HYBRID BEAMFORMING WITH TWO BIT RF PHASE SHIFTERS IN SINGLE GROUP MULTICASTING

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Özlem Tuğfe Demir, T. Engin Tuncer

Electrical and Electronics Engineering Department,
Middle East Technical University, Ankara, Turkey

{deozlem, etuncer}@metu.edu.tr

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1 Problem Statement and Motivation

- Single group multicast beamforming exploits channel state information (CSI) to steer power effectively to a group of users subscribing for the same data stream.
- This problem is studied mostly using digital beamforming where a separate RF chain is required for each antenna [2].
- Although the full capacity is achieved with digital beamforming, its cost and complexity are high.
- In this paper, we propose a special TWO BIT hybrid beamforming structure as shown in Fig. 1 to decrease hardware cost while maintaining comparable performance with respect to the completely digital beamformer [4], [5].

2 Contributions

- This is the first work which considers hybrid beamforming for single group multicasting.
- A special problem formulation is derived for two bit RF phase shifters which leads to simplicity, low cost and effective solution.
- The combinatorial optimization problem is converted to a continuous programming formulation.
- 2 IS BETTER THAN 4!

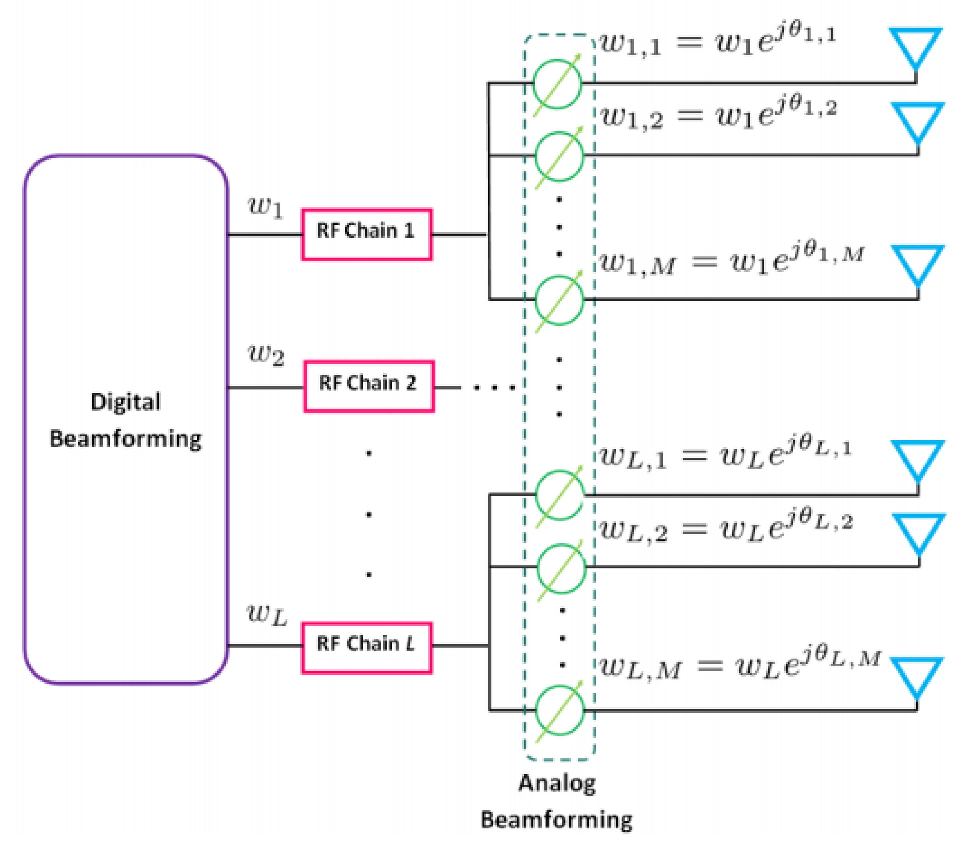

Fig. 1. Hybrid Beamforming System

3 System Model

$\mathbf{x}(t) = s(t)\mathbf{w}$

(transmitted signal (M x 1); $s(t)$: information signal with variance σ_s^2; $\mathbf{w}$: complex beamforming weight vector (M x 1))

