Study on the Use of Error Term in Parallel-form Narrowband Feedback Active Noise Control Systems

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***Abstract*—Parallel-form narrowband feedback active noise control (FBANC) system has been shown to perform better than conventional internal model control (IMC) based FBANC system in cancelling multi-tonal noise. A previous paper illustrated a novel approach in estimating the frequencies of the multi-tone noise, and using an internal tonal generator cum frequency grouping unit to increase its frequency separation in each channel of the parallel-form FBANC system based on a full-band error. This paper investigates whether it is necessary to use a narrowband error to further improve on the performance of the parallel-form narrowband FBANC through theoretical analysis. Computer simulations are also presented to validate its performance.**

I. Introduction

Active noise control (ANC) techniques have been commonly used in many applications that require the reduction of acoustic noise [1]-[2]. It is based on a simple physical principle of destructive interference by introducing a secondary anti-noise source to cancel the primary noise source. The anti-noise source must be approximately the same amplitude and 180^o out of phase with the primary noise source at the zone of cancellation. Typically, an adaptive feedforward active noise control (FFANC) system detects (through a reference microphone) the primary noise source and adaptively creates an anti-noise at the zone around the error microphone through a secondary source. However, in some instances, reference microphone may not be desired due to the feedback of the secondary source to the reference microphone or physically not feasible to be set up near the noise source. In these applications, non-acoustic sensors, such as tachometers, are used to estimate the speed (rpm) of the noisy machine and internally generates the primary reference signal and its harmonics (corresponding to the rpm) to drive the adaptive filter of the FFANC.

There is yet another approach utilizing an internal model to synthesize the reference signal without the need of a reference microphone. This approach is known as the internal model control (IMC) feedback active noise control (FBANC) [3]-[5]. The FBANC, however, is subjected to an accurate estimation of the secondary path, $S(z)$ to synthesize the reference signal. It has been found in recent studies [5] that the accuracy of the secondary path estimation of the FBANC system is much more critical compared to the FFANC system. A new FBANC system based on IMC was recently proposed in [4], which uses the idea of tonal signal generation, as deployed in the direct/parallel FFANC proposed in [6]. The direct/parallel FFANC system is used to cancel multiple noisy tones at the fundamental frequency and its harmonic frequencies, which are commonly produced by many rotating machines. The trick in the direct/parallel FFANC system lies in the partitioning of the internally generated harmonic noise sources to different channels to increase the frequency separation among the generated harmonics, before performing parallel adaptive filtering across these channels.

In a similar manner, we have proposed a new parallel-form narrowband FBANC in [7] that allows noisy tonal harmonics to be separated and adapted separately to improve its convergence and noise reduction. This new FBANC system is based on the assumption that the number of tones in the primary noise is known and the frequencies of the multi-tone noise are estimated in real time by using infinite-impulse-response (IIR) adaptive notch filter (ANF) [8]. Based on the estimated frequencies, the reference signals are internally generated and grouped to different channels so that the frequency separation in each channel is increased, and thus, the convergence rate and noise reduction are improved. Furthermore, the proposed new feedback ANC system is found to be less sensitive to impulsive noise. However, question on whether the updating error needs to be separated to facilitate parallel adaptive filter update remains unanswered. This paper attempts to answer this question and performs a theoretical analysis to investigate whether there is any difference in using a single full-band error or multiple narrowband error updates.

The rest of the paper is structured as follows. Section II gives the two variant structures of the parallel-form narrowband FBANC based on common-error and multiple-error updates. Our theoretical analysis on the use of different error terms is discussed in Section III, which is followed by the computer simulations in Section IV. Finally, we conclude this paper in Section V.

