AN IMPROVED WAVELET BASED SHOCK WAVE DETECTOR

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ABSTRACT

In this paper, the detection of shock wave that generated by supersonic bullet is considered. A wavelet based multi-scale products method has been widely used for detection. However, the performance of method decreased at low signal-to-noise ratio (SNR). It is noted that the method does not consider the distribution of the signal and noise. Thus we analyze the method under the standard likelihood ratio test in this paper. It is found that the multi-scale product method is made in an assumption that is extremely restricted, just hold for a special noise condition. Based on the analysis, a general condition is considered for the detection. An improved detector under the standard likelihood ratio test is proposed. Monte Carlo simulations is conducted with simulated shock waves under additive white Gaussian noise. The result shows that this new detection algorithm outperforms the conventional detection algorithm.

Index Terms— shock wave, detector, wavelet transform, edge detection, likelihood ratio test

1. INTRODUCTION

The gunshots detection is of topical interest in areas related to anti-terrorism, public security, and military actions. Many studies in recent years focus on gunshot theory and their potential applications [1][2][3]. Many monitoring systems of gunshots are also developed in recent years [4][5][6][7][8].These methods are based on the detection of acoustical gunshot signatures, including muzzle blast and shock wave [1][2]. Muzzle blast is caused by an explosive charge to fire the bullet, and the shock wave is propagating away from the supersonic bullet's path. The shock wave is an N-shaped wave emanating in the form of an acoustic cone trailing the projectile. As a distinguishable signature of acoustical gunshot signal, the shock wave has been widely used for shooter localization and weapon classification in the literature [6][9][10][11]. This paper focuses on the detection of shock wave. Detection of shock wave is the key part of an initial low-power processing stage. If a gunshot exists, this detector would activate an advanced algorithm including localization and classification.

Because of its practical importance, couples of algorithm are developed to accomplish the task of shock waves detection. These methods can be classified as frequency-based [12], shape-based [6] [13], and Joint time-frequency based [14]. As discussed below, we are particularly interested in shock waves detection methods that are based on the shape of shock wave. Sadler [13] proposes a multi-scale product (MSP) detector based on discrete wavelet transform. This approach enhances multi-scale peaks due to edges, while suppressing noise, by exploiting the multi-scale correlation due to the presence of the desired signal in a direct way. It has low computational complexity and robust with unknown interference. The method has been widely used in shock wave detection. However, the performance decreases at low SNR. It doesn't consider the distribution of the signal and noise. It is not under the standard likelihood ratio test.

In this paper, first we introduce the mathematic model of shock wave. The generalized likelihood ratio test (GLRT) is given based on the model. Second, we formulate the shock wave detection problem as a detection problem in the time-scale domain. We present a theoretical analysis of Multi-scale product (MSP) method. We show that the MSP method is optimal in likelihood ratio sense only when we assume that the coefficients follows log-normal distribution with some assumptions of the mean and the variance. However, such assumption of the mean and the variance is not accurate (inconsistent with the model). Then we presented a new likelihood ratio based detect method. We conduct Monte Carlo simulations using the simulated shock waves with additive white Gaussian noise. The results illustrate that the performance of new detector is better than MSP detector. The new methods can detect shock wave with lower signal to noise ratio than the former methods.

2. THE SHOCK WAVE MODEL

The acoustic shock wave from the bullet has a very fast rise to a maximum, followed by a corresponding minimum. As

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the shock wave propagates, the pressure disturbance forms an "N" shape due to the nonlinear behavior of the air. The shape is with very fast rise and fall edges, and a linear slope between the edges[1].

The mathematical model used here was introduced by Whitman [15]. The waveform is defined using (1) and (2) for calculating the atmospheric peak amplitude and the length of the shock wave. These form the Whitman model, and are defined as

$$A = \frac{0.53\phi P_0 \left(M^2 - 1\right)^{1/8}}{d^{3/4} l^{1/4}}, \tag{1}$$

$$L = \frac{1.82\phi M d^{1/4}}{c(M^2 - 1)^{3/8} l^{1/4}}, \tag{2}$$

where P_0 is the atmospheric air pressure, ϕ is the diameter of a projectile, $M = v/c$ is the Mach number, $d$is the miss distance, and l is the length of the bullet.

