A Joint Design Approach for Spectrum Sharing between Radar and Communication Systems

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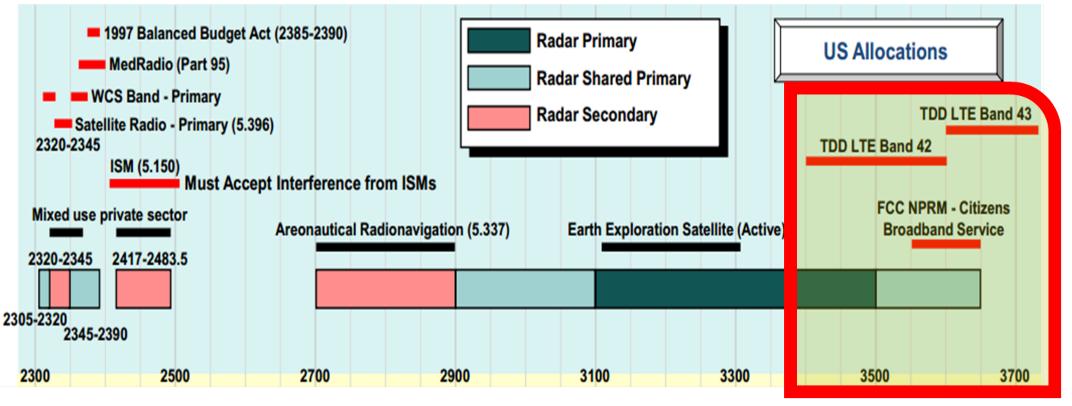
1. Overview

- We propose a joint design for the coexistence of MIMO radars and a communication system, for a scenario in which the targets fall in different range bins.
- Transmit precoding at the radar transmit antennas and adaptive communication transmission are adopted, and are jointly designed to maximize the SINR at the radar receiver subject to the communication system meeting certain rate and power constraints.
- We propose a reduced dimensionality design, which has reduced complexity without degrading radar SINR.

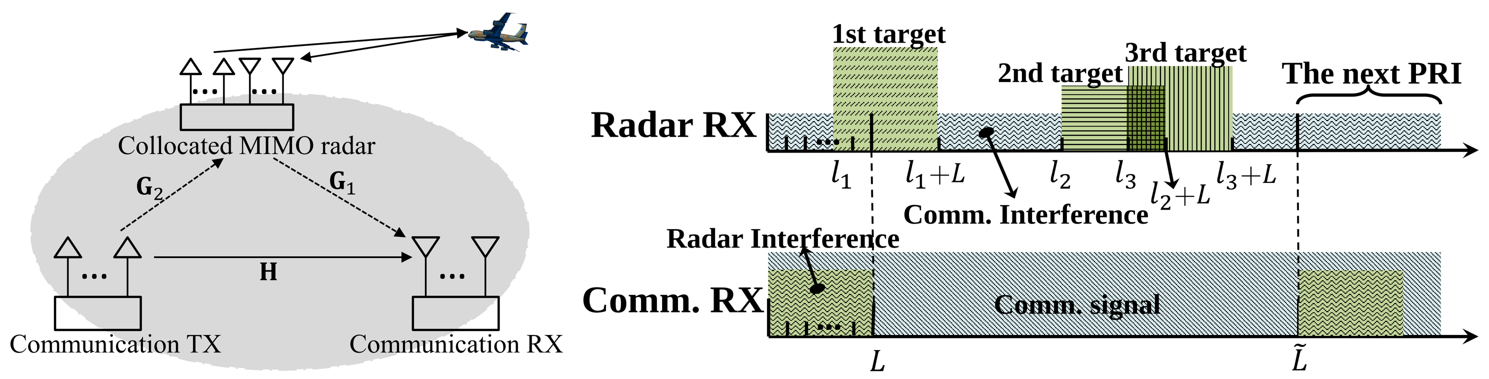
2. Motivation

- Radar and communication systems overlap in the spectrum domain thus causing interference to each other.
- Spectrum sharing can increase spectrum efficiency.