II. Parallel-form Narrowband Feedback Active Noise System

Fig. 1 illustrates the block diagram of the conventional parallel-form narrowband FBANC, where the same full-band error is used in filtered-x least-mean-square (FxLMS)

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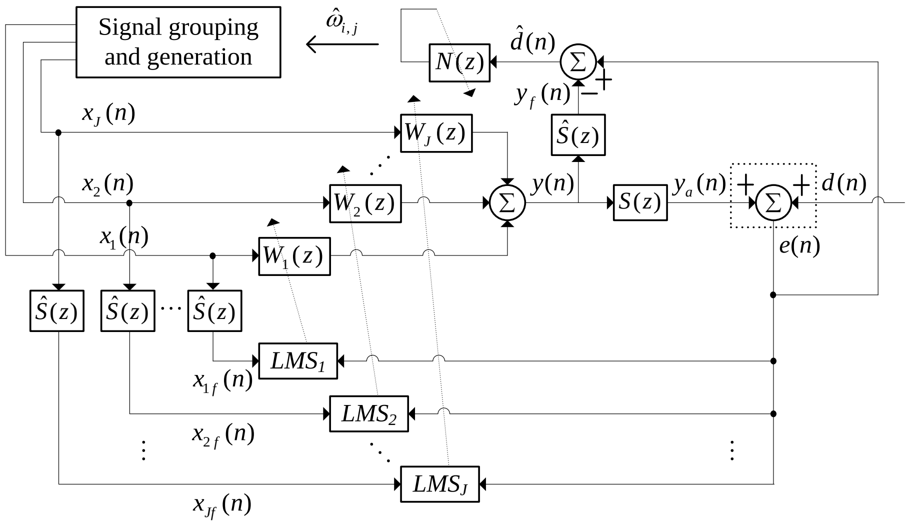

Fig. 1. A conventional parallel-form narrowband FBANC system, full-band error used in FxLMS (adapted from [7]).

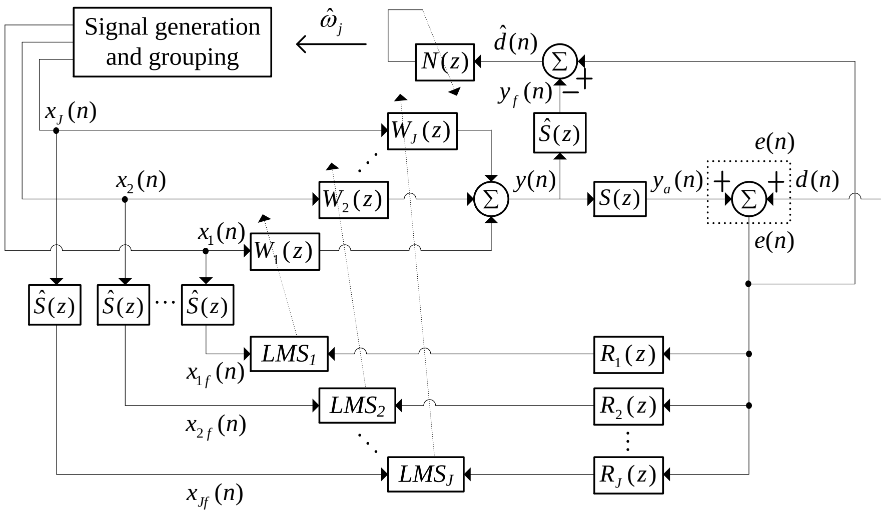

Fig. 2. A complete parallel-form narrowband FBANC system, narrowband error used in FxLMS (adapted from [7]).

algorithm to update the weights of all the channels [7]. For comparison, the complete parallel-form narrowband FBANC system is illustrated in Fig. 2, where the error term is obtained from IIR ANF $R_j(z)$ [9]. Clearly, the only difference between the two FBANC systems in Figs. 1 and 2 is the use of different error terms.

It is assumed that the primary noise $d(n)$ consists of J dominant narrowband components with frequency f_j, $j=1,2,...,J$. The primary path, secondary path, and secondary path model are expressed as $P(z)$, $S(z)$, and $\hat{S}(z)$, respectively. Each channel of the adaptive filter $W_j(z)$ consists of two weights, and all the J channels are connected in parallel. The associated sinusoidal reference signal $x_j(n)$ can be generated (synchronized by a non-acoustic sensor or obtained by estimating the frequencies using an adaptive notch filter [7], [8]). In the generation of the reference signal, it has been found that the amplitude of the sinusoids affects the convergence rate [6]. In order to further improve the convergence rate, the amplitudes of all the reference signals are set as the inverse of the magnitude response of the secondary path at the respective frequencies. Hence, the reference signal in the jth channel $x_j(n)$ is expressed as

