Multilevel Distributed Approach for DC Optimal Power Flow

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Dec 2015

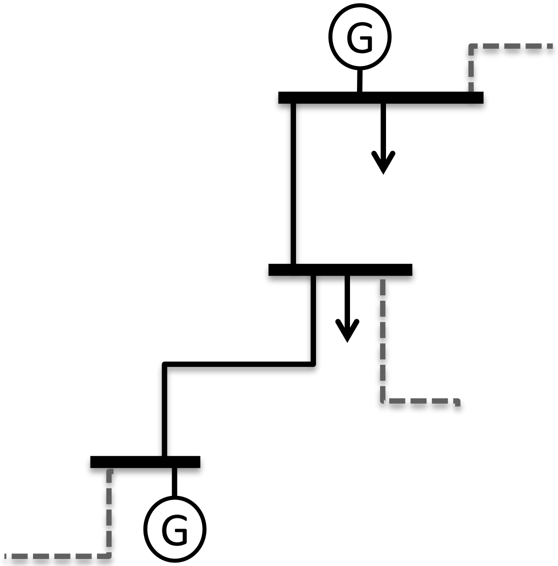
GlobalSIP 2015

Outline

- **Motivation and Background**
- Multilevel Distributed Optimal Power Flow
- Simulation Results
- Summary

Motivation

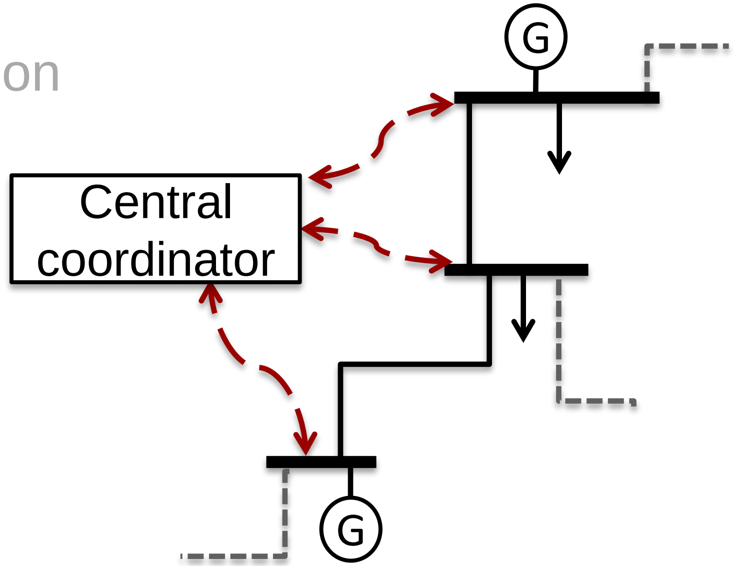
- Optimal Power Flow (OPF)



Motivation

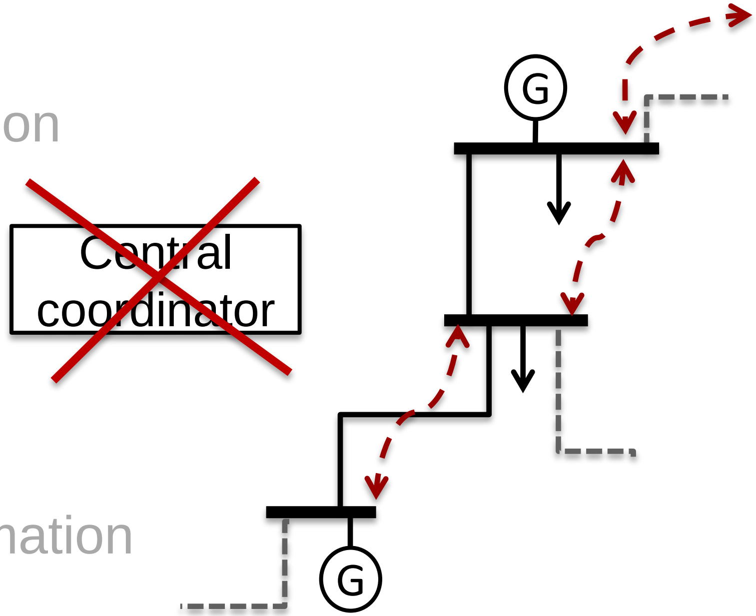
- Optimal Power Flow (OPF)

- Traditional/Centralized
  - Requires complete set of information



Motivation

- Optimal Power Flow (OPF)
- Traditional/Centralized
  - Requires complete set of information
- Distributed
  - No need for complete set of information



Existing work

- Decomposition based methods[1,2]
  - Each distributed agent in the system has to solve an optimization problem.

- Consensus based methods[3,4]
  - Each distributed agent evaluates a computationally cheap update function
  - Enabling multilevel distributed computation

[1]: T. Erseghe, "Distributed optimal power flow using ADMM," IEEE Trans. Power Syst., vol. 29, no. 5, pp. 2370–2380, Sep. 2014.
[2]: M. Kraning, E. Chu, J. Lavaei, and S. Boyd. "Dynamic network energy management via proximal message passing". Foundations and Trends in Optimization, 1(2):1–54, 2013.
[3]:Ziang Zhang and Mo-Yuen Chow. "Convergence analysis of the incremental cost consensus algorithm under different communication network topologies in a smart grid." Power Systems, IEEE Transactions on, 27(4):1761–1768, 2012.
[4]: S. Kar, G. Hug, J. Mohammadi, and J.M.F. Moura. "Distributed state estimation and energy management in smart grids: A consensus+ innovations approach." Selected Topics in Signal Processing, IEEE Journal of, 8(6):1022–1038, Dec 2014.