$y_k(t) = \mathbf{h}_k^H \mathbf{x}(t) + n_k(t) \quad k = 1, \dots N$

(received signal for the k^{th} user; $\mathbf{h}_k$: complex channel vector for the k^{th} user; $n_k(t)$: additive noise uncorrelated with the information signal with variance σ_k^2)

$SNR_k = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{h}_k|^2}{\sigma_k^2}$

(signal-to-noise ratio for the k^{th} user)

$\mathbf{w} = [\, w_{1,1} \dots w_{1,M} \; w_{2,1} \dots w_{2,M} \dots w_{L,1} \dots w_{L,M} \,]^T$ ($LM \times 1$)

$w_{l,m} = w_l e^{j\theta_{l,m}}$

(w_l: digital weight of the l^{th} RF chain; $\theta_{l,m}$: phase of the m^{th} RF phase shifter following the l^{th} RF chain)

$|w_l| = |w_{l,m}|$ for $m = 1, \dots, M$ and $l = 1, \dots, L$

$\theta_{l,1} = 0$ (i.e., $w_{l,1} = w_l$) for $l = 1, \dots, L$

$w_{l,m}/w_{l,1} \in \{1, j, -1, -j\}, m = 2, \dots, M, l = 1, \dots, L$ (2-bit phase shifts)

4 QoS-Aware Hybrid Beamforming

- Quality of service (QoS) aware multicast beamforming problem is to minimize the total transmitted power subject to received SNR constraint for each user, i.e.,

$$\min_{\mathbf{w} \in \mathbb{C}^{LM}} \mathbf{w}^H \mathbf{w}$$
$$s.t. \quad \mathbf{w}^H \mathbf{R}_k \mathbf{w} \geq \gamma_k \sigma_k^2, \quad k = 1, \dots, N$$
$$\frac{w_{l,m}}{w_{l,1}} \in \{1, j, -1, -j\}, \quad m = 2, \dots, M, \; l = 1, \dots, L$$

(γ_k: minimum SNR need of the k^{th} user) $\quad \mathbf{R}_k = \mathbf{h}_k \mathbf{h}_k^H$

- The above problem is not convex and has a combinatorial nature.

$$\min_{\mathbf{W} \in \mathbb{C}^{LM \times LM}} Tr\{\mathbf{W}\} \quad (4.a)$$
$$s.t. \quad Tr\{\mathbf{R}_k \mathbf{W}\} \geq \gamma_k \sigma_k^2, \quad k = 1, \dots, N \quad (4.b)$$
$$\frac{W_{l,l}(m,1)}{W_{l,l}(1,1)} \in \{1, j, -1, -j\}, \quad (4.c)$$
$$m = 2, \dots, M, \; l = 1, \dots, L$$
$$\mathbf{W} \succeq 0 \quad (4.d)$$
$$rank(\mathbf{W}) = 1 \quad (4.e)$$

$\mathbf{W} = \mathbf{w}\mathbf{w}^H$

$W_{l_1,l_2}(m_1, m_2)$ → (m_1,m_2)-th entry of the (l_1,l_2)-th MxM submatrix of W

- The optimization problem in (4) is still nonconvex due to (4.c) and (4.e).
- The following lemma is used to express the discrete constraints in (4.c) in terms of continuous variables.

> ***Lemma 1:*** The constraints in (4.c) can be expressed as linear equality and inequalities as follows,
>
> $$\frac{-W_{l,l}(1,1)}{\sqrt{2}} \leq \mathrm{Re}(W_{l,l}(m,1)e^{j\pi/4}) \leq \frac{W_{l,l}(1,1)}{\sqrt{2}} \quad (5.a)$$
> $$\frac{-W_{l,l}(1,1)}{\sqrt{2}} \leq \mathrm{Im}(W_{l,l}(m,1)e^{j\pi/4}) \leq \frac{W_{l,l}(1,1)}{\sqrt{2}} \quad (5.b)$$
> $$W_{l,l}(m,m) = W_{l,l}(1,1), \quad m = 2, \dots, M, \; l = 1, \dots, L \quad (5.c)$$

- When (4.c) is replaced by (5), the optimization problem in (4) can be solved using semidefinite relaxation (SDR) by dropping the rank condition [7].
- In SDR, rank one solution is not guaranteed, and it may return unacceptable solutions in certain problems including (4).
- In our previous work [6], an effective approach is presented for the semidefinite programming problems with rank one constraint.

