Blind Channel Gain Cartography

2016 IEEE Global Conference on Signal and Information Processing
Greater Washington, D.C., USA
Dec. 7th - 9th, 2016

Daniel Romero, Donghoon Lee *and* ***Georgios. B. Giannakis***

Dept. of Electrical and Computer Eng.
University of Minnesota, Twin Cities, MN, USA

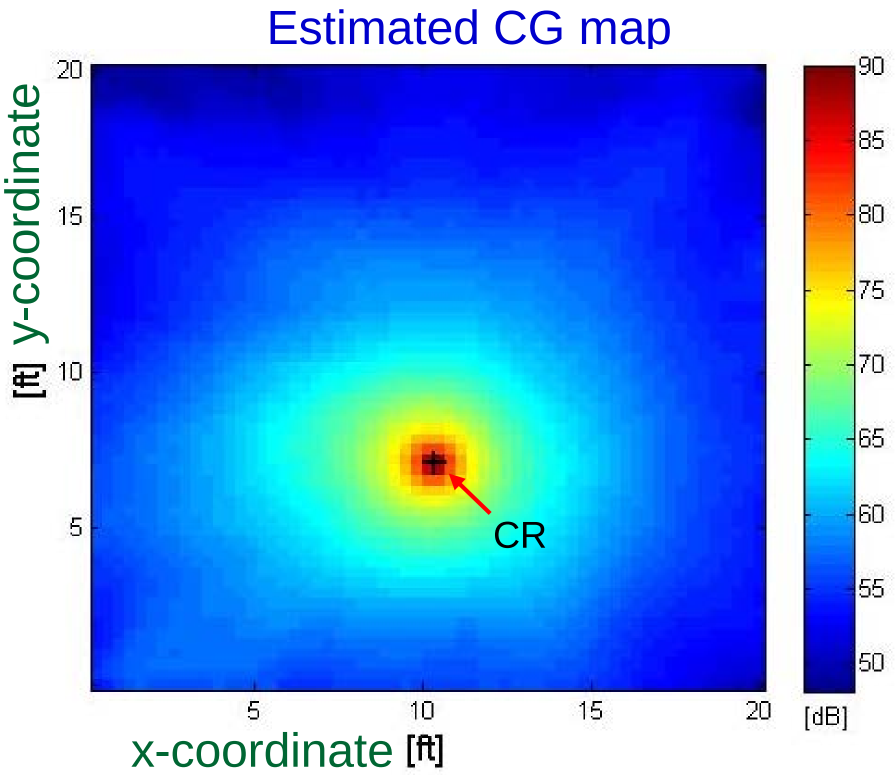
Acknowledgement: ARO W911NF-15-1-0492 and NSF-EARS 1343248

Outline

- Channel gain (CG) cartography
- Blind CG cartography formulation
- Proposed algorithms
- Numerical results
- Conclusions and future work

Channel gain cartography

- Channel gain (CG) cartography
  - Global view of any-to-any CGs
  - Instrumental in
    - cognitive radio (CR)
    - interference management



- Idea
  - Spatially close radio links exhibit similar shadowing [Agrawal-Patwari'09]
  - Interpolation of CG measurements between Tx-Rx pairs

P. Agrawal and N. Patwari, "Correlated link shadow fading in multihop wireless networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4024–4036, Aug. 2009.

Contributions in context

- Related work
  - RF tomography using the spatial loss field (SLF) model [Patwari-Wilson'08]
  - Tracking spatio-temporal dynamics of CG via KKF [Kim et al.'11]
  - Experimental validation of RF tomography models [Hamilton et al.'12]
  - CG cartography via low-rank and sparsity on SLF [Lee-Kim'14]
- Limitation
  - Existing approaches postulate ad-hoc propagation models
- Desiderata

  Joint estimation of CG map and propagation model
- Contributions
  - Blind estimator for CG map and propagation function
  - Efficient batch algorithm for large datasets

Any-to-any channel gain estimation

- Sensors located at $\mathbf{x}_n$ and $\mathbf{x}_{n'}$ over 2-D geographical area $\mathcal{A}$

- Channel gain (in dB) over the link $\mathbf{x}_n$–$\mathbf{x}_{n'}$

$$G(\mathbf{x}_n, \mathbf{x}_{n'}) = G_0 - \gamma 10 \log_{10} ||\mathbf{x}_n - \mathbf{x}_{n'}|| - s(\mathbf{x}_n, \mathbf{x}_{n'}) \qquad (1)$$