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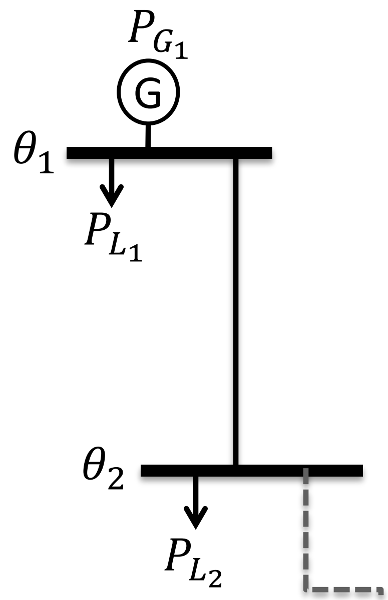
DC-OPF

- Optimal Power Flow (OPF):
  - Finding the lowest cost generation dispatch
  - And ensuring no line flow violations



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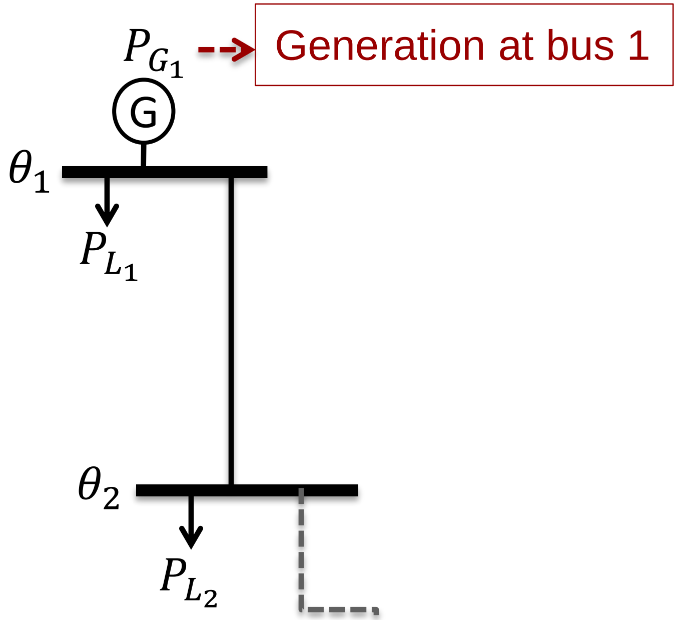

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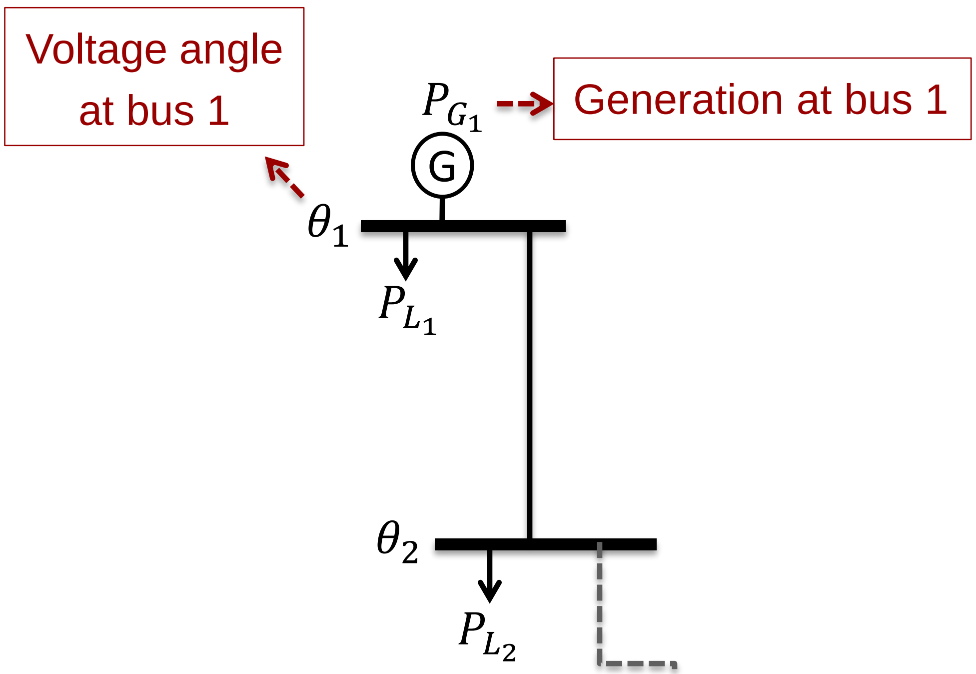