3. System Model

- Consider a MIMO communication system which coexists with a MIMO-MC radar system using the same carrier frequency. Assumptions:
  - Flat fading, narrow band radar and comm. signals;
  - Block fading: the channels remain constant for one PRI;
  - Both systems have the same symbol rate;



- The signal received at the radar and communication receivers are modeled as

$$\mathbf{y}_R(l) = \underbrace{\sum_{k=1}^{K} \beta_k \mathbf{v}_r(\theta_k)\mathbf{v}_t^T(\theta_k)\mathbf{P}\mathbf{s}(l-l_k)}_{\text{Target Echoes}} + \underbrace{\mathbf{G}_2\mathbf{x}(l)e^{j\alpha_2(l)}}_{\text{Comm. Interference}} + \underbrace{\mathbf{w}_R(l)}_{\text{Noise}}, \quad (1)$$

$$\mathbf{y}_C(l) = \underbrace{\mathbf{H}\mathbf{x}(l)}_{\text{Comm. Signal}} + \underbrace{\mathbf{G}_1\mathbf{P}\mathbf{s}(l)e^{j\alpha_1(l)}}_{\text{Radar Interference}} + \underbrace{\mathbf{w}_C(l)}_{\text{Noise}}, \ l \in \mathbb{N}_{\tilde{L}}^+, \quad (2)$$

  - $\mathbf{s}(l)$ and $\mathbf{x}(l)$ respectively denote the radar and communication waveform vector at time index l; $\mathbf{S} \triangleq [\mathbf{s}(1), \cdots, \mathbf{s}(L)]$ is a random orthonormal matrix; $\mathbf{x}(l) \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{xl})$; $\mathbf{w}_R(l) \sim \mathcal{CN}(\mathbf{0}, \sigma_R^2\mathbf{I})$; $\mathbf{w}_C(l) \sim \mathcal{CN}(\mathbf{0}, \sigma_C^2\mathbf{I})$.
  - l_k and β_k denotes the echo propagation delay and the RCS for the k-th target;
  - $\mathbf{v}_r(\theta) \in \mathbb{C}^{M_{r,R}}$ and $\mathbf{v}_t(\theta) \in \mathbb{C}^{M_{t,R}}$ are the receive and transmit steering vectors.
  - Note that $\mathbf{s}(l)$ is nonzero only for $l \in \mathbb{N}_L^+$. The echo from the k-th target appears starting from l_k and lasts for L samples.
  - $e^{j\alpha_i(l)}, i \in \{1,2\}$ denote the random phase offset between the radar and the communication systems.

4. Problem Formulation

- The joint design problem for radar and communication spectrum sharing is formulated to maximize the radar SINR, subject to satisfying the communication rate and TX power constraints
  - The **average communication rate** over $\tilde{L}$ symbols is given by

$$C_{\text{avg}}(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}) \triangleq \frac{1}{\tilde{L}} \sum_{l=1}^{\tilde{L}} \log_2 \left|\mathbf{I} + \mathbf{R}_{\text{Cin}l}^{-1}\mathbf{H}\mathbf{R}_{xl}\mathbf{H}^H\right|, \quad (3)$$

    - radar interference $\mathbf{R}_{\text{Cin}l} = \mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I}$ if $l \in \mathbb{N}_L^+$, otherwise $\mathbf{R}_{\text{Cin}l} = \sigma_C^2\mathbf{I}$.
    - $\mathbf{\Phi} \triangleq \mathbf{P}\mathbf{P}^H/L$ is positive semidefinite.
  - The **overall radar SINR** is the average of local SINRs for all K targets given by

$$\text{SINR}_k = \frac{1}{L} \sum_{l \in \mathcal{L}_k} \text{Tr}\left(\mathbf{R}_{\text{Rin}l}^{-1}\mathbf{D}_k\mathbf{\Phi}\mathbf{D}_k^H\right),$$

    - $\mathcal{L}_k \triangleq \{l_k, \cdots, l_k + L - 1\}$: time period of the k-th target echo;
    - $\mathbf{R}_{\text{Rin}l} \triangleq \mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H + \sigma_R^2\mathbf{I}$: the communication interference.
- The communication rate is maximized using *adaptive transmission.*
- We present two formulations based on the availability of target prior information.
  - **Knowledge-based spectrum sharing** with known $\{\sigma_{\beta k}^2\}$, $\{l_k\}$, and $\{\theta_k\}$:

$$(\mathbf{P}_1) \max_{\{\mathbf{R}_{xl}\} \succeq 0, \mathbf{\Phi} \succeq 0} \text{SINR, s.t. } C_{\text{avg}}(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}) \geq C, \quad (4a)$$

$$\sum_{l=1}^{\tilde{L}} \text{Tr}(\mathbf{R}_{xl}) \leq P_C, L\text{Tr}(\mathbf{\Phi}) \leq P_R, \quad (4b)$$

  - **Robust spectrum sharing with unknown** $\{\sigma_{\beta k}^2\}$ **and** $\{l_k\}$**:** The local SINR_k associated with the k-th target is relaxed to the whole PRI

$$\text{SINR}_k' = \frac{1}{\tilde{L}} \sum_{l \in \mathbb{N}_{\tilde{L}}^+} \text{Tr}\left(\mathbf{R}_{\text{Rin}l}^{-1}\mathbf{D}_k\mathbf{\Phi}\mathbf{D}_k^H\right).$$