$$x_j(n)=\mathrm{g}(f_j)\cos(2\pi f_j n), \quad j=1,2,...,J, \tag{1}$$

where $\mathrm{g}(f_j)=\frac{1}{\hat{S}(f)\big|_{f=f_j}}$. The filtered signal $x_{jf}(n)$ is computed as

$$x_{jf}(n)=\sum_{i=0}^{I}\hat{s}_i x_j(n-i), \quad j=1,2,...,J, \tag{2}$$

where $\hat{s}_i$ is the coefficient of $\hat{S}(z)$. Using FxLMS algorithm, the weight vector is updated as

$$\mathbf{w}_j(n+1)=\mathbf{w}_j(n)+2\mu_j e(n)\mathbf{x}_{jf}(n), \quad j=1,2,...,J, \tag{3}$$

where $e(n)$ is the error signal, $\mathbf{w}_j(n)=[w_{j,0}(n) \quad w_{j,1}(n)]^T$, $\mathbf{x}_{jf}(n)=[x_{jf}(n) \quad x_{jf}(n-1)]^T$, T denotes transpose operator, and μ_j is the step size of channel j.

The selection of error term in (3) has been studied recently. In [7], the full-band error that is common to all J adaptive filters has been used and validated to perform better than the conventional internal model control based FBANC system. However, according to the analysis in [9], for a complete parallel-form narrowband FBANC system, the narrowband error can be used to update the individual adaptive filters. Nevertheless, the necessity of using narrowband errors as compared to the full-band error is not fully understood. In this paper, we extend the analysis in [9], and study the difference in the use of narrowband errors and full-band error in the FBANC systems [7].

III. THEORETICAL ANALYSIS ON THE USE OF ERROR TERM

Within the scope of the narrowband FBANC, it is assumed that the primary noise $d(n)$, as well as the cancelling noise $y_a(n)$, consist of J narrowband (or tonal) components. Thus, we can express them as,

$$\begin{aligned} d(n)&=\sum_{j=1}^{J}d_j(n), \quad j=1,2,...,J, \\ y_a(n)&=\sum_{j=1}^{J}y_{aj}(n), \quad j=1,2,...,J, \end{aligned} \tag{4}$$

where $d_j(n)$ and $y_{aj}(n)$ have the same frequency f_j as the reference signal $x_j(n)$. Thus, the error signal, can also be expressed as the sum of the J narrowband errors, i.e.,

$$e(n)=\sum_{j=1}^{J}e_j(n), \quad j=1,2,...,J, \tag{5}$$

where

$$e_j(n)=d_j(n)-y_{aj}(n), \quad j=1,2,...,J. \tag{6}$$

Therefore, we can rewrite weight updating equation expressed in (3) as

$$\begin{aligned} &\mathbf{w}_j(n+1) \\ &=\mathbf{w}_j(n)+2\mu_j\left[\sum_{i=1}^{J}e_i(n)\right]\mathbf{x}_{jf}(n) \\ &=\mathbf{w}_j(n)+2\mu_j e_j(n)\mathbf{x}_{jf}(n)+2\mu_j\left[\sum_{i=1,\ i\neq j}^{J}e_i(n)\right]\mathbf{x}_{jf}(n) \\ &=\mathbf{w}_j(n)+2\mu_j e_j(n)\mathbf{x}_{jf}(n)+2\mu_j\beta_j(n)\mathbf{x}_{jf}(n), \end{aligned} \tag{7}$$

where

$$\beta_j(n)=\sum_{i=1,\ i\neq j}^{J}e_i(n), \quad j=1,2,...,J. \tag{8}$$

Clearly, the error term $\beta_j(n)$ can be considered as a disturbance to the jth channel FxLMS. Substitute (6) into (7) and take the expectation in both sides, we have

$$\bar{\mathbf{w}}_j(n+1)=\left[\mathbf{I}-2\mu_j\mathbf{R}_j\right]\bar{\mathbf{w}}_j(n)+2\mu_j\mathbf{P}_j+2\mu_j\mathbf{D}_j, \tag{9}$$

where $\mathbf{I}$ is the identity matrix, and

$$\begin{aligned} &\bar{\mathbf{w}}_j(n)=E\left[\mathbf{w}_j(n)\right], \mathbf{R}_j=E\left[\mathbf{x}_{jf}^T(n)\mathbf{x}_{jf}(n)\right]^T, \\ &\mathbf{P}_j=E\left[d_m(n)\mathbf{x}_{jf}(n)\right]^T, \text{ and } \mathbf{D}_j=E\left[\beta_j(n)\mathbf{x}_{jf}(n)\right]^T. \end{aligned} \tag{10}$$