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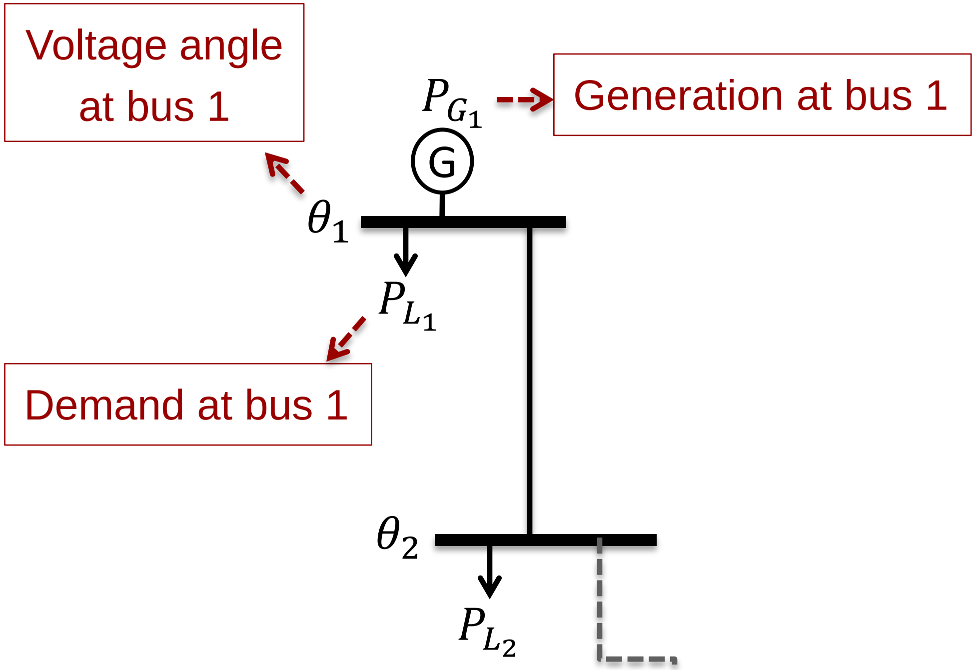

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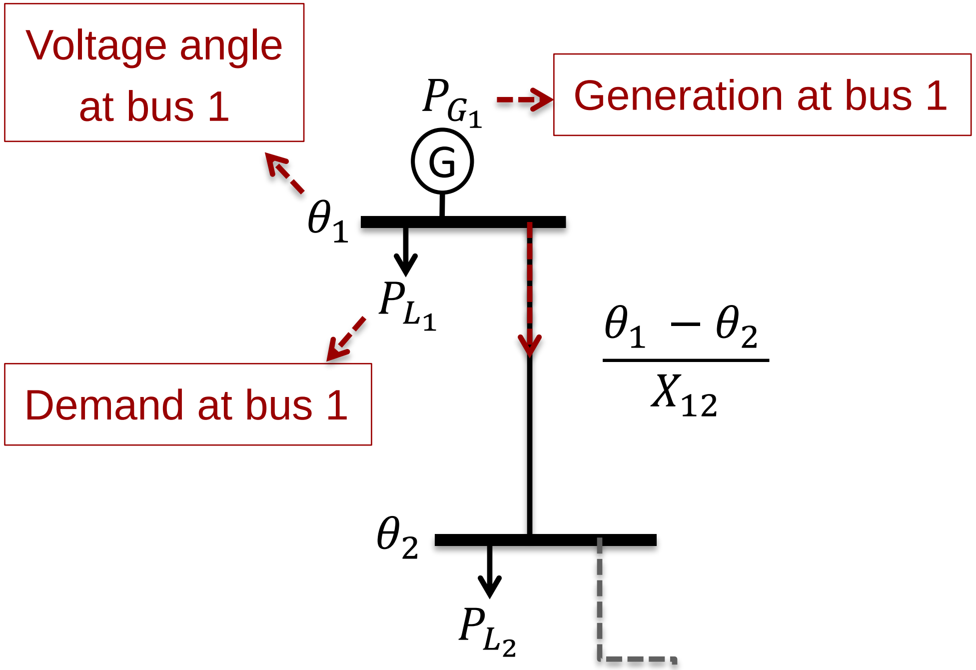

