Decentralized Coordination of Energy Resources in Electricity Distribution Networks

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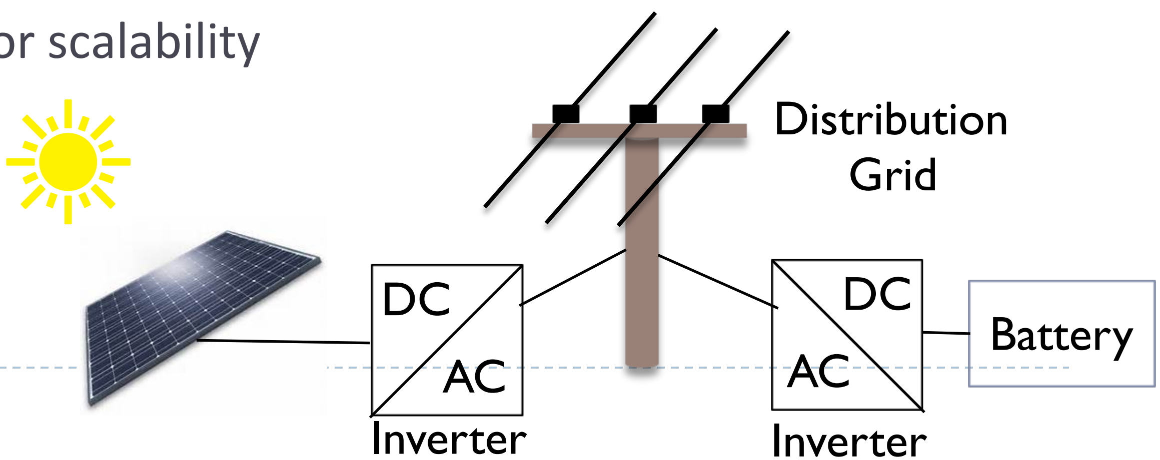
Opportunities in distribution systems

- Real power consumption of programmable loads
- Reactive power generated or consumed by photovoltaic (PV) inverters
- Distributed storage: charge/discharge and reactive power support

- Objectives
  - Thermal loss minimization, voltage regulation, end-user satisfaction
- Challenges
  - Power flow models are nonconvex
  - Uncertainty in renewable distributed generation (DG)
  - Distributed algorithms for scalability



Prior art and contributions

- Distributed algorithms for reactive power compensation by PV units; coordination with controllable loads

  [Turitsyn et al. ’10-’11] [Li et al. ’12] [Lam et al. ’12] [Bolognani et al. ’13] [Šulc et al. ’14]

  [Dall’Anese et al. ’14] [Peng-Low ’15] [Bazrafshan-Gatsis ’16]

- Coupling of storage with renewable energy
  - No network constraints ➔ no reactive power support or voltage constraints