The simulation time-domain waveform of shock wave can be modeled as

$$f(t;\theta) = A \cdot \left(1 - \frac{2\,(t-\tau)}{L}\right), \tau \le t \le \tau + L, \tag{3}$$

where t is discrete time index, A is the amplitude, L is the length of shock wave, and τ is the arrival time. $\theta = [\tau, A, L]$ is the parameter vector. It is shown in Fig.1. We remark that shock wave is formed as a constant slope N wave.

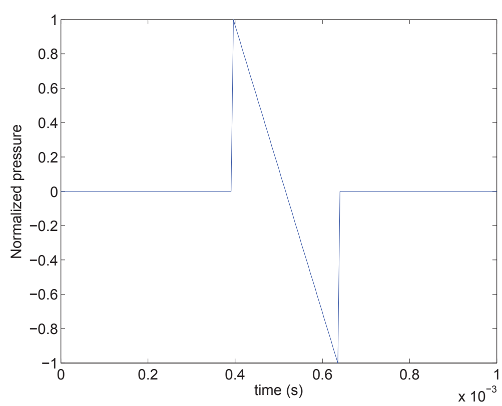


Fig. 1. Simulated ideal shock wave.

Consider the detection of a transient signal with unknown amplitude, length and arrival time. The binary hypothesis test $H_0 : x(t) = v(t)$; $H_1 : x(t) = f(t;\theta) + v(t), 0 \le t \le T$, where $v(t)$ is white Gaussian noise with variance σ^2 . A standard scheme for detection with unknown parameters is the generalized likelihood ratio test (GLRT)[16]. The form is

$$r\left(x(t)\right) = \frac{\max_{\{\tau,A,L\}} p\left(x(t)|\tau, A, L, H_1\right)}{p\left(x(t)|H_0\right)}. \tag{4}$$

If the likelihood ratio $r\left(x(t)\right)$ satisfies $r\left(x(t)\right) > \lambda$, hypothesis H_1 is accepted. λ is a threshold value that is chosen to according to a desired probability of false alarm. The GLRT corresponds to forming the maximum likelihood estimates of the parameters that are used in the likelihood ration test. A search of the parameter space creates significant computation complexity in the detector. What is more, the performance of GLRT degrades quickly for strong interference which violates the assumption of noise.

3. ANALYSIS OF WAVELET BASED MULTI-SCALE PRODUCT DETECTOR

In order to reduced the computational complexity and improve the robustness, Sadler proposed a Multi-scale product detector for shock wave edge detection[13]. This approach exploits the very fast rise and fall times of the shock wave edges. In the following, we will analysis this Multi-scale product detector.

MZ discrete wavelet transform (MZ-DWT)[17] is used to detect the edges of the signal. Denote the MZ-DWT operator by W , $X_j[k] = Wx[k]$ is the wavelet coefficient at time k in 2^j scale. Employing a multi-scale analysis can overcome choice of proper scale in some extent. The idea of a cross-correlation for edge detection is used here. The multi-scale product is

$$\Lambda\left(k\right) = \prod_{j=j_1}^{j_m} X_j(k). \tag{5}$$

The function $\Lambda\left(k\right)$ shows peaks at the N-wave edges. To detect the peak at the N-wave edge, the test statistics is designed as:

$$L_1(X) = \max_{\{k\}} \Lambda\left(k\right) = \max_{\{k\}} \prod_{j=j_1}^{j_m} X_j(k). \tag{6}$$

The use of $L_1(X)$ for detection exploits the MZ-DWT responsing to the signal and noise in a beneficial way. Singularities produce cross-scale peaks in $X_j(k)$, and these are reinforced in $L_1(X)$. If both of the two peaks corresponding to the two edges of N-wave are above the threshold, detection will be declared.

We found that the detector can be derived under the standard likelihood ratio test with certain distribution assumption. Also, it is a sub-optimal detector under that distribution. The optimal detector based on the distribution is derived in the following.

