STRUCTURE-ADAPTIVE VECTOR MEDIAN FILTER FOR IMPULSE NOISE REMOVAL IN COLOR IMAGES

Lianghai Jin, Min Jin, Xiangyang Xu, and Enmin Song*

School of Computer Science and Technology

Huazhong University of Science and Technology, China

Introduction

- This paper propose a window-adaptive vector median filter (VMF) for color image denoising.

- The new method is based on local orientation estimation.

Introduction

- To preserve image details while smoothing of noise, the smoothing should be performed along the direction of the dominant orientation.

- The stronger the strength of a local orientation, the flatter the filter window should be.

How to detect the orientation

Theorem: the power spectrum of spatial pattern lies along a line through the origin in the Fourier domain, and the direction of the line is perpendicular to the dominant spatial orientation of the pattern.

To compute the orientation of a local pattern in color images, we need to analyze the power spectrum of the color pattern.

For a color image, the power spectrum is obtained from quaternion Fourier transform.

Quaternion Fourier transform (QFT)

Let a color image $\boldsymbol{f}(x, y)$ be represented in pure quaternion form:

$$\boldsymbol{f}(x,y) = R(x,y)\,i + G(x,y)\,j + B(x,y)\,k$$

Then, the quaternion Fourier transform (QFT) of $\boldsymbol{f}(x, y)$ is expressed as:

$$F(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,e^{-\mu\,(ux+vy)}dxdy$$

where μ is a unit pure quaternion and typically set to $(i + j + k)/\sqrt{3}$.

Quaternion power spectrum images

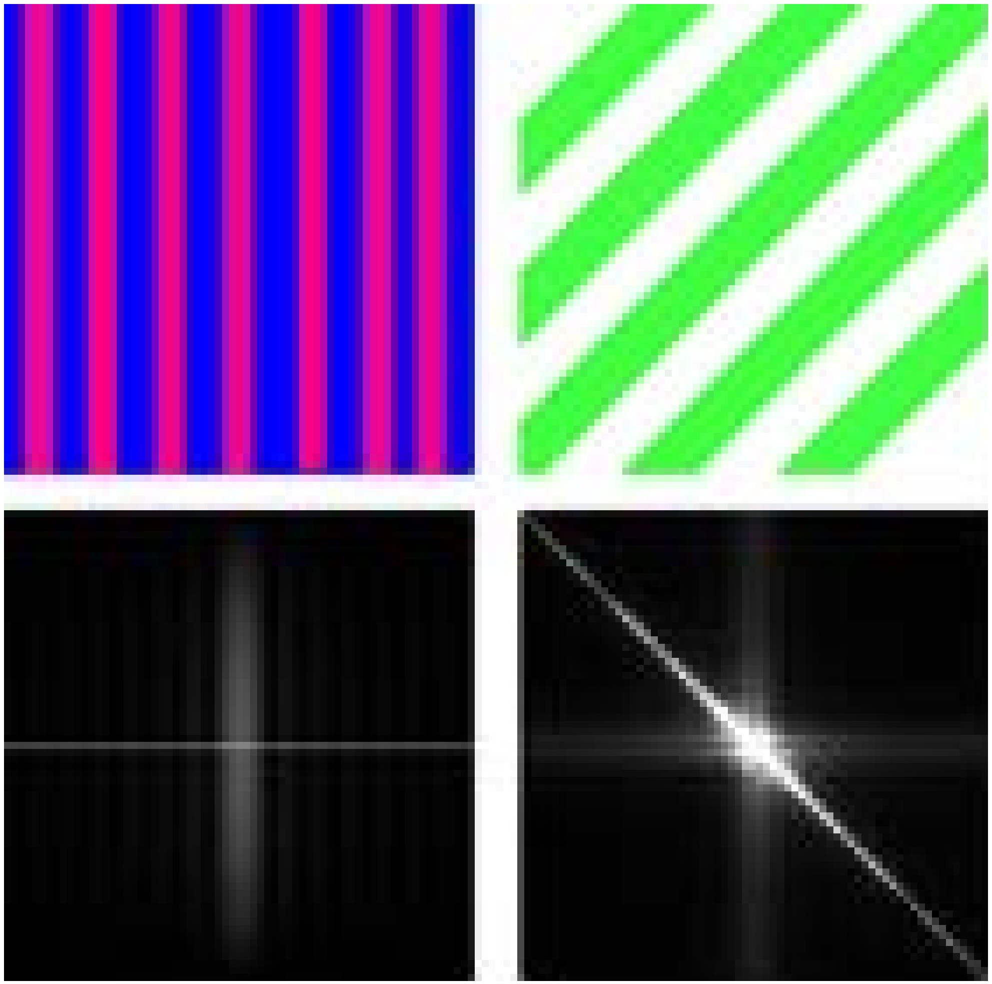

Figure. 1

Minimize Distance Sum

The detection of spatial color orientation becomes a minimization problem in QFT: finding a directional line $\boldsymbol{n}$ through the origin in the quaternion power spectrum image which minimizes the following distance sum $D(\boldsymbol{n})$:

$$D(\boldsymbol{n}) = \iint \left(d_{\boldsymbol{n}}(u,v) \left| F(u,v) \right| \right)^2 dudv$$