According to the analysis in [9], when $\mathbf{D}_j\neq\mathbf{0}$, it will act as a disturbance, which will cause misalignment of the adaptive weights, and reduce the step size bound as compared to the case when the narrowband error is used.

However, the significance of $\mathbf{D}_j$ is not well examined. According to (8), $\beta_j(n)$ is the sum of the narrowband error excluding the jth subband, and by substituting (6) into (8), we have

$$\beta_j(n)=\sum_{i=1,\ i\neq j}^{J}\left[d_i(n)-y_{ai}(n)\right], \quad j=1,2,...,J, \tag{11}$$

Thus, we can express $\mathbf{D}_j$ as

$$\begin{aligned} \mathbf{D}_j&=E\left[\sum_{i=1,\ i\neq j}^{J}\left[d_i(n)-y_{ai}(n)\right]\mathbf{x}_{jf}(n)\right]^T \\ &=\sum_{i=1,\ i\neq j}^{J}E\left[d_i(n)\mathbf{x}_{jf}(n)\right]^T-\sum_{i=1,\ i\neq j}^{J}E\left[y_{ai}(n)\mathbf{x}_{jf}(n)\right]^T \end{aligned} \tag{12}$$

It is well known that the expectation of the cross-correlation of two tones having different frequencies is equal to 0, i.e.,

$$E\left[\cos(2\pi f_i n)\cos(2\pi f_j n)\right]=0, \quad \forall f_i\neq f_j. \tag{13}$$

Based on (13), we can deduce that $\mathbf{D}_j=\mathbf{0}$. This finding indicates that the adaptive filter weights in the steady state will not differ between the two cases: one using narrowband error $e_j(n)$, and the other using full-band error $e(n)$. Without considering other effects, the above analysis indicates that there will not be much difference on the performance between the two FBANC systems (Figs. 1 and 2), in terms of step size bound, and convergence rate.

In addition, the narrowband filters are not ideal due to certain bandwidth of the bandpass filter [9]. It has been found in [10] that when the bandwidth is small, the group delay in the narrowband filters becomes a major factor that degrades the convergence of FBANC. On the other hand, if we increase the bandwidth, the maximum stable step size decreases. In other words, when the frequency separation of the primary noise is smaller, the maximum stable step size in narrowband error method is smaller. In order to make its steady-state performance be comparable to the full-band error method, a smaller step size is required,