Gain at unit distance (G_0) | Path-loss exponent (γ) | Shadowing ($s(\mathbf{x}_n, \mathbf{x}_{n'})$)

  - $||\mathbf{x}_n - \mathbf{x}_{n'}||$: distance between $\mathbf{x}_n$ and $\mathbf{x}_{n'}$

- Goal: Interpolation of channel gain $G(\mathbf{x}, \mathbf{x}')$ for an arbitrary link $\mathbf{x}$–$\mathbf{x}'$

  (as) $\{G_0, \gamma\}$ known

  - Shadowing model allows interpolation of $\hat{s}(\mathbf{x}, \mathbf{x}')$
    - Substitution of $\hat{s}(\mathbf{x}, \mathbf{x}')$ into (1) yields $G(\mathbf{x}, \mathbf{x}')$

Shadowing model

- RF tomography model [Agrawal-Patwari'09]

$$s(\mathbf{x},\mathbf{x}') = \int_{\mathcal{A}} w(\mathbf{x},\mathbf{x}',\tilde{\mathbf{x}}) f(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}} \simeq \sum_{i=1}^{N_g} w(\phi_1(\mathbf{x},\mathbf{x}'), \phi_2(\mathbf{x},\mathbf{x}',\tilde{\mathbf{x}}_i)) f(\tilde{\mathbf{x}}_i)$$

  - Spatial loss field (SLF) $f : \mathcal{A} \rightarrow \mathbb{R}$
  - Weight function w weighs $f(\tilde{\mathbf{x}})$ more heavily at $\tilde{\mathbf{x}}$ lying closer to $\mathbf{x}-\mathbf{x}'$
  - w functions in literature expressible in terms of

$$\begin{cases} \phi_1(\mathbf{x},\mathbf{x}') := \|\mathbf{x}-\mathbf{x}'\|_2 \\ \phi_2(\mathbf{x},\mathbf{x}',\tilde{\mathbf{x}}) := \|\mathbf{x}-\tilde{\mathbf{x}}\|_2 + \|\tilde{\mathbf{x}}-\mathbf{x}'\|_2 \end{cases}$$

Blind CG cartography

- Existing works select w heuristically

- Proposed work: joint estimation of f and w
  - Heuristic selection of w not required
  - Estimation of w via kernel regression

- Problem statement
  - Given: measurements $\{\hat{s}_t\}_{t=1}^{T}$ and sensor locations $\{(\mathbf{x}_{n(t)}, \mathbf{x}_{n'(t)})\}_{t=1}^{T}$
  - Estimate $\{f(\tilde{\mathbf{x}}_i)\}_{i=1}^{N_g}$ and w

Problem formulation

- Reproducing kernel Hilbert space

$$\mathcal{H} := \left\{ w(\boldsymbol{\phi}) = \sum_{i=1}^{\infty} \alpha_i \kappa(\boldsymbol{\phi}, \boldsymbol{\phi}_i) : \ \alpha_i \in \mathbb{R}; \boldsymbol{\phi}, \boldsymbol{\phi}_i \in \mathbb{R}_+^2 \right\} \Leftarrow \boldsymbol{\phi} := [\phi_1, \phi_2]^{\mathcal{T}}$$

  - Reproducing kernel $\kappa : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$ e.g., Gaussian radial basis function

- Regularized formulation

$$\text{(P1)} \quad \min_{\substack{w \in \mathcal{H}, \\ \boldsymbol{f} \in \mathbb{R}^{N_g}}} \frac{1}{T} \sum_{t=1}^{T} \left(\hat{s}_t - \sum_{i=1}^{N_g} w(\boldsymbol{\phi}_{t,i}) f(\tilde{\mathbf{x}}_i) \right)^2 + \mu_w \|w\|_{\mathcal{H}}^2 + \mu_{\boldsymbol{f}} \|\boldsymbol{f}\|_2^2$$

  - Smoothness on w promoted by $||w||_{\mathcal{H}}^2 = \sum_{i=1}^{\infty} \sum_{i'=1}^{\infty} \alpha_i \alpha_{i'} \kappa(\boldsymbol{\phi}_i, \boldsymbol{\phi}_{i'})$

- Challenge: $\mathcal{H}$ is of possibly infinite dimension

Finite-dimensional reformulation

- Representer theorem [Schölkopf and Smola 02']

$$\hat{w}(\phi) = \sum_{t=1}^{T} \sum_{i=1}^{N_g} \alpha_{t,i} \kappa(\phi, \phi_{t,i})$$