5 Equivalent Problem

- The following lemma is used to express rank constraint in a quadratic form.

> ***Lemma 2:*** For a Hermitian symmetric, positive semidefinite matrix $\mathbf{W}$, the condition in (6) necessitates $\mathbf{W}$ being a rank one matrix.
>
> $$(Tr\{\mathbf{W}\})^2 - Tr\{\mathbf{W}^2\} \leq 0 \quad (6)$$

- Using Lemma 2, the rank constraint in (4.e) can be replaced by (6).
- The only nonconvex constraint (6) can be moved into the objective function using exact penalty approach [8].

> ***Lemma 3:*** ([12], page 487): The problem in (4) is equivalent to the problem in (7) for $\mu > \mu_0$ with μ_0 being a finite positive value in the sense that any local minimum of the problem in (4), which satisfies the second order sufficiency conditions, is also a local minimum of the problem in (7).
>
> $$\min_{\mathbf{W} \in \mathbb{C}^{LM \times LM}} Tr\{\mathbf{W}\} + \mu \max(0, (Tr\{\mathbf{W}\})^2 - Tr\{\mathbf{W}^2\}) \quad (7)$$
> $$s.t. \; (4.b), (4.d), (5.a), (5.b), (5.c)$$

- (7) can be expressed as,

$$\min_{\mathbf{W} \in \mathbb{C}^{LM \times LM}} Tr\{\mathbf{W}\} + \mu((Tr\{\mathbf{W}\})^2 - Tr\{\mathbf{W}^2\}) \quad (8)$$
$$s.t. \; (4.b), (4.d), (5.a), (5.b), (5.c)$$

6 Alternating Minimization Algorithm

Hybrid Beamforming Algorithm (HBA)

Let $\lambda_{max}(\mathbf{W})$ be the maximum eigenvalue of the matrix $\mathbf{W}$.

Initialization: $k = 0$,
Solve (9) for $\mathbf{W^0}$ while fixing $\mathbf{W^{-1}}$ as zero matrix. Set a proper μ.
Iterations: $k = k + 1$
1) Solve (9) for $\mathbf{W^k}$ while fixing $\mathbf{W^{k-1}}$. If $rank(\mathbf{W^k}) = 1$ go to step 4.

$$\min_{\mathbf{W} \in \mathbb{C}^{LM \times LM}} Tr\{\mathbf{W}\} + \mu(Tr\{\mathbf{W^{k-1}}\}Tr\{\mathbf{W}\} - Tr\{\mathbf{W^{k-1}}\mathbf{W}\}) \quad (9)$$
$$s.t. \; (4.b), (4.d), (5.a), (5.b), (5.c)$$

2) If $\frac{\lambda_{max}(\mathbf{W^k})}{Tr\{\mathbf{W^k}\}} \geq \beta \frac{\lambda_{max}(\mathbf{W^{k-1}})}{Tr\{\mathbf{W^{k-1}}\}}$ (improved solution), where $\beta > 1$ is a proper positive threshold value (Ex: 1.5), keep the value of μ same. Otherwise, increase μ (Ex: $\mu \rightarrow 2\mu$)
3) Terminate if the maximum iteration number, $k = k_{max}$, is reached.
End:
4) If $rank(\mathbf{W^k}) = 1$, take the beamformer weight vector as the principal eigenvector of the matrix $\mathbf{W^k}$. Otherwise, select the elements of the beamformer weight vector as,

$$w_{l,1} = \sqrt{W_{l,l}^k(1,1)} e^{\angle(W_{l,1}^k(1,1)/W_{1,1}^k(1,1))} \quad (10.a)$$
$$w_{l,m} = w_{l,1} e^{\hat{\theta}(\angle(W_{l,l}^k(m,1)/W_{l,l}^k(1,1)))} \quad (10.b)$$
$$m = 2, \dots, M, \; l = 1, \dots, L$$

where $\hat{\theta}(\angle(W_{l,l}^k(m,1)/W_{l,l}^k(1,1)))$ is the quantized angle such that $\hat{\theta}(\angle(W_{l,l}^k(m,1)/W_{l,l}^k(1,1))) \in \{0, \pi/2, \pi, 3\pi/2\}$.
5) If necessary, scale $\mathbf{w}$ properly such that all SNR constraints are satisfied.