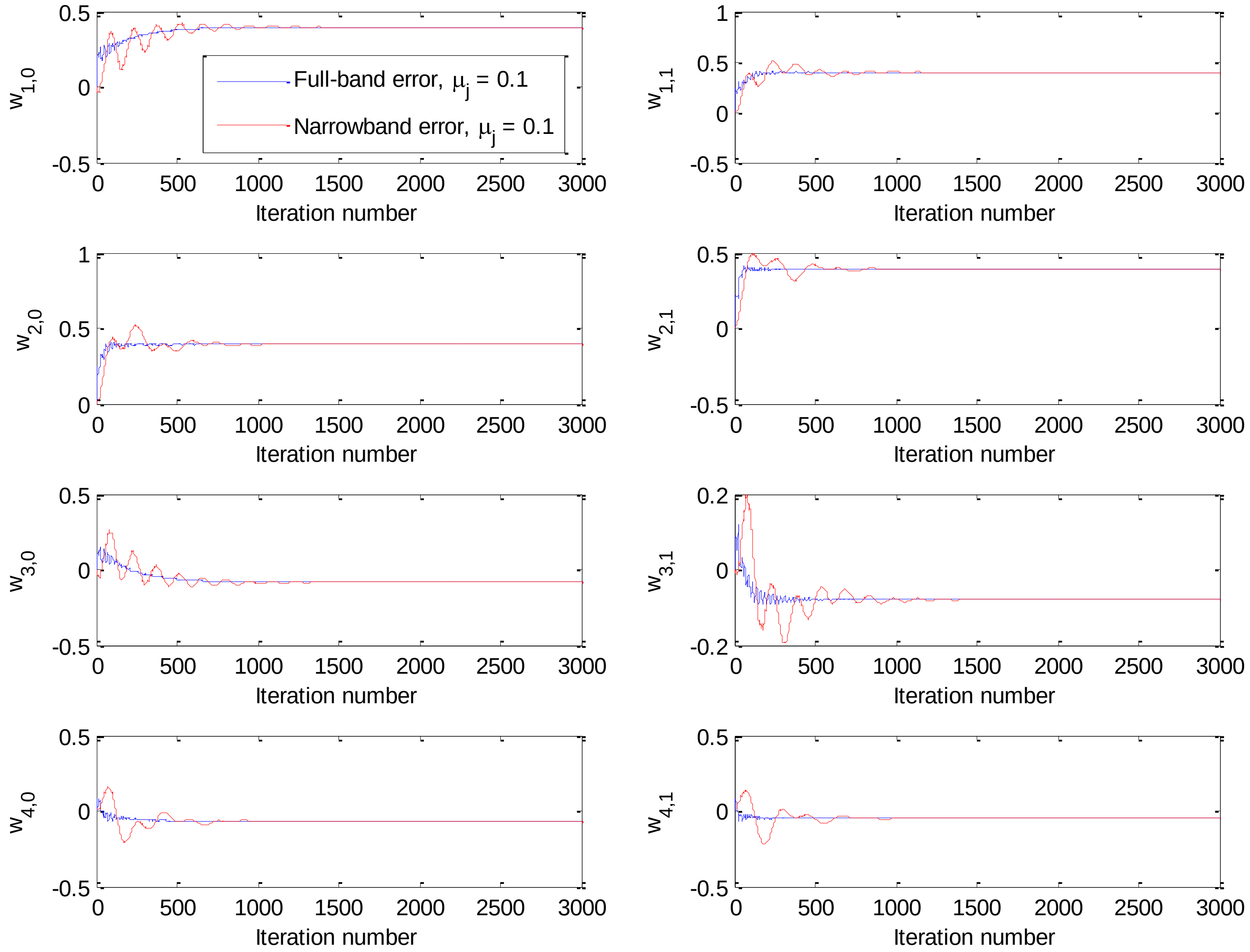

Fig. 3 Convergence of adaptive filter weights in the two methods: full-band error, and narrowband error in parallel-form narrowband FBANC.

IV. SIMULATION RESULTS

To validate the analysis in Section III, computer simulations are conducted. The secondary path estimation is assumed to be perfect, i.e., $S(z) = \hat{S}(z)$. In our simulation, we consider 4 tonal (J = 4) signal as primary noise. Their frequencies are 80, 160, 240, and 320 Hz, with sampling frequency of f_s = 2000 Hz. For the narrowband filters, we adopted the same filters described in [9] with p_m = 0.99. After 6000th iterations (i.e., 3 seconds), we compute the following estimates of (10) as:

$$\mathbf{R}_j = \frac{1}{6000}\begin{bmatrix} \sum_{n=1}^{6000}\left[x_{jf}^2(n)\right] & \sum_{n=1}^{6000}\left[x_{jf}(n-1)x_{jf}(n)\right]^T \\ \sum_{n=1}^{6000}\left[x_{jf}(n)x_{jf}(n-1)\right] & \sum_{n=1}^{6000}\left[x_{jf}^2(n-1)\right] \end{bmatrix}$$

$$\mathbf{P}_j = \frac{1}{6000}\begin{bmatrix} \sum_{n=1}^{6000}\left[d_m(n)x_{jf}(n)\right] \\ \sum_{n=1}^{6000}\left[d_m(n)x_{jf}(n-1)\right] \end{bmatrix}$$

$$\mathbf{D}_j = \frac{1}{6000}\begin{bmatrix} \sum_{n=1}^{6000}\left[\beta_m(n)x_{jf}(n)\right] \\ \sum_{n=1}^{6000}\left[\beta_m(n)x_{jf}(n-1)\right] \end{bmatrix}$$