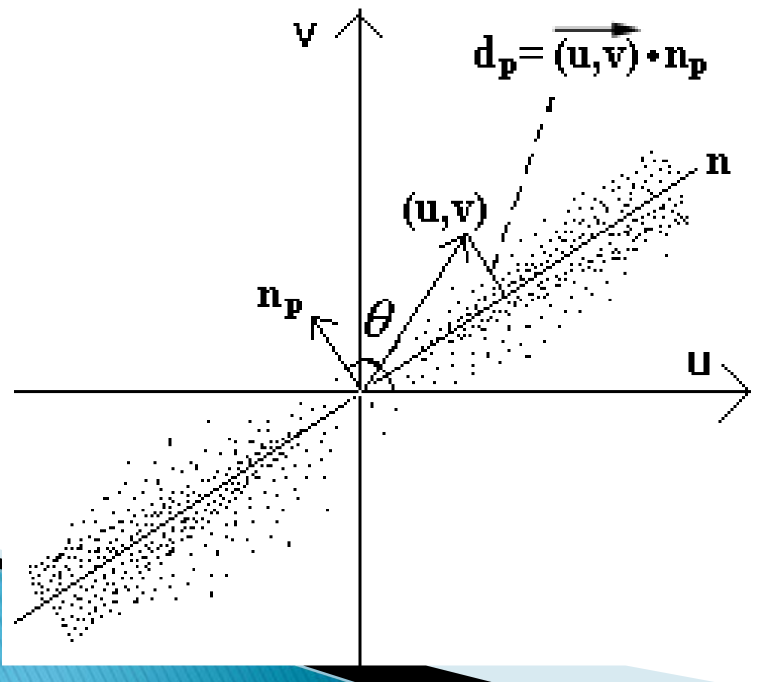

Fig. 2. The distance of point (u, v) to directional line $\boldsymbol{n}$, where $\boldsymbol{n}_p = (cos\theta\ sin\theta)$ is the normal vector of $\boldsymbol{n}$.

Orientation Strength

$$\mathbf{n}_p = (\cos\theta_{max} \;\; \sin\theta_{max}) \qquad \lambda_{max} = D(\mathbf{n}_p) = \iint \left(d_{\mathbf{n}_p}(u,v) \left| F(u,v) \right| \right)^2 du\,dv$$

$$\mathbf{n} = (\cos\theta_{min} \;\; \sin\theta_{min}) \qquad \lambda_{min} = D(\mathbf{n}) = \iint \left(d_{\mathbf{n}}(u,v) \left| F(u,v) \right| \right)^2 du\,dv$$

the orientation strength g

$$g = \left(\frac{\lambda_{max} - \lambda_{min}}{\lambda_{max} + \lambda_{min}} \right)^2$$

Support Window

- Denote the orientation and orientation strength of the pattern in neighborhood centered at any position $\boldsymbol{x}_0$ as $\boldsymbol{l}_{\boldsymbol{x}0}$ and $g(\boldsymbol{x}_0)$, then the support window $\boldsymbol{W}$ is defined as the following elliptic shape:

$$\boldsymbol{W} = \left\{ \boldsymbol{x} \mid \rho\left(\boldsymbol{x}, \boldsymbol{x}_0\right) \leq 1 \right\}$$

with

$$\rho\left(\boldsymbol{x}, \boldsymbol{x}_0\right) = \frac{\left((\boldsymbol{x} - \boldsymbol{x}_0) \cdot \boldsymbol{l}_{\boldsymbol{x0}}\right)^2}{r^2} + \frac{\left((\boldsymbol{x} - \boldsymbol{x}_0) \cdot \boldsymbol{l}_{\boldsymbol{x0}}^{\perp}\right)^2}{\left((1 - g(\boldsymbol{x}_0))\, r\right)^2}$$

METHOD

- Denote that $\boldsymbol{l} = (cos\theta\ sin\theta)$, $\boldsymbol{x}_0 = (x_0, y_0)$ and $\boldsymbol{x} = (x, y)$, then

$$\rho(\boldsymbol{x}, \boldsymbol{x}_0) = \frac{\left((x-x_0)\cos\theta + (y-y_0)\sin\theta\right)^2}{r^2} + \frac{\left(-(x-x_0)\sin\theta + (y-y_0)\cos\theta\right)^2}{\left((1-g(x_0, y_0))\,r\right)^2}$$

- Thus, the output of proposed SAVMF on position $\boldsymbol{x}_0$ of color image **I** can be expressed as follows:

$$\mathbf{y}^{\mathrm{SAVMF}}(\boldsymbol{x}_0) = \min_{\mathbf{I}(\boldsymbol{x})} \sum_{\boldsymbol{x} \in W} \left\| \mathbf{I}(\boldsymbol{x}) - \mathbf{I}(\boldsymbol{x}_0) \right\|_2$$

EXPERIMENTS

(a) Parrots is corrupted by 50% impulse noise

(b) Vector Median Filter

(c) Proposal (SAVMF)

EXPERIMENTS

(a) FAPGF

(b) FAPGF_SAVMF

(c) QSVMF

(d) QSVMF_SAVMF

EXPERIMENTS

Indexes / Methods	Lena(25%)			Parrots(50%)		
	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy image	16.40	12.90	0.1437	13.28	26.08	0.2868
VMF	29.63	3.85	0.0246	23.90	6.55	0.0473
SAVMF	**30.44**	**3.65**	**0.0233**	**25.74**	**5.76**	**0.0363**
FAPGF	30.74	2.08	0.0156	23.95	5.68	0.0454
FAPGF_SAVMF	**31.83**	**1.85**	**0.0141**	**25.90**	**4.71**	**0.0336**
QSVMF	33.64	1.42	0.0092	27.53	3.55	0.0220
QSVMF_SAVMF	**34.61**	**1.26**	**0.0084**	**27.89**	**3.40**	**0.0210**

Thanks!

Email: Lianghaijin@gmail.com