7 Simulation Results

- The minimum SNR threshold=$\gamma_k = 10$.
- Noise variance=$\sigma_k^2 = 1$.

Comparison of Full Digital, Full Analog and Two Bit Hybrid Beamformers

- Number of RF chains=L
- Number of RF phase shifters per RF chain=M
- LM=32.

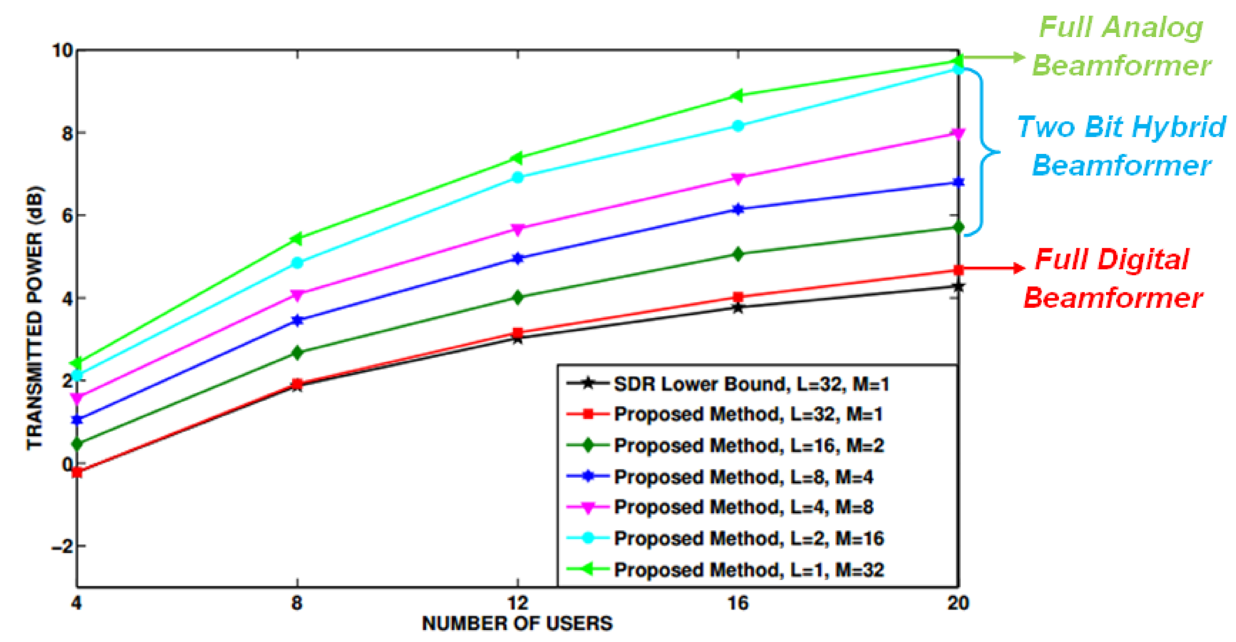

Fig. 2. Transmitted power for different number of RF chains and users for an array of $LM = 32$ antennas.

- Two bit hybrid beamformer is an effective structure to decrease the number of RF chains.

Comparison of Full Digital, Full Analog and Two Bit Hybrid Beamformers for the Number of RF Chains