(14)

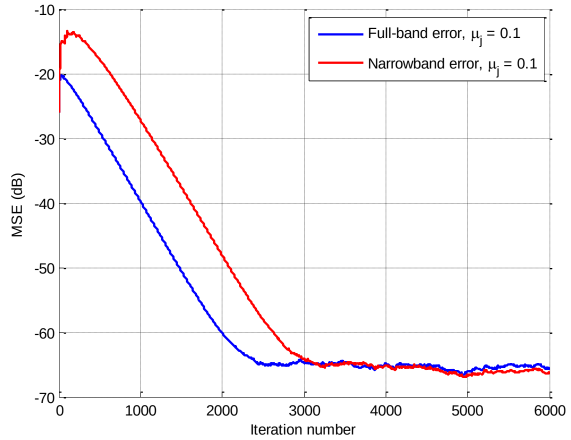

Fig. 4 Learning curves of parallel-form narrowband FBANC for the two methods: full-band error, and narrowband error. Frequencies of the tones are: 80, 160, 240, and 320 Hz.

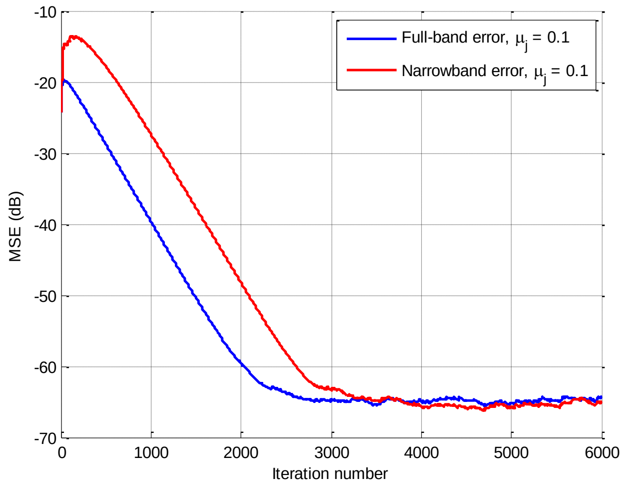

Fig. 5 Learning curves of parallel-form narrowband FBANC for the two methods: full-band error, and narrowband error. Frequencies of the tones are: 100, 200, 300, and 400 Hz.

Thus, we can calculate these terms as

$$\mathbf{R}_1=\begin{bmatrix}0.49 & 0.47\\0.47 & 0.49\end{bmatrix},\ \mathbf{P}_1=\begin{bmatrix}0.15\\0.15\end{bmatrix},\ \mathbf{D}_1=10^{-3}\times\begin{bmatrix}0.34\\0.35\end{bmatrix}$$
$$\mathbf{R}_2=\begin{bmatrix}0.45 & 0.39\\0.39 & 0.45\end{bmatrix},\ \mathbf{P}_2=\begin{bmatrix}0.15\\0.14\end{bmatrix},\ \mathbf{D}_2=10^{-3}\times\begin{bmatrix}0.01\\0.02\end{bmatrix}$$
$$\mathbf{R}_3=\begin{bmatrix}0.40 & 0.29\\0.29 & 0.40\end{bmatrix},\ \mathbf{P}_3=\begin{bmatrix}0.14\\0.11\end{bmatrix},\ \mathbf{D}_3=10^{-3}\times\begin{bmatrix}-0.13\\-0.11\end{bmatrix}$$
$$\mathbf{R}_4=\begin{bmatrix}0.36 & 0.19\\0.19 & 0.36\end{bmatrix},\ \mathbf{P}_4=\begin{bmatrix}0.13\\0.08\end{bmatrix},\ \mathbf{D}_4=10^{-3}\times\begin{bmatrix}-0.19\\-0.15\end{bmatrix} \quad (15)$$

It can be observed from (15) that as compared to $\mathbf{R}_j$, and $\mathbf{P}_j$, $\mathbf{D}_j$ is actually insignificant and can hence be ignored in (9).