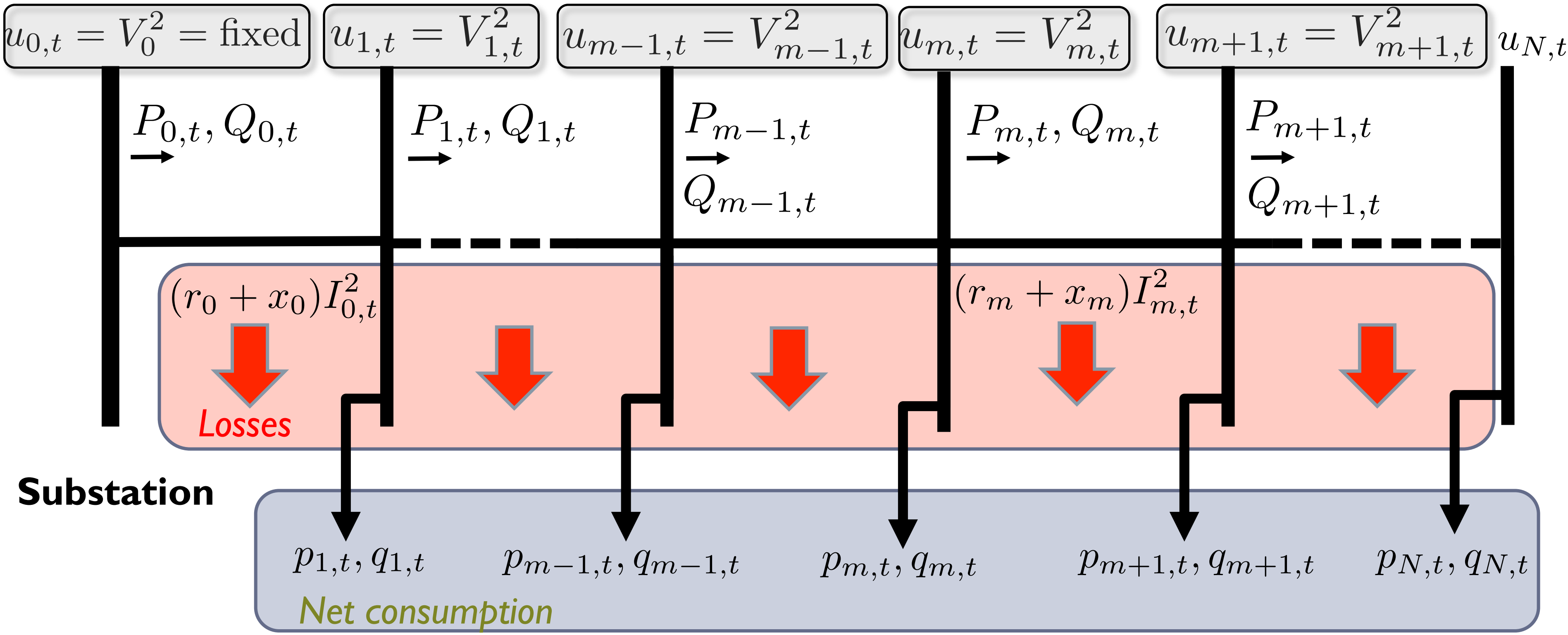
  [Marques-Gatsis ’14] [Lakshminarayana et al. ’14] [Rahbar et al. ’15] [Sun et al. ’15]

- Storage in transmission networks

  [Gayme-Topcu ’11] [Lamadrid ’15]

- **This work:** Optimal scheduling and coordination of 3 resource types
  - Controllable loads (real power), PV units (reactive power), and storage (real and reactive power)
  - Decentralized solver based on ADMM, closed-form updates

Simplified DistFlow equations



Approximations

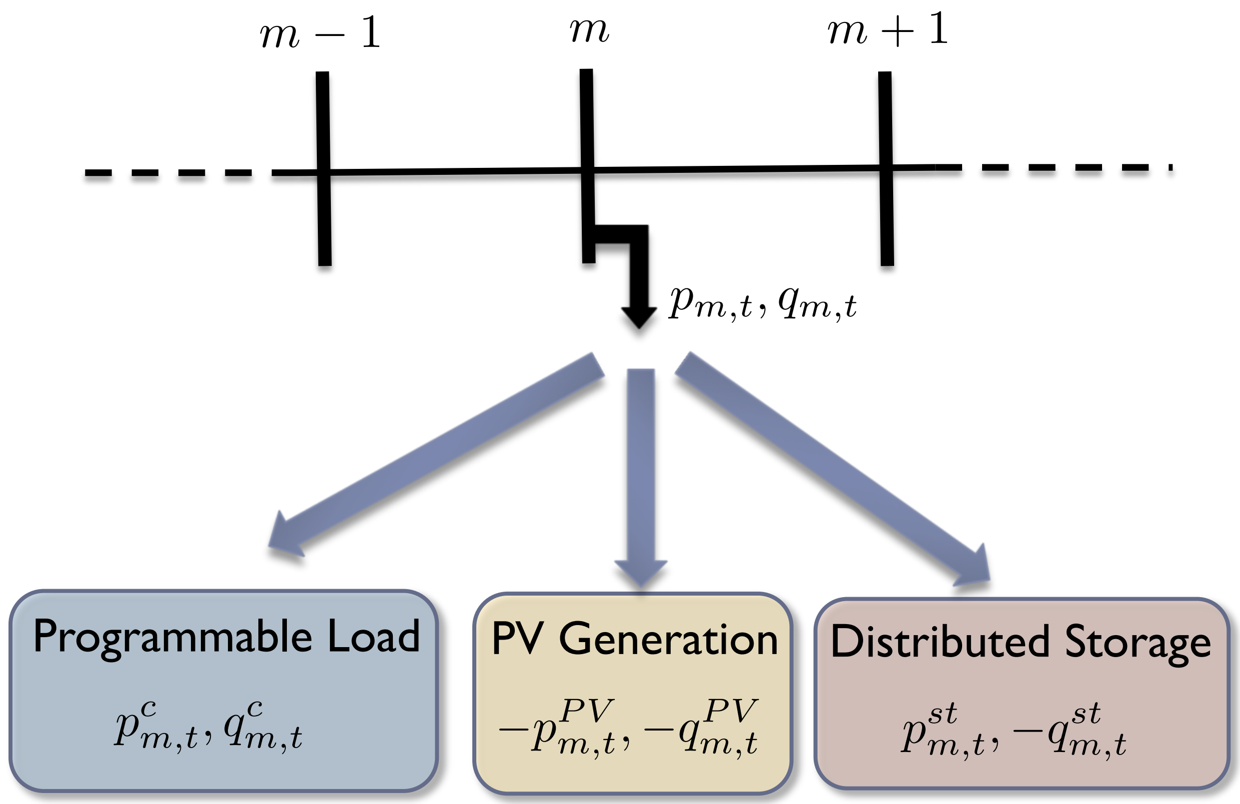
1. Losses negligible
2. Voltage drop very small

