Learned Convolutional Sparse Coding

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TEL AVIV UNIVERSITY

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Outline

- Motivation and applications
- Brief sparse coding background
- (ASC) Approximate sparse coding
- (CSC) Convolution sparse coding
- (ACSC) Approximate Convolution sparse coding

ACSC - real-time image processing

Sparse coding is a strong image processing tool

Sparse coding algorithms tend to be quite slow

Train an approximate efficient sparse coding model

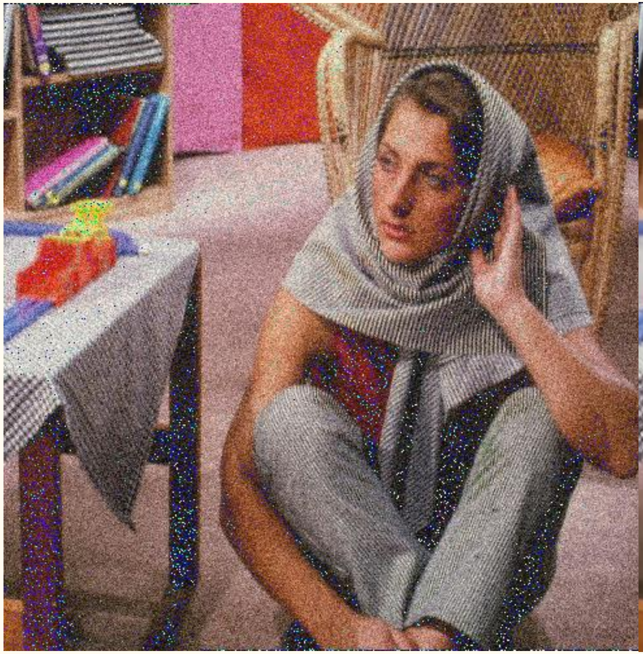
Keep the quality. **Drop** the runtime

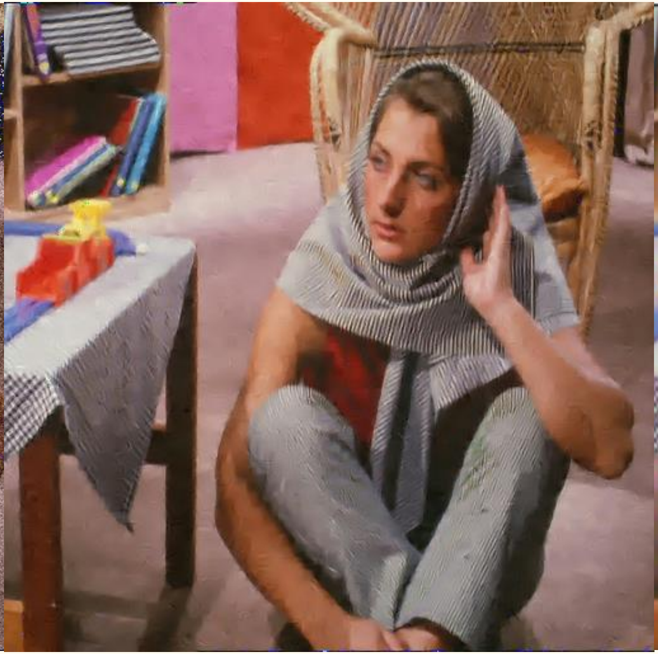
ACSC - advantages

Sparse coding has been shown to work well for Image processing task.
Yet, it tends to be slow ➡ Requires multiple minutes on CPU.

Approximate convolutional sparse coding cuts the runtime to a fraction while maintaining the sparse coding performance.
➡ Requires **less than a second** on a CPU.