  - Together w/ so-called reproducing property, implies

$$\|\hat{w}\|_{\mathcal{H}}^2 = \sum_{t,t'=1}^{T} \sum_{i,i'=1}^{N_g} \alpha_{t,i}\alpha_{t',i'}\kappa(\phi_{t,i}, \phi_{t',i'}) = \boldsymbol{\alpha}^{\mathcal{T}}\mathbf{K}\boldsymbol{\alpha}$$

- $(\mathrm{P1})$ becomes

$$(\mathrm{P2}) \quad \min_{\boldsymbol{\alpha}, \boldsymbol{f}} \frac{1}{T}\|\hat{\mathbf{s}} - (\mathbf{I}_T \otimes \boldsymbol{f}^{\mathcal{T}})\mathbf{K}\boldsymbol{\alpha}\|_2^2 + \mu_w \boldsymbol{\alpha}^{\mathcal{T}}\mathbf{K}\boldsymbol{\alpha} + \mu_{\boldsymbol{f}}\|\boldsymbol{f}\|_2^2$$

B. Schölkopf and A. J. Smola, "Learning with Kernels: Support Vector Machines, Regularization, Optimization, and Beyond", MIT Press, 2002.

Batch algorithm

- Alternating minimization (AM) approach

Set μ_w, $\mu_{\boldsymbol{f}}$, κ, and initialize $\boldsymbol{f}[0]$ at random

for $k = 0, 1 \ldots$

[S1] $\boldsymbol{\alpha}[k+1] = \left[\mathbf{K}^{\mathcal{T}}(\mathbf{I}_T \otimes \boldsymbol{f}[k]\boldsymbol{f}^{\mathcal{T}}[k])\mathbf{K} + \mu_w T\mathbf{K}\right]^{-1}\mathbf{K}^{\mathcal{T}}(\mathbf{I}_T \otimes \boldsymbol{f}[k])\hat{\mathbf{s}}$

Set $\mathbf{A}_{\mathbf{K}}[k] := \sum_{t=1}^{T}(\mathbf{e}_t \otimes \boldsymbol{\alpha}^{\mathcal{T}}[k]\mathbf{K}_t^{\mathcal{T}})$

[S2] $\boldsymbol{f}[k+1] = (\mathbf{A}_{\mathbf{K}}^{\mathcal{T}}[k]\mathbf{A}_{\mathbf{K}}[k] + \mu_{\boldsymbol{f}} T\mathbf{I}_{N_g})^{-1}\mathbf{A}_{\mathbf{K}}^{\mathcal{T}}[k]\hat{\mathbf{s}}$

- Complexity can be reduced by clustering $\{\phi_{t,i}\}_{t,i}$

Synthetic dataset

- Simulation setting
  - 80 sensors uniformly deployed over $\mathcal{A} = [0.5, 15.5] \times [0.5, 15.5]$
  - Dataset ($T = 3,160$) generated with inverse area model [Hamilton et al' 14]

$$w(\phi_1, \phi_2) := \begin{cases} 0, & \text{if } \phi_2 > \phi_1 + \frac{\lambda}{2} \\ \min(\Omega(\phi_1, \phi_2), \Omega(\phi_1, \phi_1 + \delta)) & \text{otherwise} \end{cases}$$

where $\Omega(\phi_1, \phi_2) := 4/\pi\phi_2\sqrt{\phi_2^2 - \phi_1^2}$, $\lambda = 3.5$, and $\delta = 0.5$

  - ➢ Corrupted by white Gaussian noise w/ $\sigma^2 = 10^{-3}$

  - $N_c = 1,500$ with random sampling
  - Gaussian kernel with $\sigma_\kappa = 0.15$ adopted

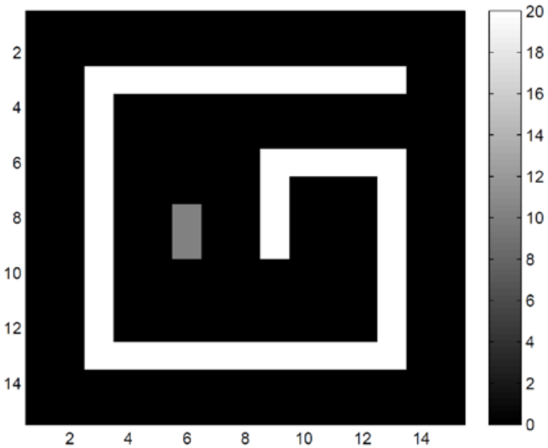
$$\kappa(\boldsymbol{\phi}, \boldsymbol{\phi}') = \exp\left(-\frac{\|\boldsymbol{\phi} - \boldsymbol{\phi}'\|_2^2}{2\sigma_\kappa^2}\right)$$