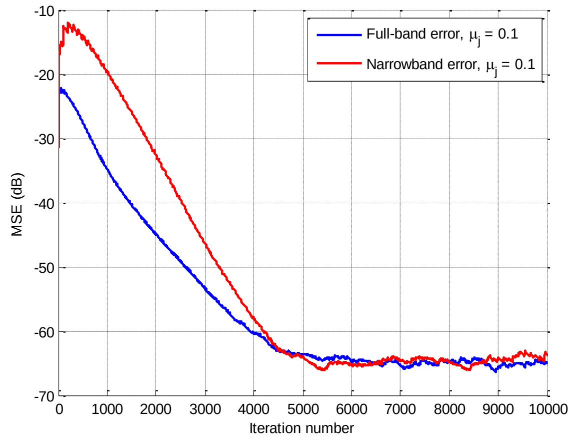

Fig. 6 Learning curves of parallel-form narrowband FBANC for the two methods: full-band error, and narrowband error. Frequencies of the tones are: 40, 80, 120, and 160 Hz.

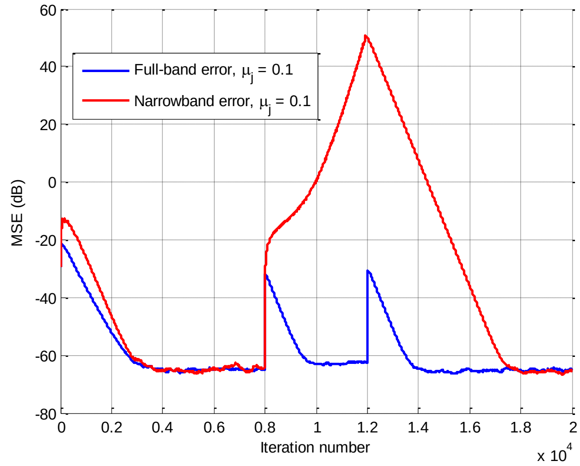

Fig. 7 Learning curves of parallel-form narrowband FBANC for the two methods: full-band error, and narrowband error. Frequencies of the tones are varying from: 50, 100, 150, and 200 Hz to 100, 200, 300, and 400 Hz.

Next, we show the convergence of the weights in these two methods in Fig. 3. Using the same step size of 0.1, the full-band error method converges approximately at the same rate as the narrowband error method. There is no misalignment of the final weights in the full-band error method as compared to the narrowband method, and hence no difference in noise reduction. As a result, the final MSE of these two methods are very similar, as shown in Fig. 4. However, the convergence of the full-band error method is faster than that of the narrowband error method. This is due to the group delay incurred by the narrowband filters in the narrowband error method, as pointed out in [10].

Furthermore, we have tested the primary noise with other frequencies in simulations and presented the results of learning curves. In Fig. 5, the frequencies for the four tones are set at 100, 200, 300, and 400 Hz with other settings remain the same. In Fig. 6, the frequencies are decreased to 40, 80, 120, and 160 Hz with p_m = 0.995 to ensure the convergence. In Fig. 7, the frequencies of the tones are set to 50, 100, 150 and 200 Hz for the first 4 seconds, and increase linearly in the following 2 seconds all the way to 100, 200, 300, and 400 Hz, respectively. It is obvious that the MSE performance of the full-band error structure is much better than the narrowband error structure in the parallel-form narrowband FBANC.

V. CONCLUSIONS

In this paper, we studied the use of the error term in parallel-form narrowband FBANC systems. Both full-band error, which is common in all adaptive filters; and narrowband error, which is obtained using IIR ANF, are considered. Our theoretical analysis suggested that the use of full-band error performs equally well compared to the narrowband error. This is because in each channel, the disturbance error term is orthogonal to the filtered reference signal. Thus, no misalignment in the weight vector is incurred and the maximum step-size bound remains the same in both cases. However, besides the additional computational cost, the use of the narrowband error inherently introduces group delay in the secondary path and slows down the convergence of the FBANC system. In conclusion, the full-band error is recommended in the adaptation of the parallel-form narrowband FBANC system.

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