ACSC - Image denoising example



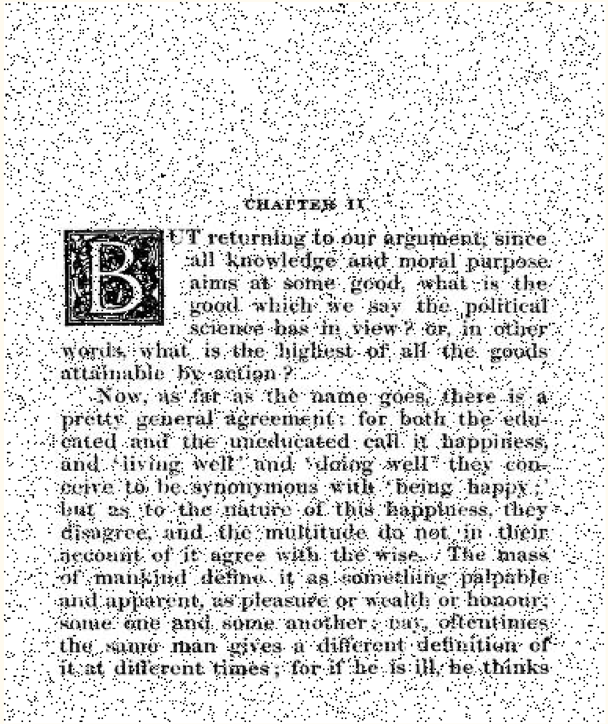


ACSC (0.7 sec) PSNR 29.88 dB

KSVD (70 sec) PSNR 28.67 dB

x100 speedup compared to OMP sparse coding in KSVD

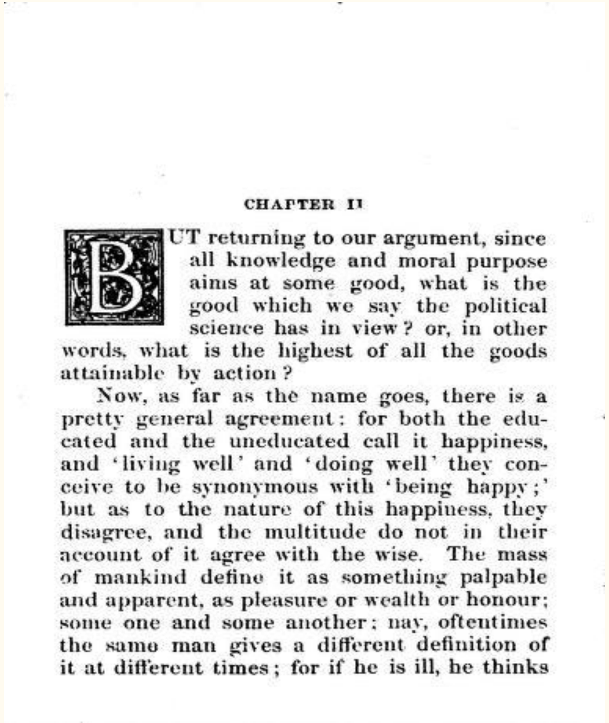
ACSC - Document S&P denoising



CHAPTER II

BUT returning to our argument, since all knowledge and moral purpose aims at some good, what is the good which we say the political science has in view? or, in other words, what is the highest of all the goods attainable by action?

Now, as far as the name goes, there is a pretty general agreement: for both the educated and the uneducated call it happiness, and 'living well' and 'doing well' they conceive to be synonymous with 'being happy;' but as to the nature of this happiness, they disagree, and the multitude do not in their account of it agree with the wise. The mass of mankind define it as something palpable and apparent, as pleasure or wealth or honour; some one and some another; nay, oftentimes the same man gives a different definition of it at different times; for if he is ill, he thinks



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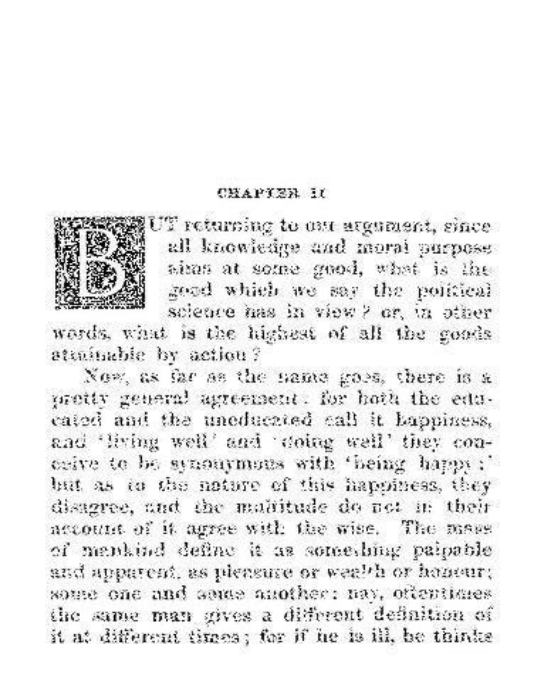
ACSC (0.43 sec) PSNR 30.2 dB

ACSC - Document inpainting



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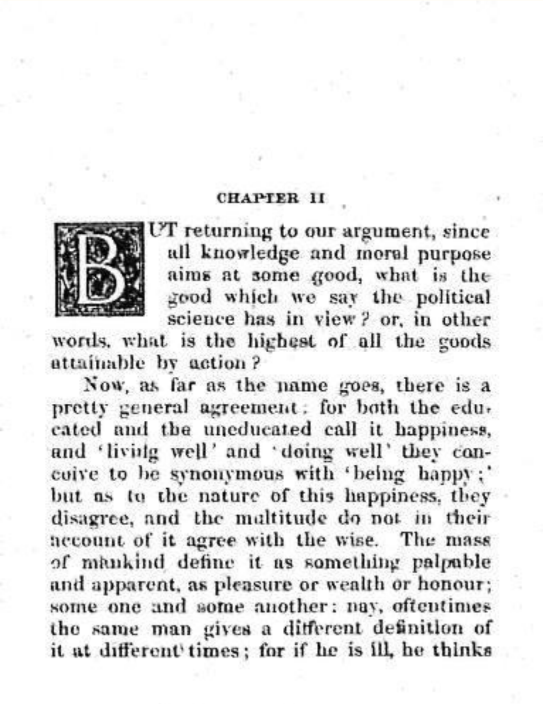

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ACSC (0.43 sec) PSNR 30.13 dB

Sparse Code problem setup

Given An input X and an over complete matrix D

Find Z^* such that:

$$X = DZ^* \quad ||Z^*||_0 < ||Z||_0 \;\; \forall Z \in \{Z|X = DZ\}$$

Sparse Code problem setup

Given An input X and an over complete matrix D

Find Z^* such that:

$$X = DZ^* \quad ||Z^*||_0 < ||Z||_0 \;\; \forall Z \in \{Z|X = DZ\}$$

Solving the above is intractable due to the ℓ_0 norm.