The detection problem is formed in the wavelet domain,

$$\begin{array}{ll} H_0 : X_j[k] \sim p(X_j[k]|H_0) & k = 0, 1, ..., N-1, \\ H_1 : X_j[k] \sim p(X_j[k]|H_1) & k = 0, 1, ..., N-1 \end{array},$$

where $x[n]$ represents a noisy observation at a discrete time

n. The likelihood ratio detector is

$$L(X) = \frac{p(X|H_1)}{p(X|H_0)} = \frac{\prod_{k=1}^{N} p(X(k)|H_1)}{\prod_{k=1}^{N} p(X(k)|H_0)}, \quad (7)$$

where $X(k) = \{X_{j_1}(k), X_{j_2}(k)..., X_{j_m}(k)\}$. Here we assumed that the coefficients are assumed independent with each other.

Consider the edges detection of shock wave by tracking maxima across scales. For simplicity, first we consider the maxima across scales correspond to one edge of shock waves. We focus on the maxima across scales. Under the two hypothesizes, we have

$$\begin{aligned} H_0 &: X_j[k] \sim p\left(X_j[k]\,|H_0\right) \quad k = 1, 2, ..., N \\ H_1 &: X_j[k] \sim p\left(X_j[k]\,|H_1\right) \quad k = i_E, \\ &\quad X_j[k] \sim p\left(X_j[k]\,|H_0\right) \quad k \neq i_E, \end{aligned}$$

where i_E denotes the time location of the edge.

Then the detector becomes:

$$\begin{aligned} L(X) &= p(X(k)|H_1)|_{k=i_E} \cdot \frac{\prod_{k=1,...N,k\neq i_E} p(X(k)|H_0)}{\prod_{k=1,...N} p(X(k)|H_0)} \\ &= \left.\frac{p(X(k)|H_1)}{p(X(k)|H_0)}\right|_{k=i_E} \end{aligned}$$

Since $i_E$is time location of the only edge under H_1, it is straightforward to have $\left.\frac{p(X(k)|H_1)}{p(X(k)|H_0)}\right|_{k=i_E} > \left.\frac{p(X(k)|H_1)}{p(X(k)|H_0)}\right|_{k\neq i_E}$. In other words, at the time location i_E , the likelihood ratio has maximal value. It is reasonable to argue that the time location $i_E = \arg\max_k \frac{p(X(k)|H_1)}{p(X(k)|H_0)}$. Thus the detector becomes:

$$L(X) = \left.\frac{p(X(k)|H_1)}{p(X(k)|H_0)}\right|_{k=i_E} = \max_{\{k\}} \frac{p(X(k)|H_1)}{p(X(k)|H_0)}. \quad (8)$$

In order to achieve the form of the Multi-scale product detector, we assume that under H_1 and H_0 , the absolute wavelet coefficients follow Log normal distribution. The distributions are independent across scales. For simplicity, we denote the absolute coefficients $|X_j(k)|$ as $\dot{X}_j(k)$. We has $p\left(\dot{X}_j(k)|H_1\right) \sim LN(\mu_j^1(k), \sigma_j^2(k))$ and $p\left(\dot{X}_j(k)|H_0\right) \sim LN(\mu_j^0(k), \sigma_j^2(k))$, where $\mu_j^1(k)$ and $\mu_j^0(k)$ represent the mean value of scale j under H_1 and H_0 respectively. Under both of the two hypotheses, the variance is $\sigma_j^2(k)$.

The probability density function of log-normal distribution is

$$f_Y(y; \mu, \sigma) = \frac{1}{y\sigma\sqrt{2\pi}} e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}, y > 0. \quad (9)$$

Suppose the wavelet coefficients from scale 2^{j_1} to 2^{j_m} are considered, we have