[Baran-Wu '89]

$$P_{m-1,t} = P_{m,t} + p_{m,t} \quad (m = 0, \ldots, N-1;\ t = 1, \ldots, T)$$
$$Q_{m-1,t} = Q_{m,t} + q_{m,t}$$
$$u_{m+1,t} = u_{m,t} - 2(r_m P_{m,t} + x_m Q_{m,t})$$

$$u_{0,t} = V_0^2;\ P_{N,t} = Q_{N,t} = 0 \quad (t = 1, \ldots, T)$$

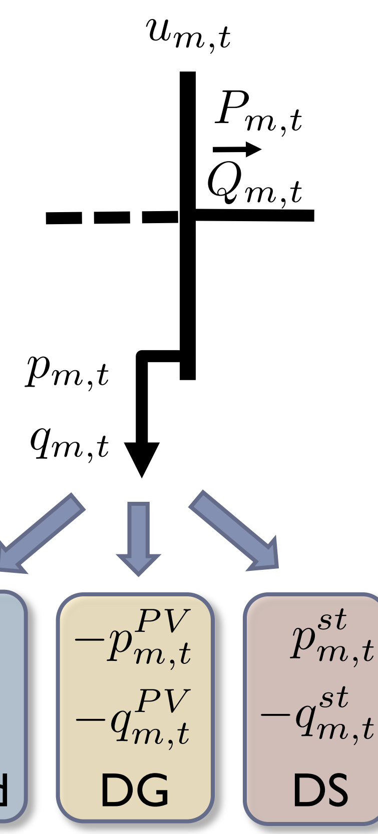
Distributed Energy Resources



User consumption model

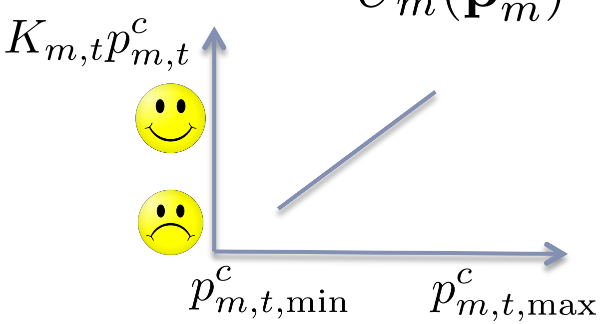
- Real and reactive power consumption $p^c_{m,t}, q^c_{m,t}$
- Real power limits $p^c_{m,t,\min} \leq p^c_{m,t} \leq p^c_{m,t,\max}$
- Reactive power determined through the power factor

$$q^c_{m,t} = p^c_{m,t}\sqrt{\frac{1}{\mathrm{PF}^2_m} - 1}$$



- Concave utility function models user happiness, e.g.,

$$U_m(\mathbf{p}^c_m) = \sum_{t=1}^{T} K_{m,t}p^c_{m,t}$$



PV injection model

- Maximum real and apparent power capacities $p^{PV}_{m,\max}, S^{PV}_{m,\max}$

$$p^{PV}_{m,t} \leq p^{PV}_{m,\max} \quad (m = 1, \ldots, N;\ t = 1, \ldots, T)$$