Can be approximated by $\boldsymbol{\ell_1}$ relaxation

Leads to the LASSO minimization problem

$$Z^*_{lasso} = \mathbf{argmin}_Z ||X - DZ||^2_2 + \lambda ||Z||_1$$

Sparse Code setup - solving LASSO

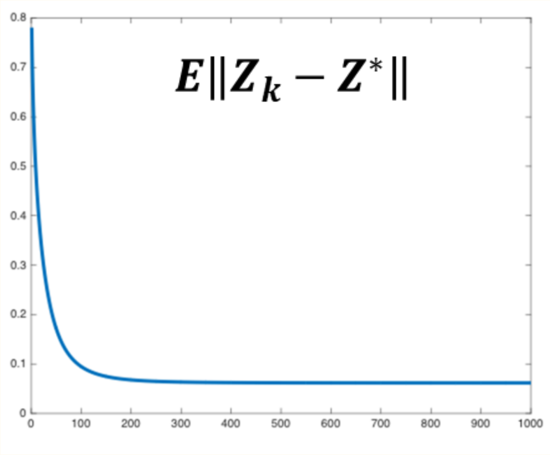
Use the proximal gradient technique -

$$f(x) = g(x) + h(x)$$

$$x^{(k)} = \mathbf{prox}_{t_k,h}(x^{(k-1)} - t_k \nabla g(x^{(k-1)}))$$

ISTA

$$Z_{k+1} = S_{\lambda/L}(Z_k + D^T(X - DZ_k))$$



This is an iterative method

Many iterations may be required for convergence!

Sparse Code setup - solving LASSO

Long convergence time of ISTA can be an issue for time sensitive applications

→ An acceleration strategy that approximates the solution is required

Learned ISTA (LISTA): Replaces linear operations and threshold in ISTA with learned ones

$$Z_{k+1} = S_{\lambda/L}((I - D^T D)Z_k + D^T X)$$

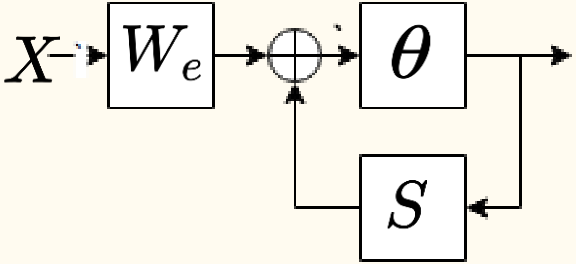
LISTA

$$Z_{k+1} = S_\theta(SZ_k + W_e X)$$

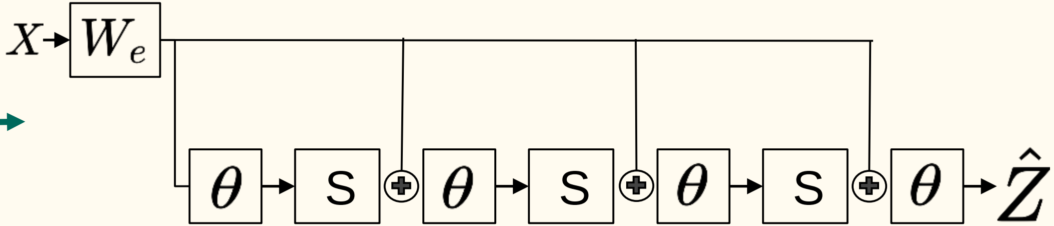
Using Learned-ISTA as a SC approximation

Given a fixed number of iterations learn the matrices S and W and the thresholds θ

$$Z_{k+1} = S_\theta(SZ_k + W_e X)$$



recurrent structure

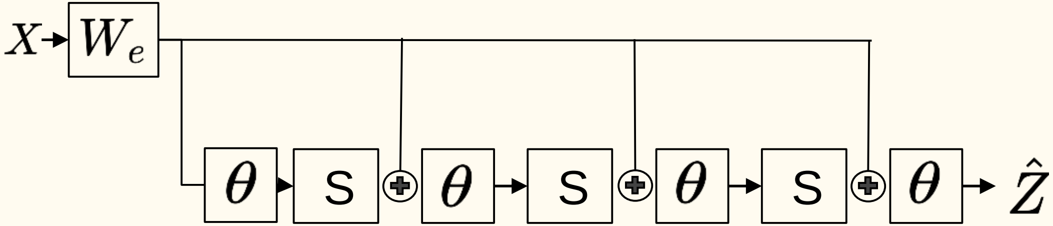


trained by unfolding

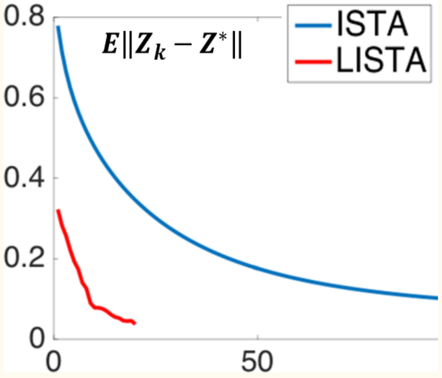
$$L(W, X^p) = \frac{1}{2}||Z^{*p} - f_e(W, X^p)||_2^2$$

LISTA training

LISTA iteration: $Z_{k+1} = S_\theta(SZ_k + W_e X)$



trained by unfolding



Train using loss function:

$$L(W, X^p) = \frac{1}{2}||Z^{*p} - f_e(W, X^p)||_2^2$$

Where Z^{*p} is the sparse code of X^p

Learning Fast Approximations of Sparse Coding - Karol Gregor and Yann LeCun

LISTA disadvantages

LISTA is a patch based method.