$$\begin{aligned} &\max_{\{k\}} \frac{p(\dot{X}_{j_1}(k), \dot{X}_{j_2}(k), ..., \dot{X}_{j_m}(k)|H_1)}{p(\dot{X}_{j_1}(k), \dot{X}_{j_2}(k), ..., \dot{X}_{j_m}(k)|H_0)} \\ &= \max_{\{k\}} \frac{\exp\left(\sum_{j=j_1}^{j_m} -(\ln \dot{X}_j(k) - \mu_j^1)^2 / 2\sigma_j^2\right)}{\exp\left(\sum_{j=j_1}^{j_m} -(\ln \dot{X}_j(k) - \mu_j^0)^2 / 2\sigma_j^2\right)} \\ &= \max_{\{k\}} \prod_{j=j_1}^{j_m} \left(\dot{X}_j(k)\right)^{\left(\mu_j^1-\mu_j^0\right)/\sigma_j^2} \cdot e^{\sum_{j=s_1}^{s_m} \left(\left(\mu_j^0\right)^2 - \left(\mu_j^1\right)^2\right)/2\sigma_j^2} \\ &= \max_{\{k\}} \prod_{j=j_1}^{j_m} \left(\dot{X}_j(k)\right)^{\left(\mu_j^1-\mu_j^0\right)/\sigma_j^2} \cdot \xi \end{aligned}$$

where

$$\xi = e^{\sum_{j=s_1}^{s_m} \left(\left(\mu_j^0\right)^2 - \left(\mu_j^1\right)^2\right)/2\sigma_j^2}.$$

Compare these two detection statistics $L_1(X)$ and $L(X)$ we found that only if $\left(\mu_j^1 - \mu_j^0\right)/\sigma_j^2 = 1, j = j_1, j_2, ...j_m$, test statistics $L(X)$ is

$$L(X) = \max_{\{k\}} \prod_{j=j_1}^{j_m} \left(\dot{X}_j(k)\right) \cdot \xi > \gamma. \quad (10)$$

Then $L(X)$ and $L_1(X)$ are the same form except for ξ which has no relation with the observation $X_j(k)$. Under the limitation of the mean and variance, it becomes Multi-scale product detector. However, this limitation may not be accurate because the distributions of coefficients across scales are not the same. Furthermore, the variance is usually not equal to $\mu_j^1 - \mu_j^0$. In other words, the Multi-scale product detector is a sub-optimal detector based on the assumption of log normal Gaussian distribution.

4. AN IMPROVED DETECTOR

Based on the analysis of the section 3, it is straightforward to propose the new detector

$$L(X) = \max_{\{k\}} \prod_{j=j_1}^{j_m} \left(\dot{X}_j(k)\right)^{\left(\mu_j^1-\mu_j^0\right)/\sigma_j^2}. \quad (11)$$

From the derivation, we can find that this is optimal detector under log normal distribution assumption. Compared to the MSP detector, it considers the difference of mean and variance between different scales. In the following, we give the details of the implementation.

In order to estimate the parameters of the probability density functions more accurately, we separate the coefficients corresponding to the edge form the observations in wavelet domain. Since the edge can be characterized by maxima of the wavelet coefficients across scales [18], the maxima modulus across scales are extracted. We roughly consider the maxima belong to the subset of signal. The rest of the coefficients are assumed to be the subset of noise. With this separation, the mean and variance of log normal distribution can be estimated more accurately.

The mean μ_j^1 under hypothesis H_1 is estimated from the subset of signal. To get a more accurate estimation, the relationship of the maxima modulus across scales is used. If a function has an isolated singularity of order$\alpha \leq r$ atn_0, there exists a maxima line $M(s) \rightarrow n_0$ such that at fine scales$|X_s(M(s))| \sim s^{\alpha+1/2}$ [18]. Then, $\hat{\mu}_j^1$ can be estimated by using an approximation $|X_j(k)| = B \cdot \left(2^j\right)^{\alpha+1/2}$. Since the Maximum likelihood estimation of parameters for joint log-normal distribution is $\hat{\mu}_j^1 = \frac{\sum \ln \dot{x}_j}{n}$, we have the equation $\hat{\mu}_j^1 = \hat{B} + \left(\hat{\alpha} + \frac{1}{2}\right) \cdot \ln\left(2^j\right)$. For convenient, we first estimate $\hat{B}$ and $\hat{\alpha}$ through the maxima line by using linear regression. The maxima line estimation algorithm is based on the ridge-finding algorithm in [19]. Note that we assume the variances are the same under two hypothesizes. Since most of the coefficients contain noise only, it is more accurate to estimate the variance from the noise subset.