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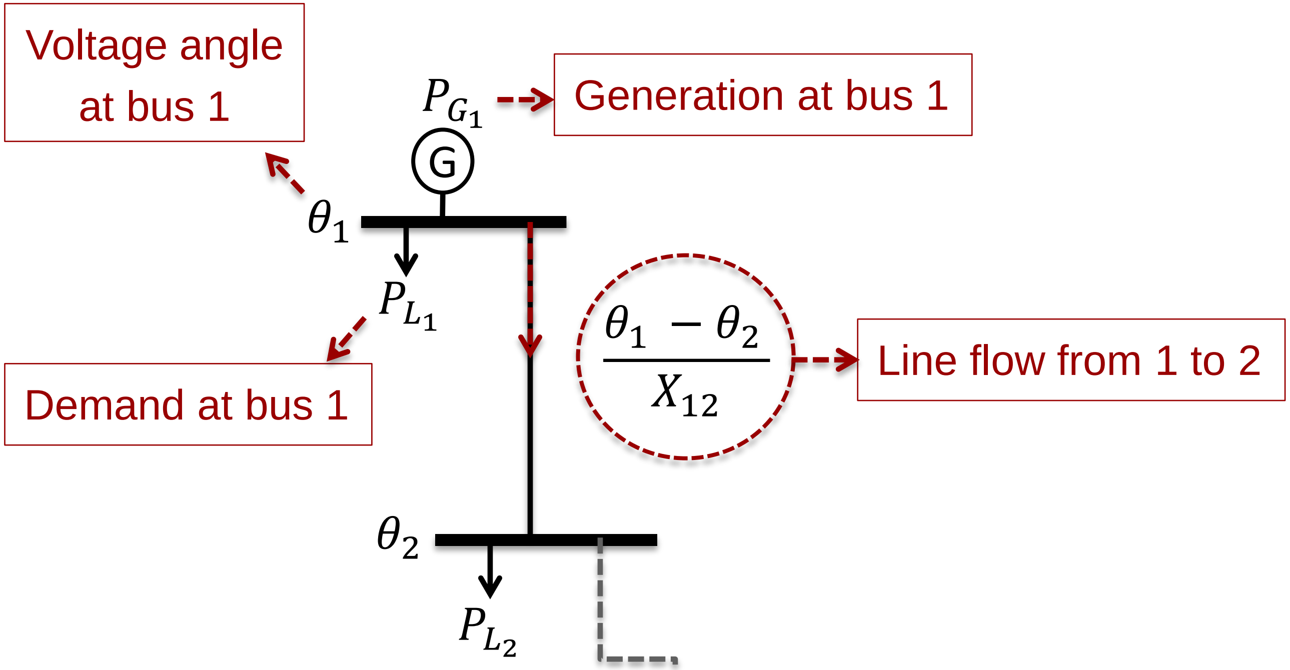

P_{G_1}, θ_1, P_{L_1}, $\frac{\theta_1 - \theta_2}{X_{12}}$, θ_2, P_{L_2}

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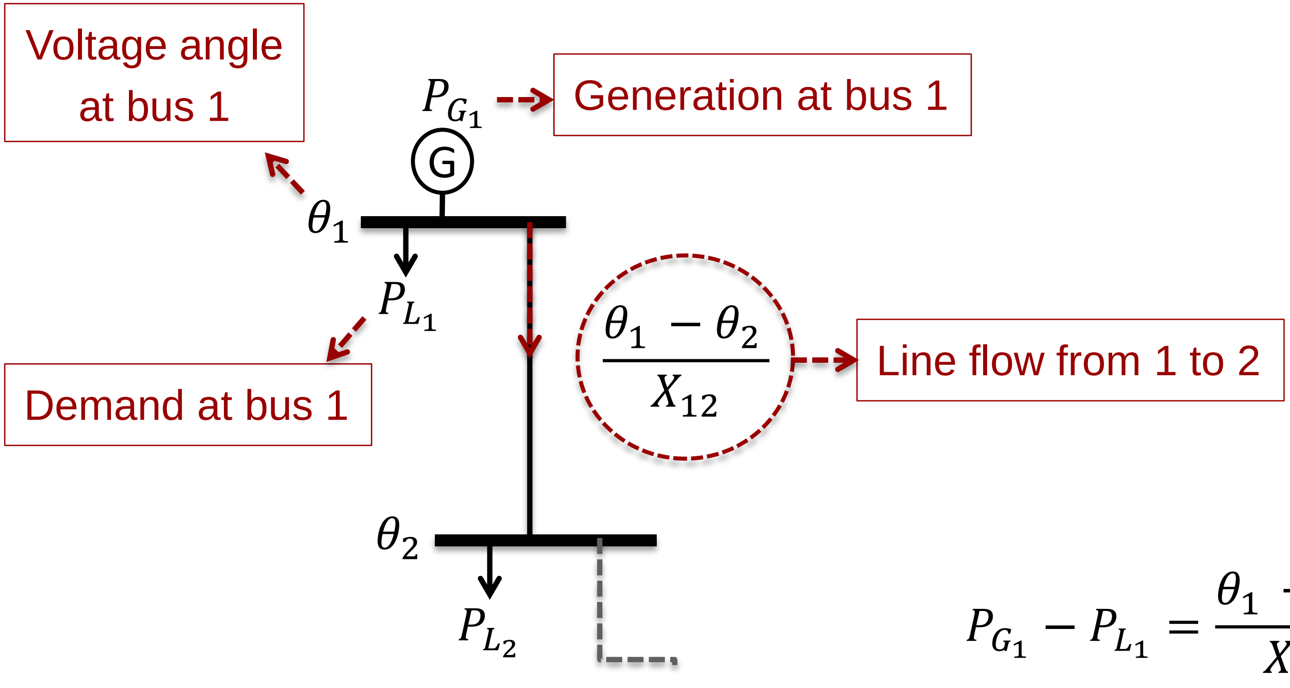

$$P_{G_1} - P_{L_1} = \frac{\theta_1 - \theta_2}{X_{12}}$$

DC-OPF

$$\min_{P_{G_n}} \sum_{n \in \Omega_G} (a_n P_{G_n}^2 + b_n P_{G_n} + c_n)$$

s.t.