Therefore, we have

- Loss of spatial information.
- Inefficient for large patches
- Image properties not incorporated (i.e. translation invariance).

Solution: Use convolutional structure.

Approximate Convolutional Sparse Coding (ACSC)

Learn a **convolutional** sparse coding of the whole image instead of patches

➡ A global algorithm:

- Image is processed as whole.
- Efficient.
- Convolution based → inherently incorporates image properties.

This overcomes the disadvantages of LISTA being a patch based method:

- Loss of spatial information.
- Not Inefficient for large patches
- Image properties not incorporated (i.e. translation invariance)

Standard Convolutional Sparse Coding (CSC)

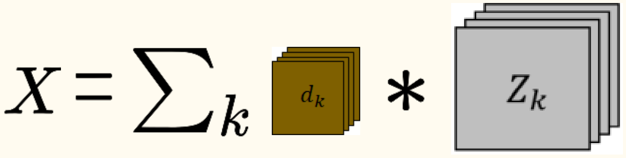
In convolutional sparse coding we replace the regular dictionary with a convolutional one:

Given input X, solve:

$$\mathbf{argmin}_{d,z} = ||X - \textstyle\sum_{k=1}^{K} d_k * Z_k||_2^2 + \beta \textstyle\sum_{k=1}^{K} ||Z_k||_1 \quad s.t. ||d_k||_2 \leq 1$$

In other word --

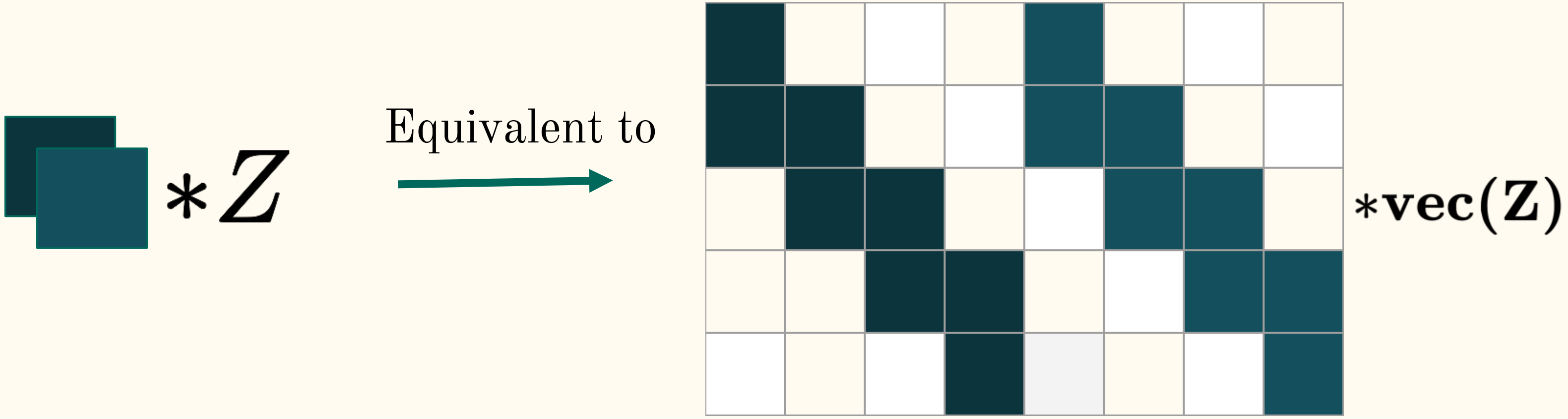
Find a set of kernels d that reconstruct X with sparse feature maps Z



Convolutional Sparse Coding as Regular Sparse Coding

Convolutional Sparse Coding (CSC) ⟷ Regular Sparse Coding (SC)

A convolutional dictionary is a concatenation of toeplitz matrices



From SC to CSC

Rewrite ISTA as a solution to CSC problem

Assuming **D** is a concatenation of toeplitz matrices we can replace matrix multiplication with convolution

D is a toeplitz matrix

$$Z_{k+1} = S_{\lambda/L}((I - D^T D)Z_k + D^T X)$$

Convolutional ISTA

$$\hat{d} = d^T$$

$$Z_{k+1} = S_{\lambda/L}(Z_k - \hat{d} * (d * Z_k) + \hat{d} * X)$$

$$d \in \mathbb{R}^{s \times s \times M \times C} \qquad \hat{d} \in \mathbb{R}^{s \times s \times C \times M}$$