Estimation of variance and mean under hypothesisH_0 is straightforward. Maximum likelihood estimation of parameters are $\hat{\mu}_j^0 = \frac{\sum \ln \dot{x}_j}{n}$ and $\hat{\sigma}_j^2 = \frac{\sum\left(\ln \dot{x}_j - \hat{\mu}_j^0\right)^2}{n}$ [20]. The coefficients of the noise subset are used for estimation.

We can further reduce the computation complexity by observing that the time shift of maxima converges toward to the time location of the edge in fine scales. Therefore, we expect that the time shift of maxima in fine scales is likely to correspond to the time location of the edge i_E. Instead of computing $L(X)$ for all values of k to find the maximal value, we constrain the computation to the time shift of maxima across scales.

5. SIMULATIONS

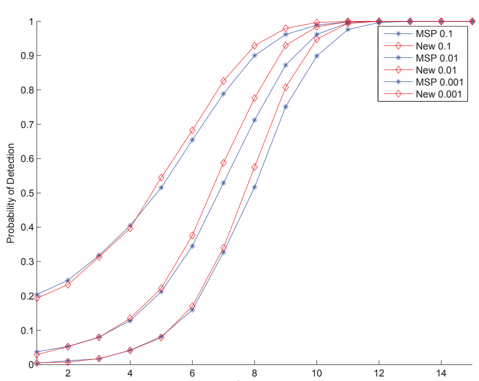
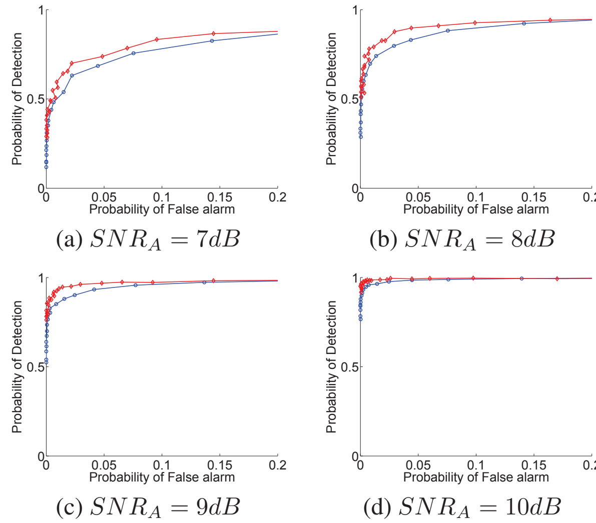


Fig. 2. Performance of detectors for method MSP (blue line) and the proposed method (red line): probability of detection versus SNR_A for $P_{fa} = 10^{-1}, 10^{-2}, 10^{-3}$.



(a) $SNR_A = 7dB$ (b) $SNR_A = 8dB$

(c) $SNR_A = 9dB$ (d) $SNR_A = 10dB$

Fig. 3. ROC curves for the MSP detector(blue line) and the proposed detector(red line).

The performance of the multi-scale product detector and the proposed detector is evaluated for simulated shock waves in additive Gaussian white noise. The simulated shock waves are constructed by using Eq.(3). Length L is fixed with $L = 46$. The value of L is established by using the method described in[13]. The signal-to-noise ratio was defined as $SNR_A = 10\log_{10}\frac{A^2}{\sigma_v^2}$, where A is the shock amplitude and σ_v^2 is the noise variance. The noisy signal was then go through MZ-DWT. Detection was carried out using equation with $j_1 = 1$ and $j_m = 3$. Data records of length are 1024. Detection thresholds were established yielding the false alarm rate $10^{-1}, 10^{-2}, 10^{-3}$ over 10000 times Monte Carlo simulation. Results in Fig.2 are averages over 5000 Monte Carlo trials for each value of SNR. Results in Fig.3 show receiver operating characteristic (ROC)curves evaluated for fixed S-NR. The results show that the proposed detector is better than the MSP detector. It reflects the benefits of considering the difference of the distribution across scales.

6. CONCLUSION

In this paper, we have analyzed the Multi-scale product detector. We derived the detector under log normal distribution and found that the detector can be derived with limitations of the parameters of the distribution. It is a sub-optimal detector under that distribution. We proposed the optimal detector under the distribution without the limitations of the parameters. The parameters of distribution can be estimated by tracking the maxima of the wavelet coefficients across scales. The simulation shows that the performance of proposed detector is better than the MSP detector.

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