$$P_{G_i} - P_{Li} = \sum_{j \in \Omega_i} \frac{\theta_i - \theta_j}{X_{ij}}, \qquad \forall i \in \Omega_B$$

$$\underline{P}_{G_n} \leq P_{G_n} \leq \overline{P}_{G_n}, \qquad \forall n \in \Omega_G$$

$$-\overline{P}_{ij} \leq \frac{\theta_i - \theta_j}{X_{ij}} \leq \overline{P}_{ij}, \quad \forall ij \in \Omega_L$$

P_{G_n}	Power output of generator n
θ_i	Voltage angle at bus i
P_{L_i}	Load at bus i
$\overline{P}_{G_n}$	Maximum generation of generator n
$\underline{P}_{G_n}$	Minimum generation of generator n
Ω_G	Set of all generators

Ω_L	Set of all lines in the grid
Ω_B	Set of buses in the grid
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X_{ij}	Reactance of line ij
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DC-OPF

Minimize generation cost ⟺

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Optimality conditions of OPF

The first order optimality conditions for the OPF optimization problem

$$\frac{\partial L}{\partial P_{G_n}} = 2a_n P_{G_n} + b_n - \sum_{c\in\Omega_C} \lambda_n + \mu_n^+ - \mu_n^- = 0$$

$$\frac{\partial L}{\partial \theta_i} = \lambda_i \sum_{j\in\Omega_i} \frac{1}{X_{ij}} - \sum_{j\in\Omega_i} \lambda_j \frac{1}{X_{ij}} + \sum_{j\in\Omega_i} (\mu_{ij} - \mu_{ji}) \frac{1}{X_{ij}} = 0$$

$$\frac{\partial L}{\partial \lambda_i} = -P_{G_i} + P_{Li} - \sum_{j\in\Omega_i} \frac{\theta_i - \theta_j}{X_{ij}} = 0$$

$$\frac{\partial L}{\mu_n^+} = P_{G_n} - \overline{P}_{G_n} \leq 0$$

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$$\frac{\partial L}{\mu_{ij}} = \frac{\theta_i - \theta_j}{X_{ij}} - \overline{P}_{ij} \leq 0$$

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Lagrange multipliers coupling with neighbors

$$\frac{\partial L}{\partial \lambda_i} = -P_{G_i} + P_{Li} - \sum_{j\in\Omega_i} \frac{\theta_i - \theta_j}{X_{ij}} = 0$$

$$\frac{\partial L}{\mu_n^+} = P_{G_n} - \overline{P}_{G_n} \leq 0$$

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Local power balance equation

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Each of these equations only involve local information

Distributed updates

- Iterative procedure
  - Solve first order optimality conditions
  - Each of these conditions only involves local information

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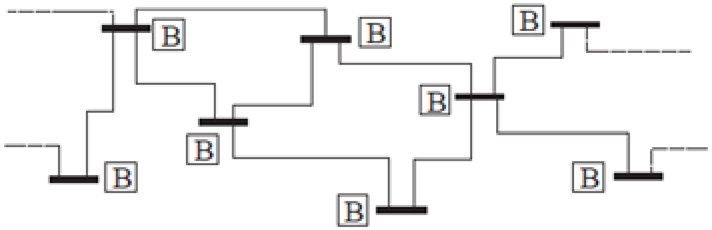

Fully Distributed

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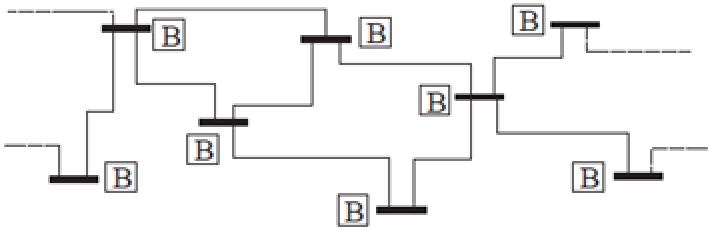

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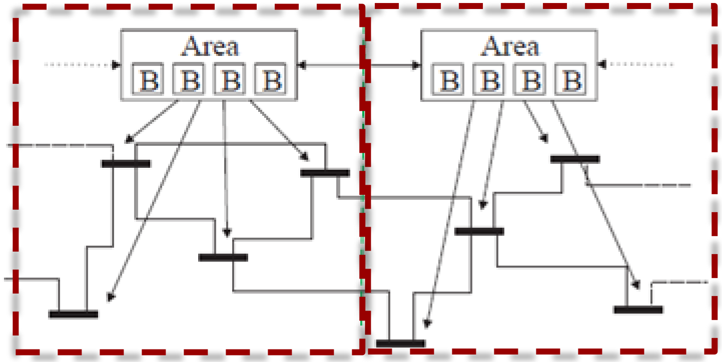

Multilevel Distributed

Distributed updates

The proposed update for λ_i :

$$\lambda_i(k+1) = \lambda_i(k) - \beta(\lambda_i(k) \sum_{j \in \Omega_i} \frac{1}{X_{ij}} - \sum_{j \in \Omega_i} \lambda_j(\eta_{ij}) \frac{1}{X_{ij}} + \sum_{j \in \Omega_i} (\mu_{ij}(k) - \mu_{ji}(\eta_{ij})) \frac{1}{X_{ij}})$$
$$- \alpha(\sum_{n \in \Omega_{G_i}} P_{G_n}(k) - P_{L_i} - \sum_{j \in \Omega_i} \frac{\theta_i(k) - \theta_j(\eta_{ij})}{X_{ij}}),$$