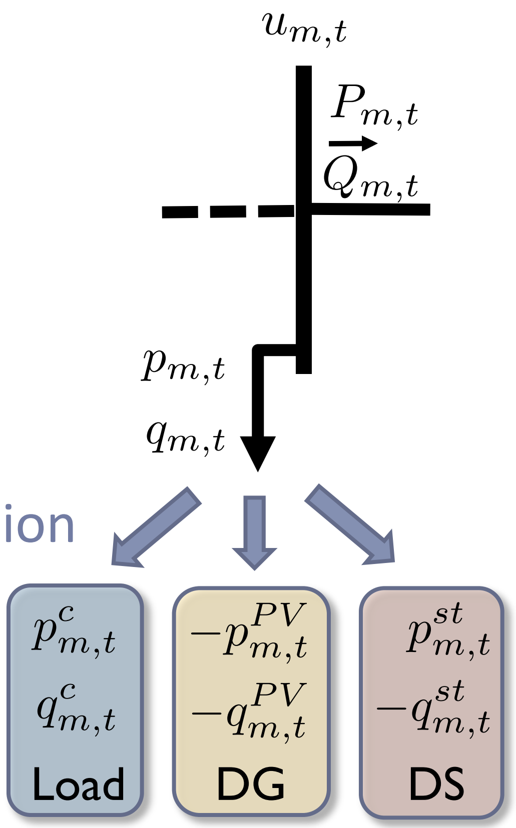
- Inverter sizing to effect reactive power control

$$S^{PV}_{m,\max} > p^{PV}_{m,\max}$$

[Turitsyn, Šulc, Backhaus, Chertkov '10-'11]

- Real powers $\{p^{PV}_{m,t}\}_{m,t}$ assumed known
  - Forecasted in advance of horizon $\{1, \ldots, T\}$
  - Intra-hour solar forecasting is rapidly developing
- Reactive power $q^{PV}_{m,t}$ *generated or consumed*: decision

$$|q^{PV}_{m,t}| \leq \sqrt{(S^{PV}_{m,\max})^2 - (p^{PV}_{m,t})^2}$$

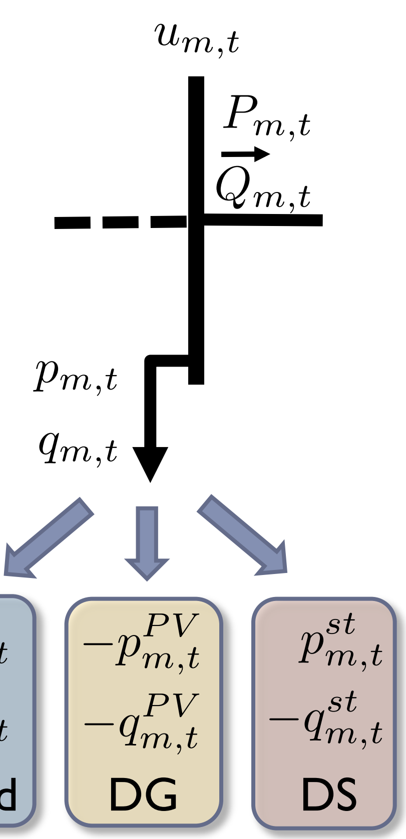


Storage model

- Charge or discharge with limits $-p^{st}_{m,\max} \leq p^{st}_{m,t} \leq p^{st}_{m,\max}$
- Time slot duration δ; energy stored in the beginning of slot $b_{m,t}$

$$b_{m,t+1} = b_{m,t} + \delta p^{st}_{m,t}$$

- Initial condition $b_{m,1}$ known
- Storage capacity limit $0 \leq b_{m,t} \leq b_{m,\max}$
- Terminal constraint $b_{m,T+1} \geq \underline{b}_m$