From CSC to Approximate CSC

Approximate Convolutional sparse coding (ACSC): Replaces convolutional operations and threshold in Convolutional ISTA solution with learned ones

$$Z_{k+1} = S_{\lambda/L}(Z_k - \hat{d} * (d * Z_k) + \hat{d} * X)$$

ACSC

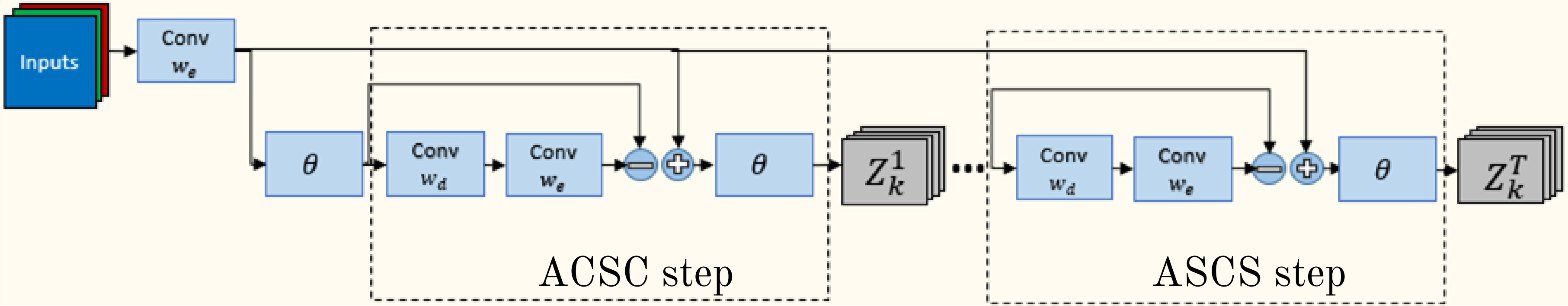
$$Z_{k+1} = S_{\theta}(Z_k - w_e * (w_d * Z_k) + w_e * X)$$

$w_d \in \mathbb{R}^{s \times s \times M \times C}$ $w_e \in \mathbb{R}^{s \times s \times C \times M}$

Approximate CSC architecture

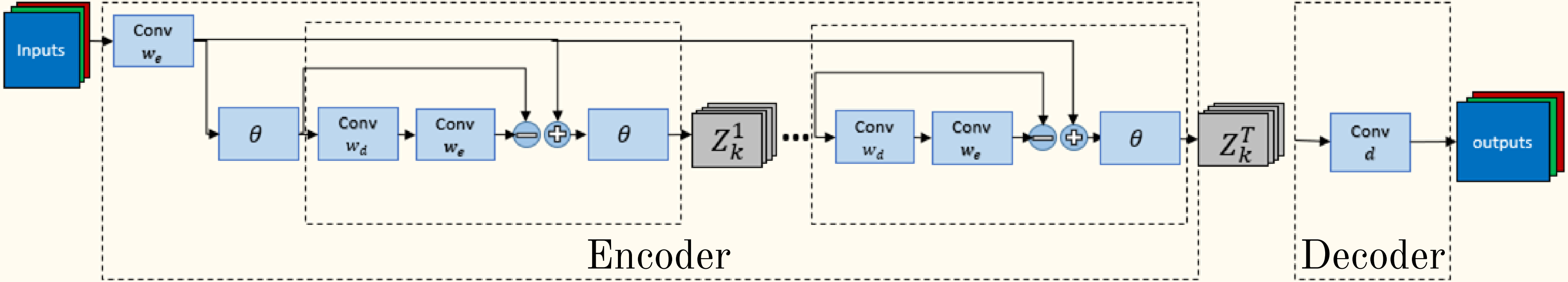
$$Z_{k+1} = S_\theta(Z_k - w_e * (w_d * Z_k) + w_e * X)$$

Proposed recurrent architecture for CSC with unfold of T steps



Approximate CSC with dictionary learning

Train end to end as a sparse autoencoder



Convolutional sparse encoder

Convolutional sparse dictionary

We learn in our framework both the sparse coding and the dictionary

In LISTA, only the sparse coding is being learned

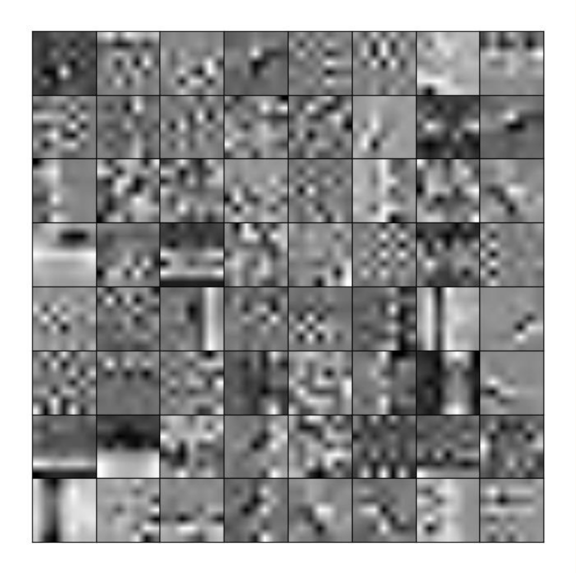
Training ACSC autoencoder

- Initialization is important $\longrightarrow$ $w_d = d = \mathbf{flipud}(\mathbf{fliplr}(w_e))$
- We found kernel of size 7x7 to give best results.
- We found Unfolding 3 time step to be sufficient
- Use loss function $L(x, \hat{x}) = \alpha(1 - \mathbf{ms\_ssim}(x, \hat{x})) + (1 - \alpha)||x - \hat{x}||_1$