  Now, the spectrum sharing problem can be formulated as

$$(\mathbf{P}_2) \max_{\{\mathbf{R}_{xl}\} \succeq 0, \mathbf{\Phi} \succeq 0} \text{SINR}', \text{ s.t. same constraints as in } (\mathbf{P}_1).$$

- Both $(\mathbf{P}_1)$ and $(\mathbf{P}_2)$ are nonconvex w.r.t. $(\{\mathbf{R}_{xl}\}, \mathbf{\Phi})$.

5. Iterative algorithm for solving $(\mathbf{P}_2)$

- A solution can be obtained via alternating optimization. Let $(\{\mathbf{R}_{xl}^n\}, \mathbf{\Phi}^n)$ be the variable at the n-th iteration.
- *First*, we solve $\{\mathbf{R}_{xl}^n\}$ while fixing $\mathbf{\Phi}$ to be $\mathbf{\Phi}^{n-1}$:

$$(\mathbf{P}_\mathbf{R}) \max_{\{\mathbf{R}_{xl}\} \succeq 0} \frac{1}{K} \sum_{k=1}^{K} \text{SINR}_k'(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}^{n-1})$$
$$\text{s.t. } C_{\text{avg}}(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}^{n-1}) \geq C, \textstyle\sum_{l=1}^{\tilde{L}} \text{Tr}(\mathbf{R}_{xl}) \leq P_C. \quad (5)$$

  - Rewrite the objective as $\sum_{l=1}^{\tilde{L}} f(\mathbf{R}_{xl})$, with $f(\mathbf{R}_{xl}) \triangleq \text{Tr}\left(\left(\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H + \sigma_R^2\mathbf{I}\right)^{-1}\mathcal{D}^{n-1}\right)$. It can be shown $(\mathbf{P}_\mathbf{R})$ is nonconvex w.r.t. $\mathbf{R}_{xl}$.
  - $(\mathbf{P}_\mathbf{R})$ can be approximated by a convex problem $(\tilde{\mathbf{P}}_\mathbf{R})$ using first order Taylor series approximation of $f(\mathbf{R}_{xl})$. The original problem $(\mathbf{P}_\mathbf{R})$ could be solved via several iterations of solving $(\tilde{\mathbf{P}}_\mathbf{R})$.
- *Second*, the obtained $\{\mathbf{R}_{xl}^n\}$ are used to solve the following problem for $\mathbf{\Phi}^n$:

$$(\mathbf{P}_\Phi) \max_{\mathbf{\Phi} \succeq 0} \text{Tr}(\mathbf{Q}^n\mathbf{\Phi})$$
$$\text{s.t. } C_{\text{avg}}(\{\mathbf{R}_{xl}^n\}, \mathbf{\Phi}) \geq C, L\text{Tr}(\mathbf{\Phi}) \leq P_R,$$