- Storage inverter sizing for reactive power support

$$S^{st}_{m,\max} > p^{st}_{m,\max}$$

- Reactive power provided by storage unit $q^{st}_{m,t}$

$$(p^{st}_{m,t})^2 + (q^{st}_{m,t})^2 \leq (S^{st}_{m,\max})^2$$

Coordination of DERs

−User Utility | Cost of power procurement | Thermal losses

$$\min \quad -\sum_{m=1}^{N} U_m(\mathbf{p}_m^c) + \sum_{t=1}^{T} \text{Cost}(P_{0,t}) + \sum_{t=1}^{T}\sum_{m=0}^{N-1} \frac{r_m}{V_0^2}\left[(P_{m,t})^2 + (Q_{m,t})^2\right]$$

Power flow equations

$$P_{m+1,t} = P_{m,t} - (p_{m+1,t}^c - p_{m+1,t}^{PV} + p_{m+1,t}^{st})$$

$$Q_{m+1,t} = Q_{m,t} - \left(p_{m+1,t}^c\sqrt{(1/\text{PF}_m^2) - 1} - q_{m+1,t}^{PV} - q_{m+1,t}^{st}\right)$$

$$u_{m+1,t} = u_{m,t} - 2(r_m P_{m,t} + x_m Q_{m,t})$$

Voltage regulation

$$V_0^2(1-\epsilon)^2 \le u_{m,t} \le V_0^2(1+\epsilon)^2$$

Storage dynamical equation

$$b_{m,t+1} = b_{m,t} + \delta p_{m,t}^{st}$$

Storage energy limits

$$0 \le b_{m,t} \le b_{m,\max}$$

$$\underline{b}_m \le b_{m,T+1} \le b_{m,\max}$$

Reactive power limits

$$-\sqrt{(S_{m,\max}^{PV})^2 - (p_{m,t}^{PV})^2} \le q_{m,t}^{PV} \le \sqrt{(S_{m,\max}^{PV})^2 - (p_{m,t}^{PV})^2}$$

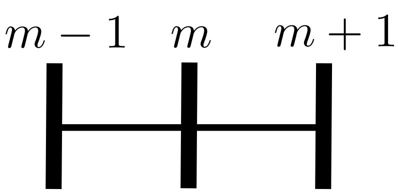
$$(p_{m,t}^{st})^2 + (q_{m,t}^{st})^2 \le (S_{m,\max}^{st})^2$$

Real power limits

$$p_{m,t,\min}^c \le p_{m,t}^c \le p_{m,t,\max}^c$$

$$-p_{m,\max}^{st} \le p_{m,t}^{st} \le p_{m,\max}^{st}$$

Decentralization: Auxiliary variables



$$\hat{P}_{m-1,t} = P_{m,t} + (p^c_{m,t} - p^{PV}_{m,t} + p^{st}_{m,t})$$

$$\hat{Q}_{m-1,t} = Q_{m,t} + (p^c_{m,t}\sqrt{\frac{1}{\mathrm{PF}^2} - 1} - q^{PV}_{m,t} - q^{st}_{m,t})$$

$$\hat{u}_{m+1,t} = u_{m,t} - 2(r_m P_{m,t} + x_m Q_{m,t})$$

$$b_{m,t+1} = b_{m,t} + \delta p^{st}_{m,t}$$

$$(t = 1, \ldots, T)$$

"Estimates" maintained at node m

Variables $\mathbf{x}_m$

Linear equality constraints
$\mathbf{C}_m \mathbf{x}_m = \mathbf{d}_m$

$$\tilde{P}_{m,t} = P_{m,t}, \quad \tilde{P}_{m,t} = \hat{P}_{m,t}$$

$$\tilde{Q}_{m,t} = Q_{m,t}, \quad \tilde{Q}_{m,t} = \hat{Q}_{m,t}$$

$$\tilde{u}_{m,t} = u_{m,t}, \quad \tilde{u}_{m,t} = \hat{u}_{m,t}$$

$$\tilde{p}^c_{m,t} = p^c_{m,t}$$

$$\tilde{p}^{st}_{m,t} = p^{st}_{m,t}$$

$$\tilde{q}^{st}_{m,t} = q^{st}_{m,t}$$

$$\tilde{q}^{PV}_{m,t} = q^{PV}_{m,t}$$

Variables $\mathbf{z}_m$

Coupling constraints
$\mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m \mathbf{z}_m = 0$