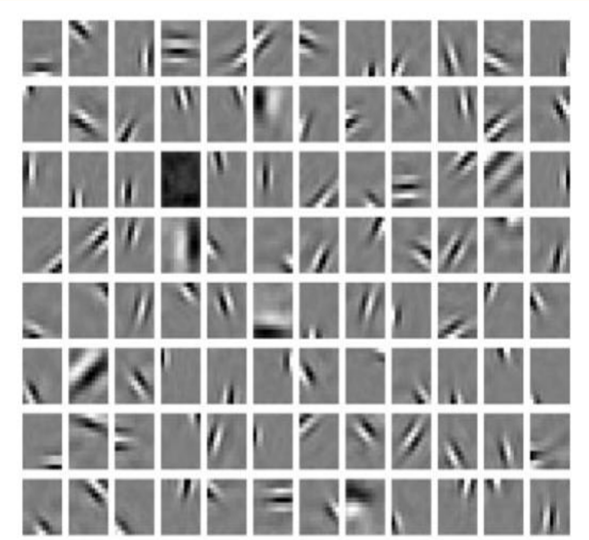
  H. Zhao, O. Gallo, I. Frosio, and J. Kautz, “Loss functions for image restoration with neural networks,”

Approximate CSC and dictionary learning

Learned convolution dictionary



Classical sparse dictionary



Classical approach tends to learn the same atom with a translation.

Convolution dictionary a single atom covers all translation.