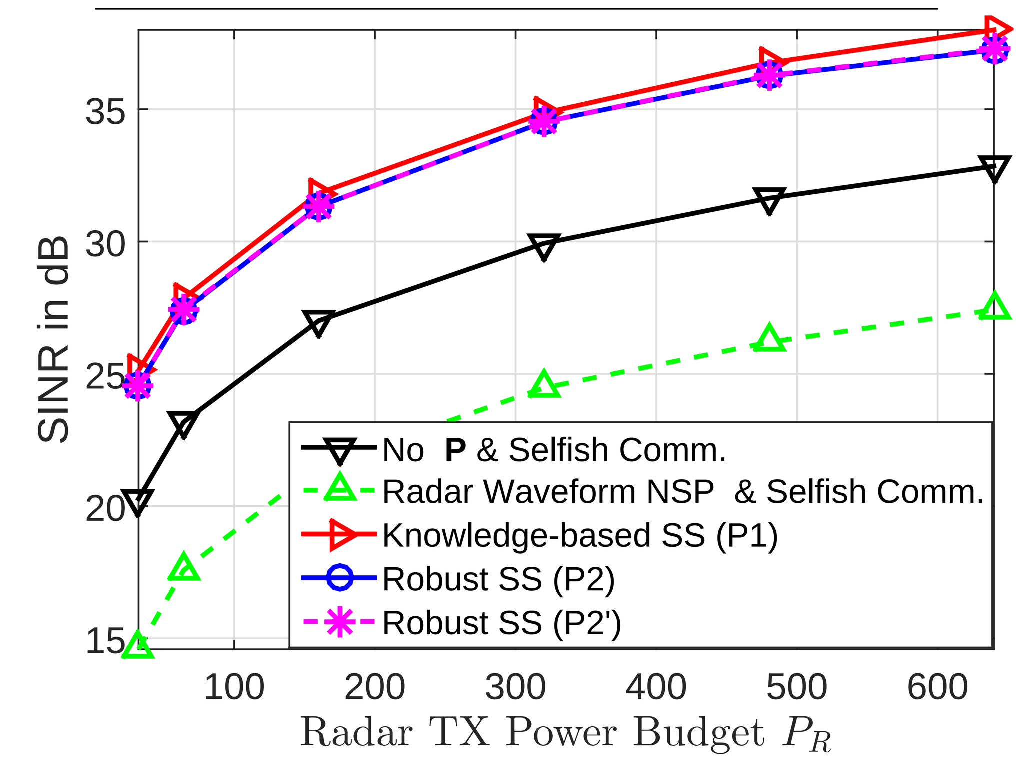
  where $\mathbf{Q}^n$ only depends on $\{\mathbf{R}_{xl}^n\}$.
  - It can be shown that $(\mathbf{P}_\Phi)$ is nonconvex.
  - We introduce a slack variable $\mathbf{\Psi}$ to overcome the non-convexity and apply alternating optimization again as an inner iteration.
- The complete proposed spectrum sharing algorithm alternately solves $(\mathbf{P}_R)$ and $(\mathbf{P}_\Phi)$. It is easy to show that the algorithm converges.

6. Reduced Dimentionality Design

Proposition 1. *Suppose that* $\{\mathbf{R}_{xl}\}$ *is initialized by* $\{\mathbf{R}_{xl}\} \equiv \mathbf{R}_x^0$. *Then, the optimal value of* $(\mathbf{P}_R)$ *in every iteration of the proposed algorithm could be achieved by* $\{\mathbf{R}_{xl}^n\}$ *such that for any* $l, l' \in \mathbb{N}_L^+$ *(or* $l, l' \in \mathbb{N}_{\tilde{L}}^+ \setminus \mathbb{N}_L^+$*), it holds that* $\mathbf{R}_{xl}^n = \mathbf{R}_{xl'}^n$.

- It suffices to solve a reduced dimensionality problem $(\mathbf{P}_2')$, which involves only two matrix variables as the communication transmission covariance matrices respectively for two periods, the one during which radar transmits and the one during which radar only receives.
- The above chioce of $\mathbf{R}_{xl}$ reasonable: the achieved radar SINR would be constant across different range bins, thus avoiding abrupt SINR degradation for certain target range bin.

7. Numerical Results



- We set $\tilde{L} = 32$, $L = 8$, $\sigma_C^2 = \sigma_R^2 = 0.01$, $M_{t,R} = M_{r,R} = M_{t,C} = M_{r,C} = 4$.
- There are three stationary targets at angles $-60°$, $0°$ and $60°$ w.r.t. to the arrays, and the corresponding propagation delays are $6, 18$ and 22.
- We take $C = 24$ bits/symbol and $P_C = \tilde{L}M_{t,C}$ (the power is normalized by the power of the radar waveform). $\mathbf{G}_1$ and $\mathbf{G}_2$ have i.i.d. entries $\mathcal{CN}(0, 0.01)$. $\mathbf{H}_{ij} \sim \mathcal{CN}(0, 1)$.
- For comparison, we implement the uniform precoding method and the null space projection (NSP) precoding method, which projects the radar waveform onto the null space of $\mathbf{G}_1$.
  - The highest SINR, as expected, is acheived by $(\mathbf{P}_1)$ in which pretty much everything is known about the targets.
  - The design of $(\mathbf{P}_2)$, which uses no knowledge about the targets, incurs an SINR loss of 1 dB only.
  - Interestingly, the low complexity spectrum sharing method of $(\mathbf{P}_2')$ achieves the same SINR performance as $(\mathbf{P}_2)$. For this particular example, as compared to $(\mathbf{P}_2)$, in $(\mathbf{P}_2')$ the number of matrix variables is reduced from 33 to 3.
  - The selfish communication schemes with no precoding achieves much worse performance. The projection-type method performs worst, because targets may fall in the row space of $\mathbf{G}_1$.

8. Conclusion

- Simulation results have validated the effectiveness of the proposed joint design approach for radar and communication spectrum sharing.
- Radar and communication coexistence is a new line of work, which calls for cooperation across public and private sectors on regulation and policy revision.