Box constraints
$\mathbf{l}_m \leq \mathbf{z}_m \leq \mathbf{u}_m$
for most variables

e.g., $V_0^2(1-\epsilon)^2 \leq \tilde{u}_{m,t} \leq V_0^2(1+\epsilon)^2$

Objective

$$\text{minimize} \quad \sum_{m=0}^{N} [f_m(\mathbf{x}_m) + g_m(\mathbf{z}_m)]$$

Closed-form ADMM updates

- $\mathbf{x}_m$-update: Linear equality constrained quadratic program (QP)
- $\mathbf{z}_m$-update: Two cases
  1. For all variables except $\tilde{p}^{st}_{m,t}, \tilde{q}^{st}_{m,t}$, single-variable QP with a box constraint
  2. For storage control variables $\tilde{p}^{st}_{m,t}, \tilde{q}^{st}_{m,t}$, QCQP in 2 variables

$$\text{minimize}_{\tilde{p}^{st}_{m,t}, \tilde{q}^{st}_{m,t}} \quad \frac{\rho}{2}(\tilde{p}^{st}_{m,t})^2 + \frac{\rho}{2}(\tilde{q}^{st}_{m,t})^2 - \tilde{p}^{st}_{m,t}(\rho p^{st}_{m,t} + \eta_{m,t}) - \tilde{q}^{st}_{m,t}(\rho q^{st}_{m,t} + \nu_{m,t})$$

subject to:

$$(\tilde{p}^{st}_{m,t})^2 + (\tilde{q}^{st}_{m,t})^2 \leq (S^{st}_{m,\max})^2$$

$$-p^{st}_{m,\max} \leq \tilde{p}^{st}_{m,t} \leq p^{st}_{m,\max}$$

x-variables fixed for z-update

ADMM Multipliers

Multiplier κ

Multipliers $\underline{\kappa}, \bar{\kappa}$

Write the KKT conditions, and check the 8 combinations of Lagrange multipliers $\kappa, \underline{\kappa}, \bar{\kappa}$ being 0 or not

$$(\rho + 2\kappa)\tilde{p}^{st}_{m,t} + \bar{\kappa} - \underline{\kappa} = \rho p^{st}_{m,t} + \eta$$

$$(\rho + 2\kappa)\tilde{q}^{st}_{m,t} = \rho q^{st}_{m,t} + \nu$$

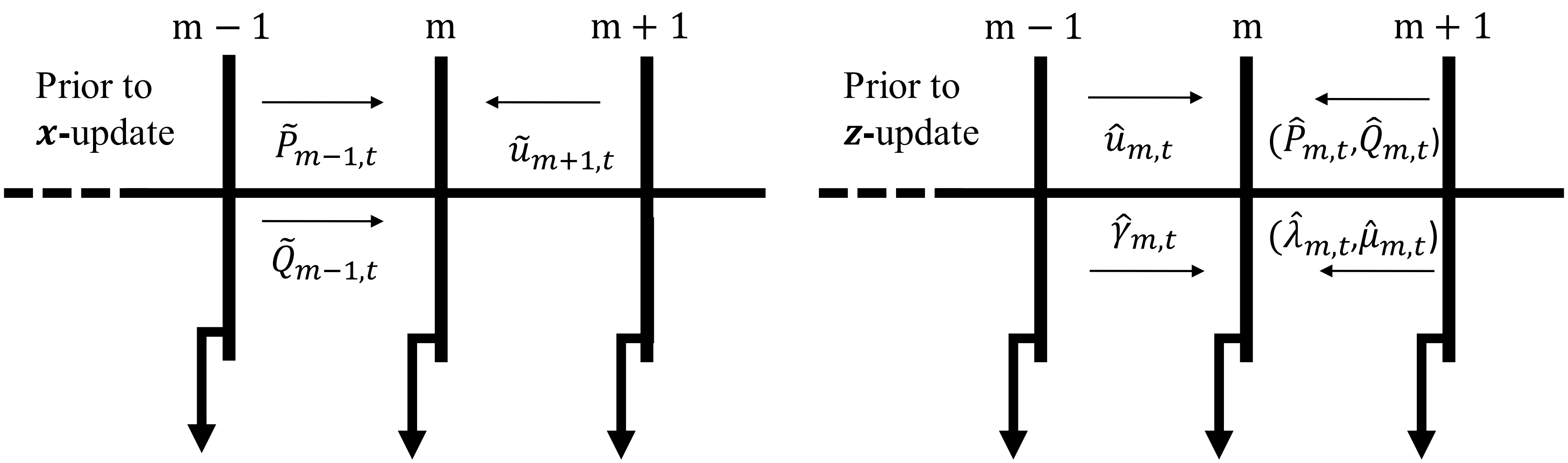
$$\kappa[(\tilde{p}^{st}_{m,t})^2 + (\tilde{q}^{st}_{m,t})^2 - (S^{st}_{m,\max})^2] = 0$$