The proposed update for P_{G_i} :

$$P_{G_i}(k+1) = \frac{\lambda_i(k+1) - b_n}{2a_n} \rightarrow \begin{cases} \overline{P}_{G_i} & \overline{P}_{G_i} < P_{G_i} \\ P_{G_i} & \underline{P}_{G_i} < P_{G_i} < \overline{P}_{G_i} \\ \underline{P}_{G_i} & P_G < \underline{P}_{G_i} \end{cases}$$

Distributed updates

The proposed update for λ_i :

Neighborhood's Lagrange multipliers coupling

$$\lambda_i(k+1) = \lambda_i(k) - \beta\Big(\lambda_i(k)\sum_{j\in\Omega_i}\frac{1}{X_{ij}} - \sum_{j\in\Omega_i}\lambda_j(\eta_{ij})\frac{1}{X_{ij}} + \sum_{j\in\Omega_i}(\mu_{ij}(k) - \mu_{ji}(\eta_{ij}))\frac{1}{X_{ij}}\Big)$$

$$-\alpha\Big(\sum_{n\in\Omega_{G_i}} P_{G_n}(k) - P_{L_i} - \sum_{j\in\Omega_i}\frac{\theta_i(k) - \theta_j(\eta_{ij})}{X_{ij}}\Big),$$

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Distributed updates

The proposed update for θ_i :

$$\theta_i(k+1) = \theta_i(k) - \gamma(-\sum_{n\in\Omega_{G_i}} P_{G_n}(k) + P_{L_i} + \sum_{j\in\Omega_i} \frac{\theta_i(k) - \theta_j(\eta_{ij})}{X_{ij}})$$

The proposed update for μ_{ij} :

$$\mu_{ij}(k+1) = \mathbb{P}\left(\mu_{ij}(k) - \delta\left(\overline{P}_{ij} - \frac{\theta_i(k) - \theta_j(\eta_{ij})}{X_{ij}}\right)\right)$$

Distributed updates

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The proposed update for θ_i :

$$\theta_i(k+1) = \theta_i(k) - \gamma\Big(- \sum_{n \in \Omega_{G_i}} P_{G_n}(k) + P_{L_i} + \sum_{j \in \Omega_i} \frac{\theta_i(k) - \theta_j(\eta_{ij})}{X_{ij}}\Big)$$

Local power balance equation

The proposed update for μ_{ij} :

$$\mu_{ij}(k+1) = \mathbb{P}\Big(\mu_{ij}(k) - \delta\Big(\overline{P}_{ij} - \frac{\theta_i(k) - \theta_j(\eta_{ij})}{X_{ij}}\Big)\Big)$$

Partitioning

1. Use graph optimization algorithm to find a dense community of nodes
2. Apply the same algorithm to find dense communities in the rest of the nodes
3. Merge communities as needed

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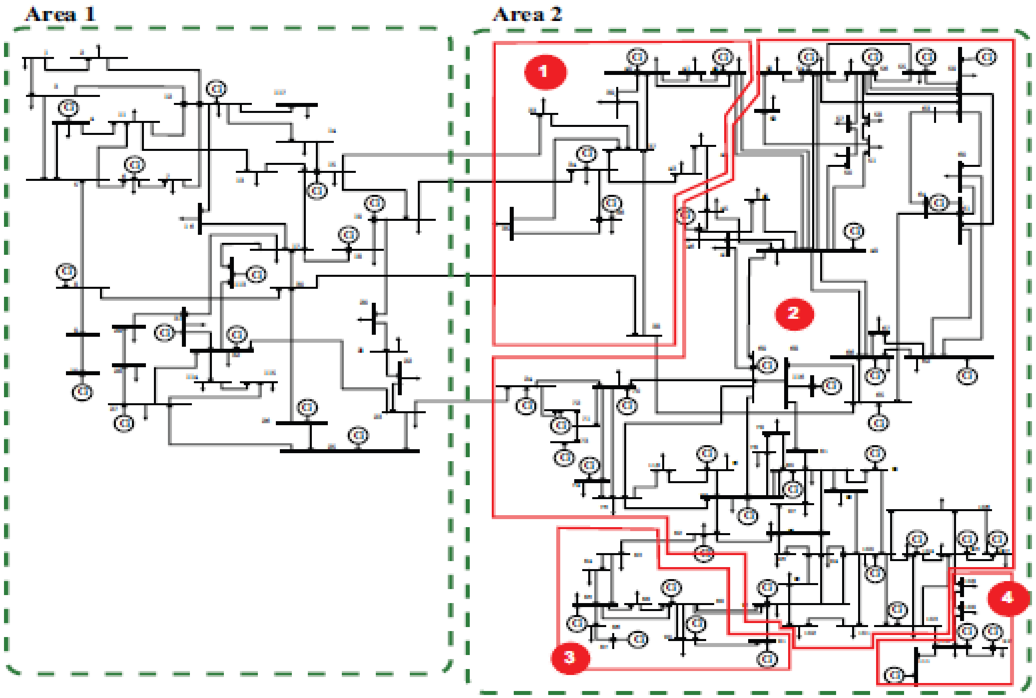