Approximate CSC Tasks

Example result on denoising task

ACSC (0.56 sec) PSNR 32.11 dB KSVD (57 sec) PSNR 32.09 dB

x100 speedup compared to OMP sparse coding in KSVD

Approximate CSC Tasks

Example result on denoising task

ACSC (0.6 sec) PSNR 30.14 dB KSVD (64 sec) PSNR 30.05 dB

x100 speedup compared to OMP sparse coding in KSVD

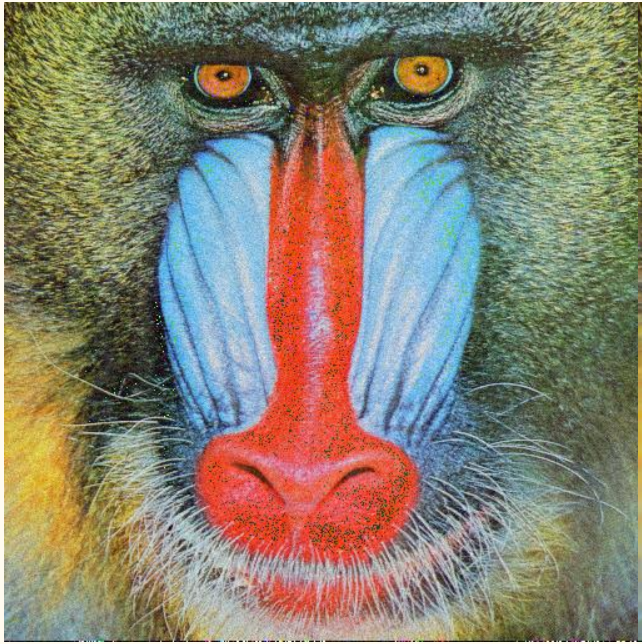
Approximate CSC Tasks

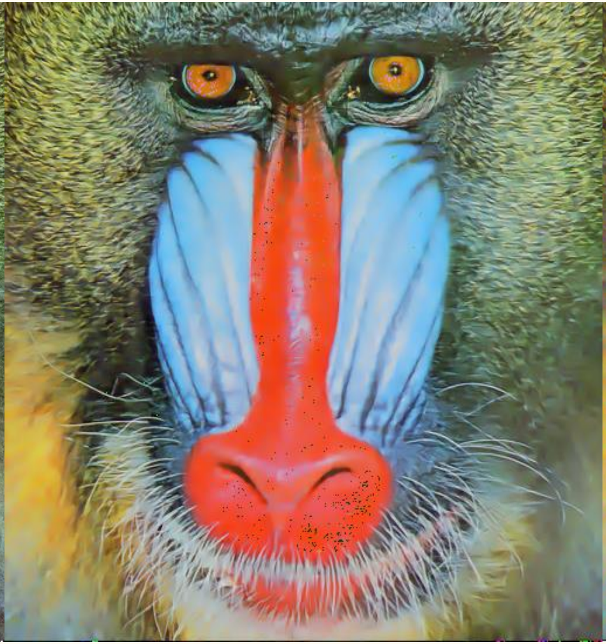
Example result on inpainting task

ACSC (0.57 sec) PSNR 32.3 dB

Approximate CSC Tasks

Example result on denoising task





ACSC (0.83 sec) PSNR 30.32 dB

KSVD (78 sec) PSNR 30.21 dB

x100 speedup compared to OMP sparse coding in KSVD

Approximate CSC Tasks

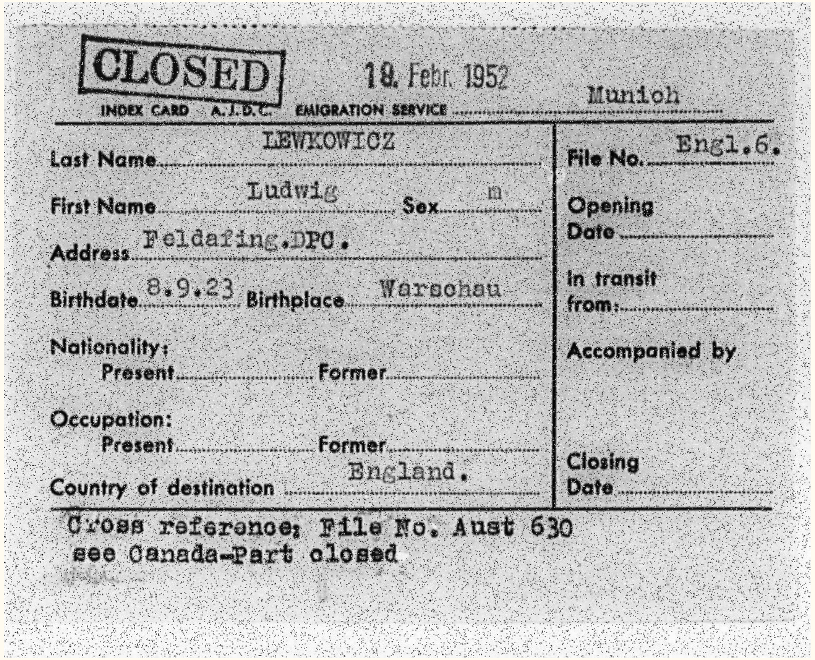
Example result on inpainting task

ACSC (0.48 sec) PSNR 30.32 dB   Heide et al.(320 sec) PSNR 30.21 dB

x650 speedup compared to Heide et al. (Fast and Flexible Convolutional sparse coding)

Approximate CSC Tasks

Example result on document denoising task


CLOSED

19. Febr. 1952

INDEX CARD A.J.D.C. EMIGRATION SERVICE Munich

Last Name LEWKOWICZ

File No. Engl.6.

First Name Ludwig Sex m

Opening Date

Address Feldafing.DPC.

Birthdate 8.9.23 Birthplace Warschau

In transit from:

Nationality:
Present Former

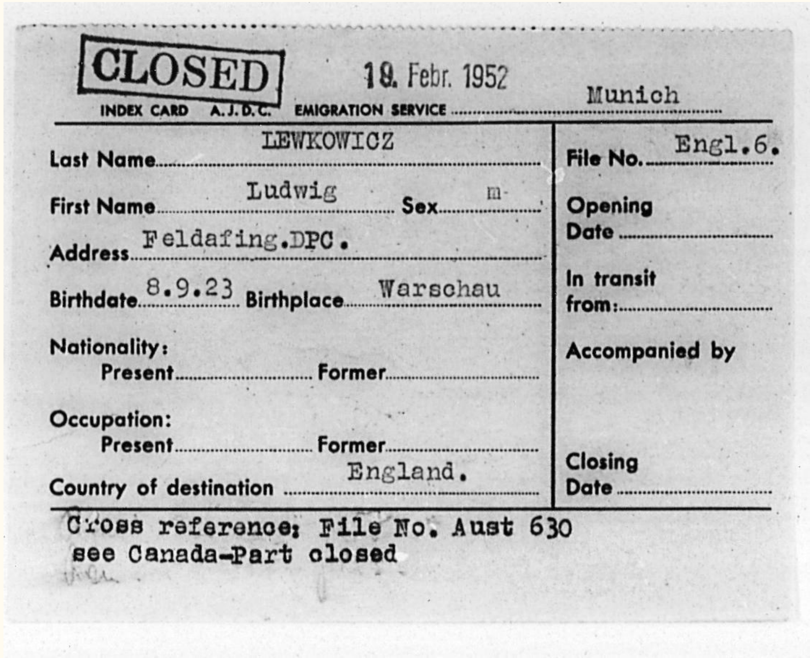
Accompanied by

Occupation:
Present Former

Closing Date

Country of destination England.

Cross reference; File No. Aust 630
see Canada-Part closed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Closing Date

Country of destination England.

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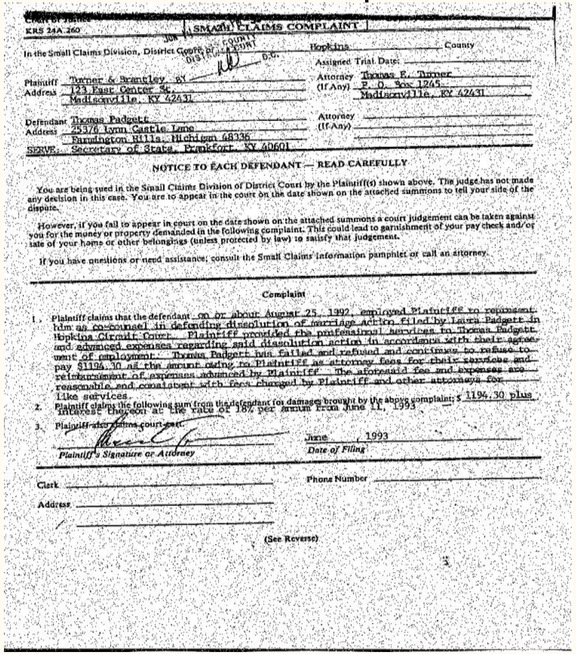
see Canada-Part closed