  - $\mu_{\boldsymbol{f}} = 10^{-4}$ and $\mu_w = 0.2$

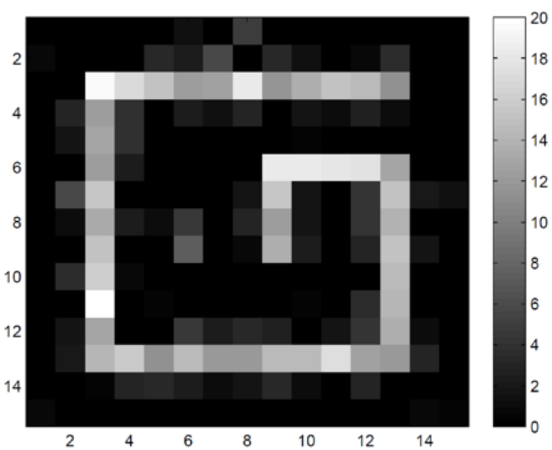
B. R. Hamilton, X. Ma, R. J. Baxley, and S. M. Matechik, "Propagation modeling for radio frequency tomography in wireless networks," *IEEE J. Sel. Topics Sig. Proc.*, vol. 8, no. 1, pp. 55–65, Feb. 2014.

Numerical results

True SLF

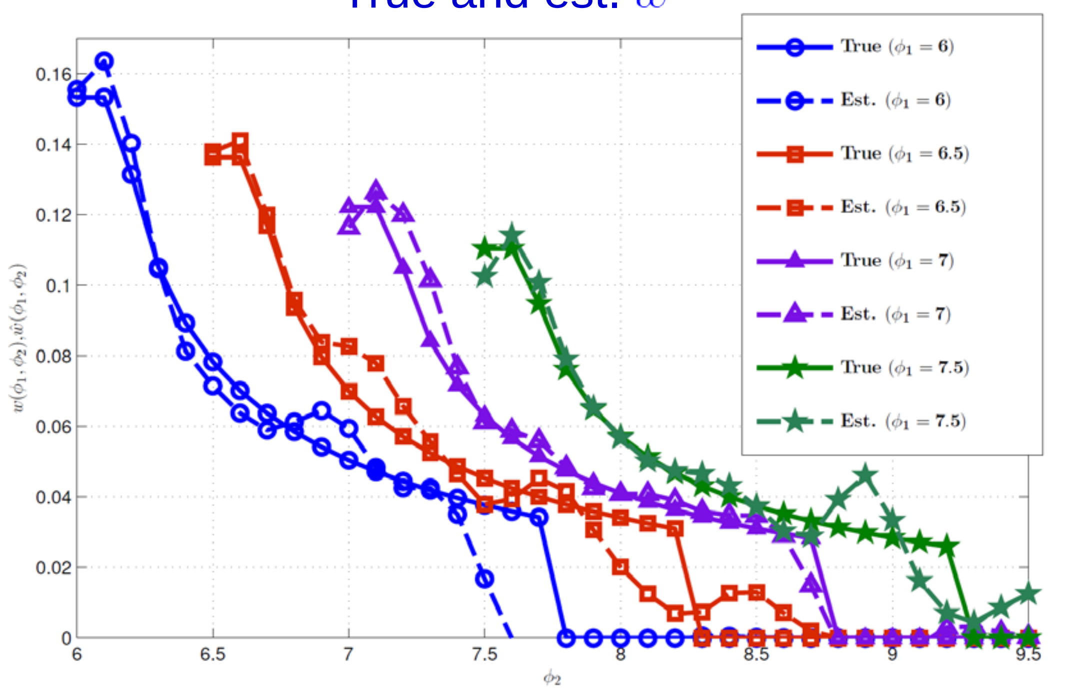


Estimated SLF



- ❑ Accurate reconstruction of the SLF
- ❑ $\hat{w}$ fits well on smooth parts of w

True and est. w



Conclusions and future work

- Blind CG map estimation via nonparmetric kernel regression
- Heuristics not required to select w
- Clustering algorithms incorporated to reduce complexity

- Future work
  - Leverage prior information on f
  - Real data tests

Thank You