Outline

- Motivation and Background
- Multilevel Distributed Optimal Power Flow
- **Simulation Results**
- Summary

- Solving a coupled system of equations
- Computationally cheap update function
- Multilevel implementation

Test System & Performance

- IEEE 118-bus test system
- Relative distance
  - Relative distance of the objective function from its optimal value over the iterations:

$$rel = \frac{|f - f^*|}{f^*}$$

    - f^* is the optimal objective function value of centralized method

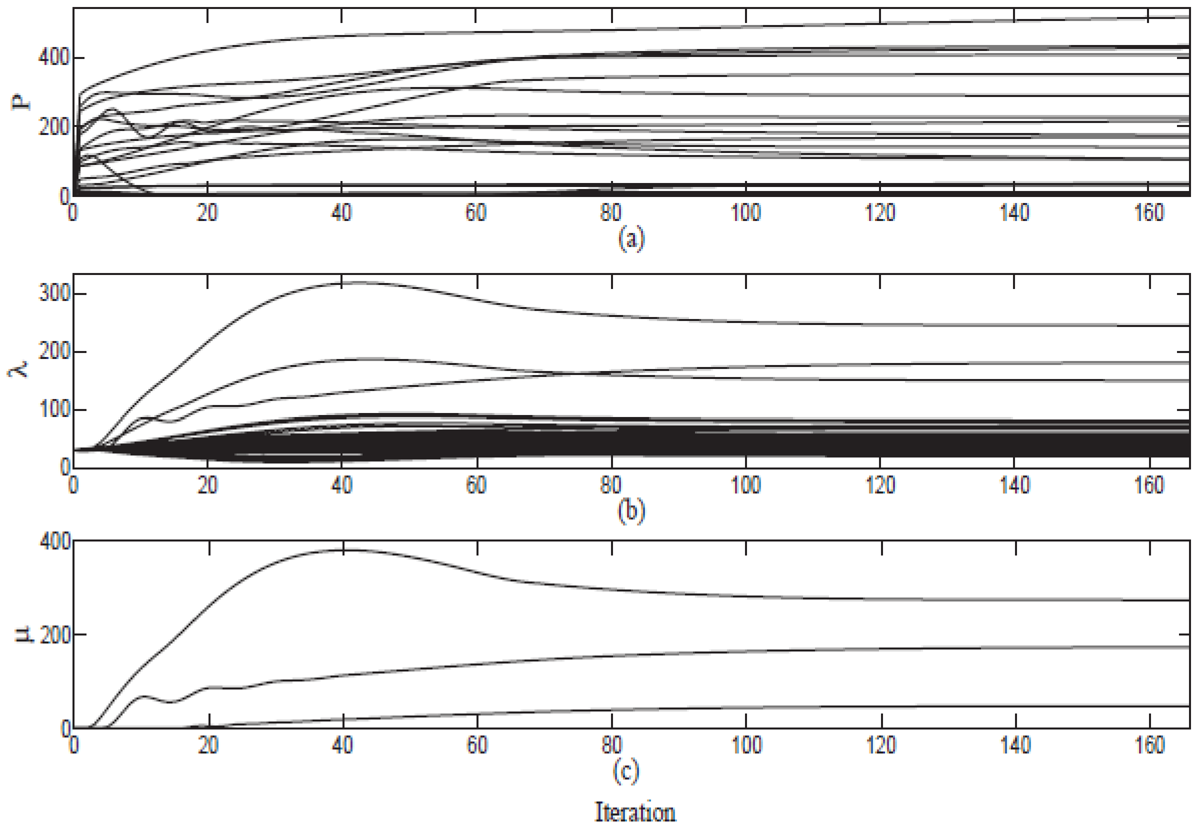
- Power balance residual
  - The sum over the residuals of all power flow equations over the course of iterations:

$$res = \sqrt{\sum_i (g_i)^2}$$

    - g_i is local power balance equation at bus i

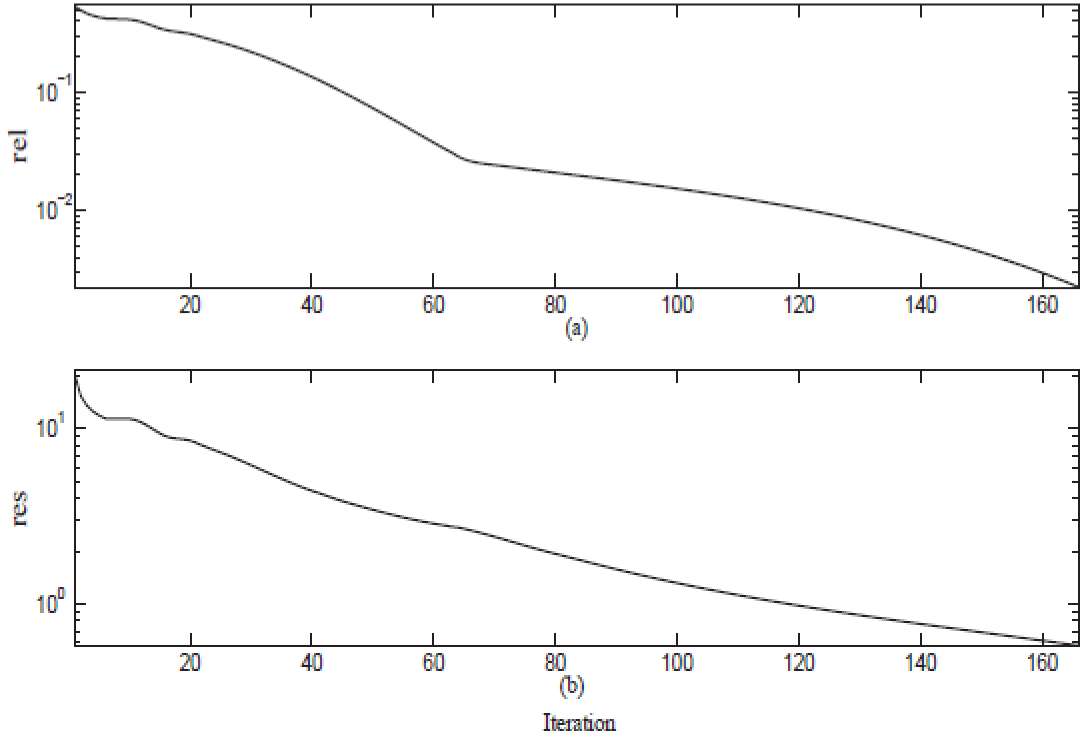
OPF; 2 Areas

- Variables (inter-area gap:20)



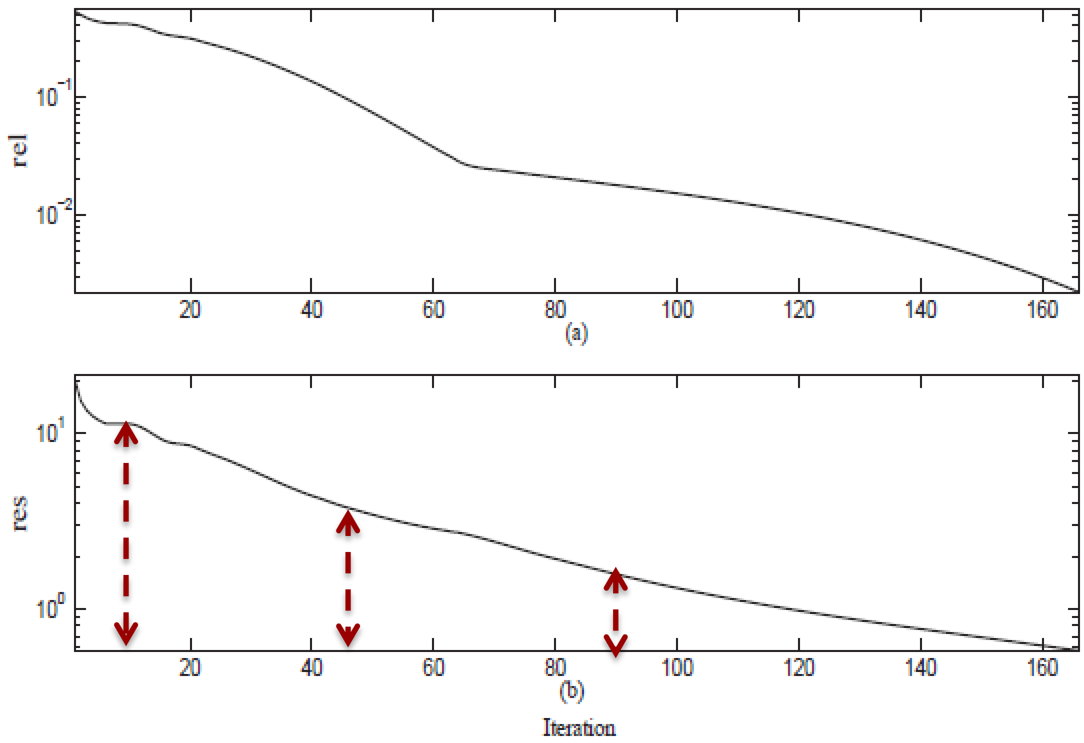
OPF; 2 Areas

- Convergence measures (inter-area gap:20)



OPF; 2 Areas

- Convergence measures (inter-area gap:20)



OPF

- Convergence measures

ACHIEVED res AND rel UNDER DIFFERENT PORTIONING METHODS, DISTRIBUTED AREA-SUBAREA IMPLEMENTATION

Partitioning method	Inter area gap	Inter subarea gap	rel	res
Random	20	10	0.0005	0.3693
Structural	20	10	0.0004	0.3567
Random	30	5	0.0009	0.3756
Structural	30	5	0.0004	0.3623
Random	80	20	0.0062	0.4808
Structural	80	20	0.0037	0.4348
Random	200	100	0.1772	5.0231
Structural	200	100	0.0442	1.2413

Outline

- Motivation and Background
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- **Summary**

Summary

- Proposed distributed solution:
  - Distributes the computation among different entities.
  - No need to share information about generation cost parameters or generation settings.
  - Each entity exchanges limited information with a few other entities.
  - Entity may represent a single bus or a collection of physically connected buses

Questions?