ACSC (0.44 sec) PSNR 29.8dB

Approximate CSC Tasks

Example result on document denoising task



Court of Justice
KRS 24A.260

SMALL CLAIMS COMPLAINT

In the Small Claims Division, District Court of Hopkins County

Assigned Trial Date:

Plaintiff Address: Turner & Brantley, 123 East Center St., Madisonville, KY 42431

Attorney (If Any): Thomas E. Turner, P. O. Box 1245, Madisonville, KY 42431

Defendant Address: Thomas Padgett, 25376 Lynn Castle Lane, Farmington Hills, Michigan 48336

SERVE: Secretary of State, Frankfort, KY 40601

Attorney (If Any):

NOTICE TO EACH DEFENDANT — READ CAREFULLY

You are being sued in the Small Claims Division of District Court by the Plaintiff(s) shown above. The judge has not made any decision in this case. You are to appear in the court on the date shown on the attached summons to tell your side of the dispute.

However, if you fail to appear in court on the date shown on the attached summons a court judgement can be taken against you for the money or property demanded in the following complaint. This could lead to garnishment of your pay check and/or sale of your home or other belongings (unless protected by law) to satisfy that judgement.

If you have questions or need assistance, consult the Small Claims Information pamphlet or call an attorney.

Complaint

1. Plaintiff claims that the defendant on or about August 25, 1992, employed Plaintiff to represent him as co-counsel in defending dissolution of marriage action filed by Laura Padgett in Hopkins Circuit Court. Plaintiff provided the professional services to Thomas Padgett and advanced expenses regarding said dissolution action in accordance with their agreement of employment. Thomas Padgett has failed and refused and continues to refuse to pay $1194.30 as the amount owing to Plaintiff as attorney fees for their services and reimbursement of expenses advanced by Plaintiff. The aforesaid fee and expenses are reasonable and consistent with fees charged by Plaintiff and other attorneys for like services.
2. Plaintiff claims the following sum from the defendant for damages brought by the above complaint: $ 1194.30 plus interest thereon at the rate of 18% per annum from June 11, 1993
3. Plaintiff also claims court cost.

Plaintiff's Signature or Attorney

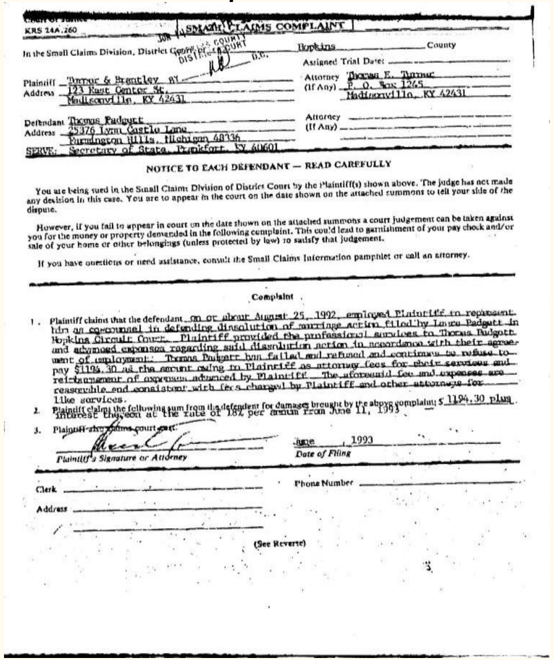
June , 1993
Date of Filing

Clerk

Phone Number

Address

(See Reverse)



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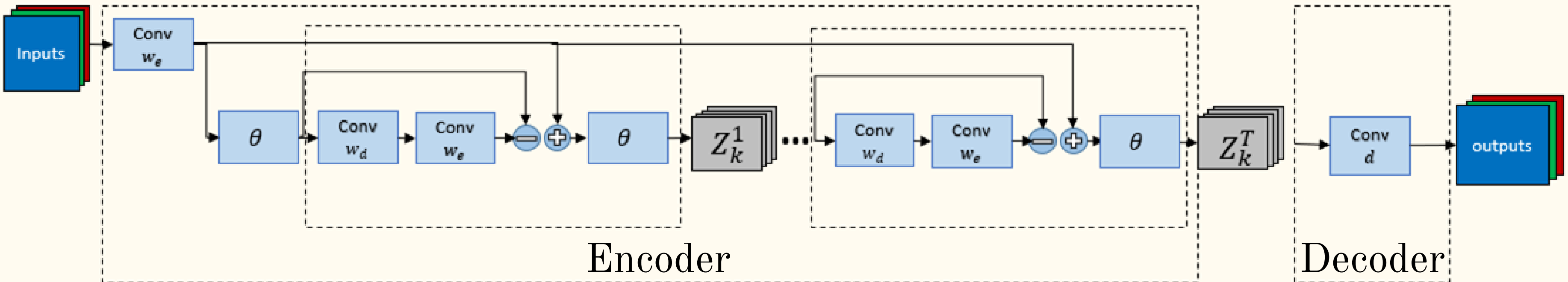
(See Reverse)

ACSC (0.38 sec) PSNR 29.4dB

Conclusion

- LISTA is good for accelerating sparse coding for patches
- Yet, it is slow when working on images as a whole
- Our proposed learned CSC is good for processing the whole image
- Provide comparable performance to other sparse coding based techniques on images with up to x100 speedup

Thank You!



Convolutional sparse encoder

Convolutional sparse dictionary