$$\bar{\kappa}(-p^{st}_{m,\max} + \tilde{p}^{st}_{m,t}) = 0$$

$$\underline{\kappa}(-p^{st}_{m,\max} - \tilde{p}^{st}_{m,t}) = 0$$

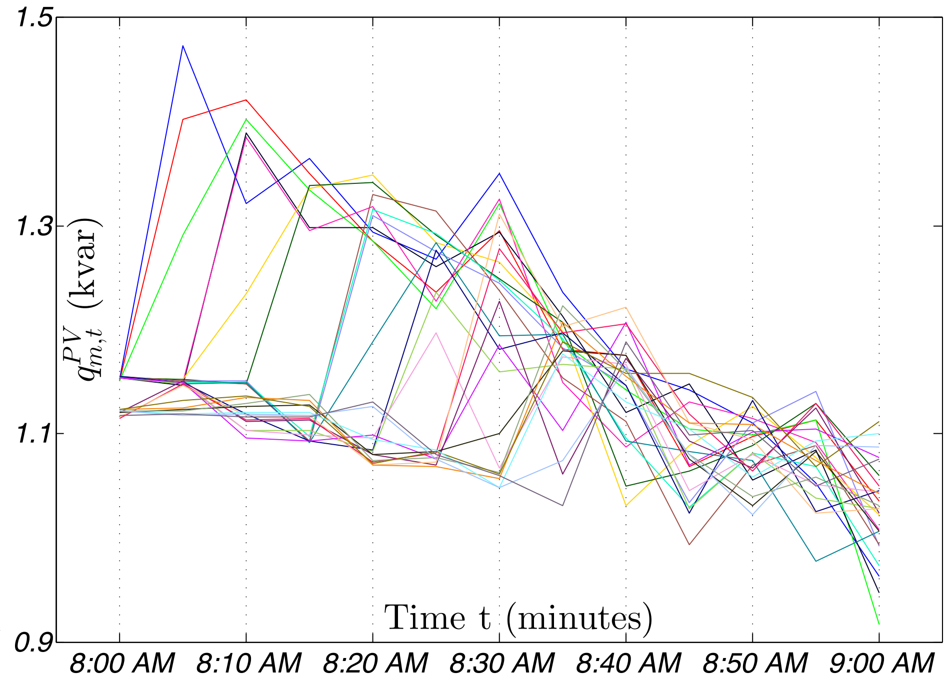
Algorithm merits

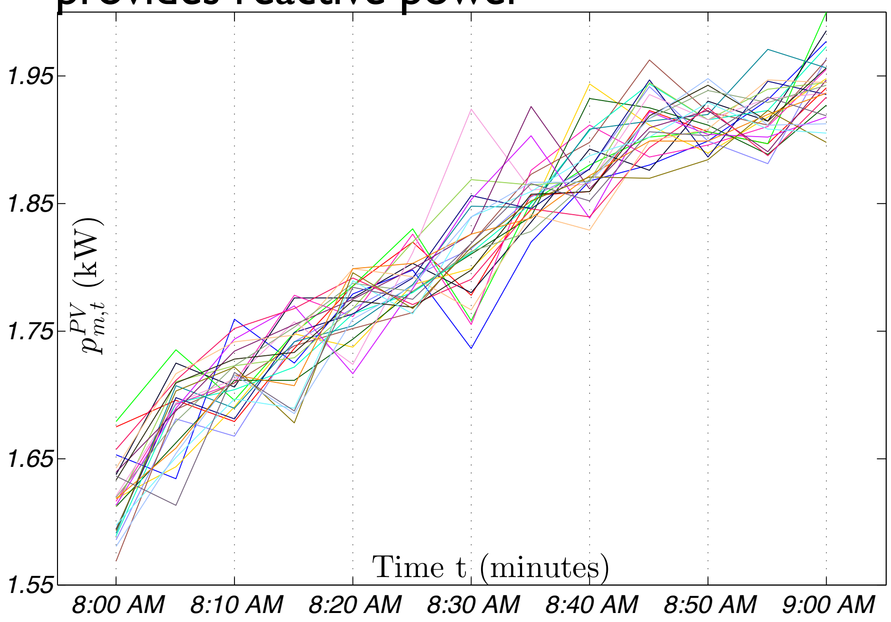
- Updates decouple *per node*
  - z-update also parallelized across t
- Updates are *in closed form*
- Communication between neighboring nodes only
- User parameters $U_m(\mathbf{p}_m)$, $p^c_{m,t,\min}$, $p^c_{m,t,\max}$, PF_m remain with node m

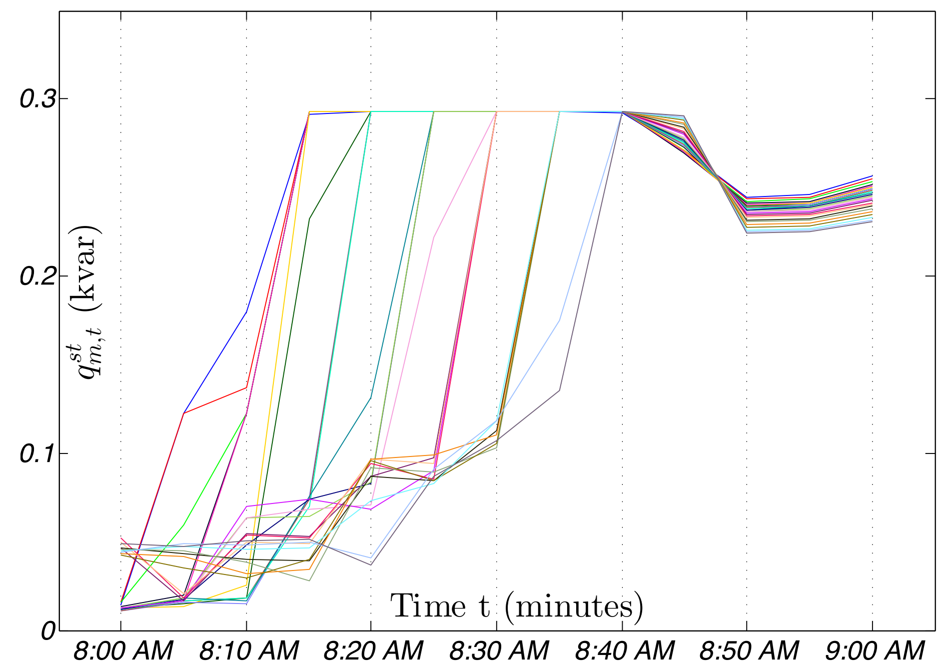


Numerical tests

- Network with $N = 25$ nodes
- Randomized PV profile from NREL data (April 4, 2006); $\delta = 5 \text{ min}$
- $K_{m,t} \uparrow$ in time $\Rightarrow p^{c}_{m,t} \uparrow$
- Increasing reactive power provided by PV units in the beginning to account for increasing reactive power load
- As PV generation increases, reactive power capability of PV decreases, and storage provides reactive power







Summary

- Coordination of DERs in distribution networks
  - Real power from programmable loads
  - Reactive power from PV inverters
  - Storage charge/discharge and reactive power support
  - Decentralized algorithm based on ADMM; closed-form updates

- Future directions
  - Impact of storage lifetime models on optimal scheduling
  - Accounting for uncertainty in solar generation

Thank you!

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