}^N (y^{\{i,k+1\}} + u_i^k) \\ & z^{k+1} = u_i^k + y^{\{i,k+1\}} - z^{k+1} \end{aligned}$$

$$(7)$$

• De-centralized approach Over a graph  $G = (\nu, \varepsilon)$ 

$$\min_{y} \sum_{i \in \nu} f_i(y^{\{i\}}) \text{ s.t. } y^{\{i\}} = y^{\{j\}} \,\forall (i,j) \in \varepsilon$$
(8)

### **Bi-ADMM** solution:

$$\begin{aligned} &+1\} = \arg\min_{y^{\{i\}}} [f_i(y^{\{i\}}) - y^{\{i\}}^T \\ &\sum_{(i)} \operatorname{sign}(j-i)\lambda_{j|i}^k) + \sum_{j \in N(i)} \frac{1}{2} ||y^{\{i\}} - y^{\{j,k\}}||_2^2 \\ &= \lambda_{j|i}^k + \gamma \operatorname{sign}(j-i)(y^{\{j,k\}} - y^{\{i,k+1\}}) \end{aligned}$$

$$(9)$$

- 2. Distributed processing of the parameter update:
  - Both fusion-center and de-centralized approach update the parameters locally as:

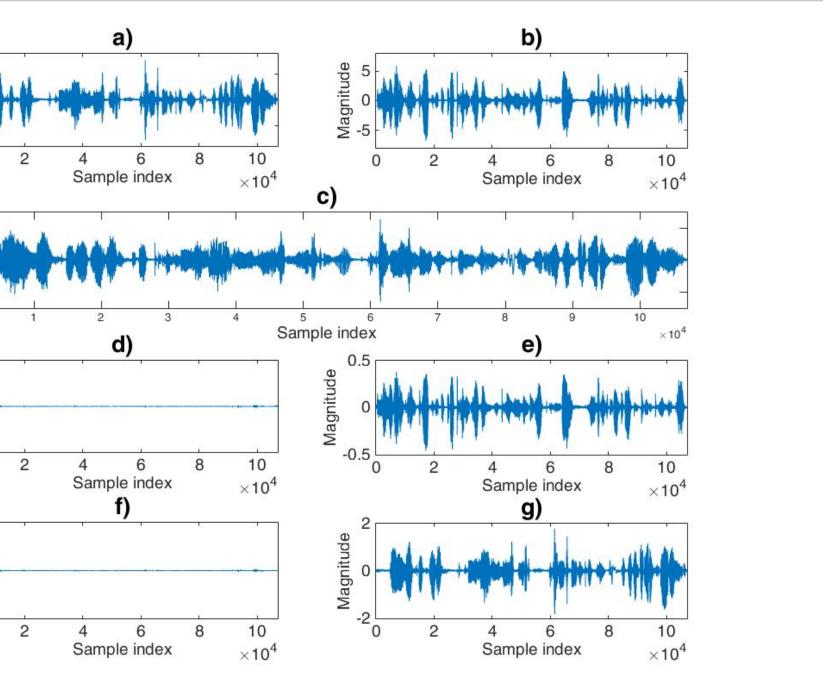
$$A_i(t+1) = A_i(t) - \mu A_i(t) F[y^{\{i,k\}}(t)]$$
(10)

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## H SIGNALS SEPARATION



### **CONSUMPTION**

	$1 \times 10^{-4}$	$4 \times 10^{-6}$	$3 \times 10^{-7}$	$1 \times 10^{-8}$
ed	$375\epsilon$	$675\epsilon$	$1025\epsilon$	$1825\epsilon$
er	$1296\epsilon$	$2376\epsilon$	$3240\epsilon$	$5400\epsilon$

## **ACROSS NOISE LEVELS**

	Fusion-center		De-centralized		Centralized	
$\frac{2}{r}$	S1	S2	S1	S2	S1	S2
	59.39	21.73	49.80	21.60	49.98	21.71
$0^{-3}$	22.65	15.21	22.63	15.20	22.59	14.94
$0^{-3}$	19.79	11.77	19.79	11.76	19.82	12.02
$0^{-3}$	18.08	9.84	18.09	10.01	18.04	10.31
$0^{-3}$	16.83	8.39	16.84	8.51	16.82	9.16

## LUSIONS

distributed adaptive linear BSS algons were proposed

- y benefit from fully shared computation de-centralized algorithm:
- can be implemented over any graph is scalable
- requires low transmission power