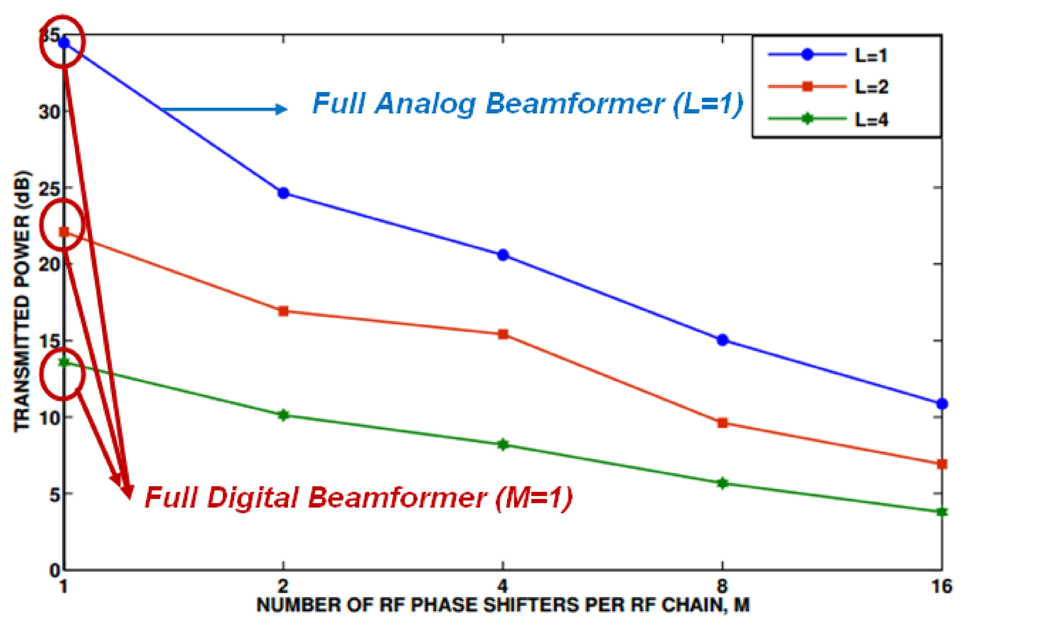

Fig. 4. Transmitted power for different number of RF chains and phase shifters for $N = 12$ users.

- $L = 2$ RF chain with $M = 8$ phase shifters per RF chain, has better performance than full digital beamformer with $L = 4$ RF chains.

Comparison with Antenna Selection

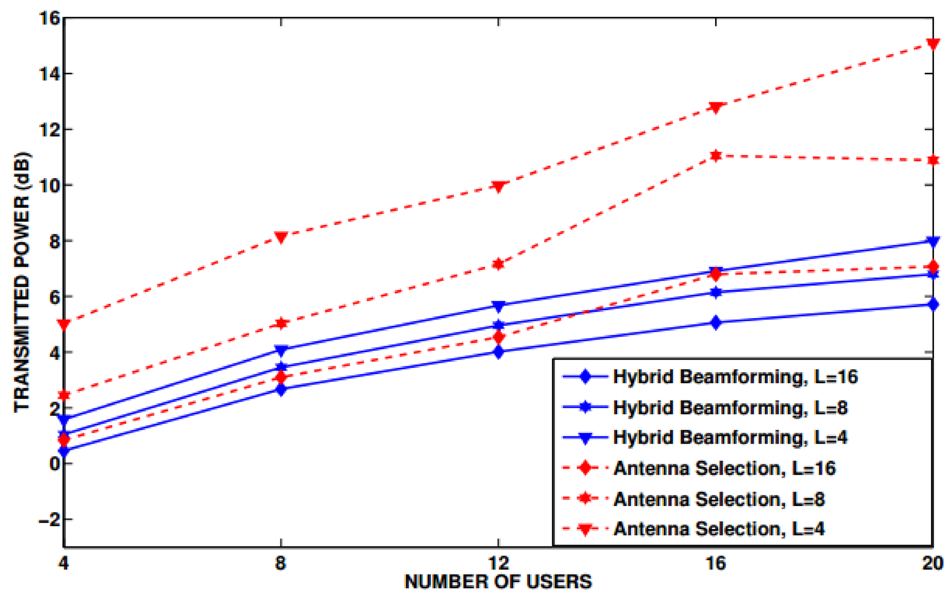

Fig. 5. Comparison of hybrid beamforming and antenna selection in terms of transmitted power.

- Hybrid beamforming results significant power saving with the use of cost efficient and simple two bit phase shifters.

CONCLUSIONS

- In this paper, a special hybrid beamforming structure is proposed for single group multicasting and the joint design of analog and digital beamformers is considered.
- The combinatorial optimization problem is converted to a quadratic-cost problem with linear constraints over a semidefinite matrix.
- The proposed method is efficient in terms of both hardware complexity and the performance.
- Simulation results show that it is a good low-cost alternative to full digital beamforming.
- The proposed method designs hybrid beamformer effectively and it performs better than antenna selection.

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