RSS-Based Sensor Localization in Underwater Acoustic Sensor Networks

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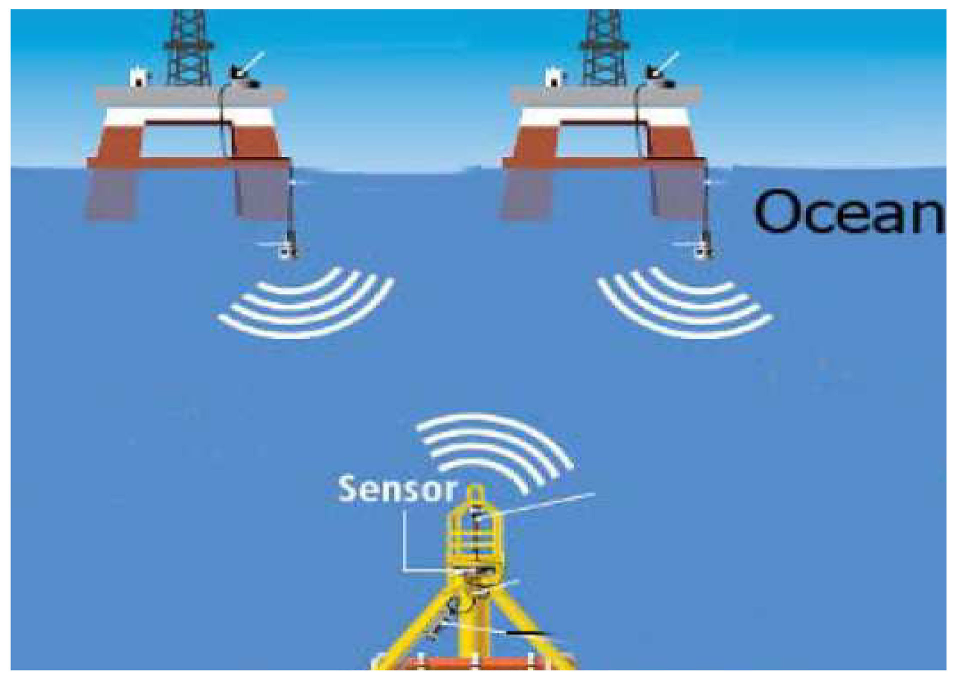
Objectives

1. Underwater acoustic (UWA) sensor localization approaches
2. received signal strength (RSS) measurement

Introduction

- **Location-awareness** is important task for underwater acoustic sensor networks.
- Measurements for localization:
  - **TOA** and **TDOA**: Synchronization is expensive, unpredictable velocity;
  - **AOA**: no line-of-sight (LOS) ;
  - ✓**RSS**: practical simplicity of implementation;
- However, **complicated** underwater channels and hence very **few** attention:
- For certain water depths, the **Urick** propagation model shall be considered;
- **The transmission loss**: $\psi_{\text{TL}}(f,d) = 10\beta\log_{10}d + \alpha^{(f)}d + \xi^{(f,d)}$
  - β is the path-loss exponent (PLE);
  - $\alpha^{(f)}$ is the absorption coefficient (dB/km) and obtained by Thorp's empirical formula $0.11\frac{f^2}{1+f^2} + 44\frac{f^2}{4100+f^2} + 2.75\times10^{-4}f^2 + 0.003$;
  - $\xi^{(f,d)}$ contains residual factors;
- **The RSS measurement**: $P_i^{(f)} = P_0^{(f)} - 10\beta\log_{10}\frac{d_i}{d_0} - \alpha^{(f)}(d_i - d_0) + n_i^{(f)}$,
  - $d_i = \|\mathbf{x} - \mathbf{s}_i\|_2 > 0$ and d_0 is the reference distance, normally $d_0 = 1\ m$
  - $P_0^{(f)}$ is measured power at d_0, i.e., the transmit power.
  - $n_i^{(f)} = \xi^{(f,0)} - \xi^{(f,d)}$ is the *Gaussian* distributed measurement noise;



Optimization Problems

Non-convex but Maximum likelihood (ML) problem:

$$\underset{\mathbf{x}\in\mathbb{R}^M}{\text{argmin}} \sum_i^N (P_i^{(f)} + 10\beta\log_{10}d_i + \alpha^{(f)}d_i)^2$$

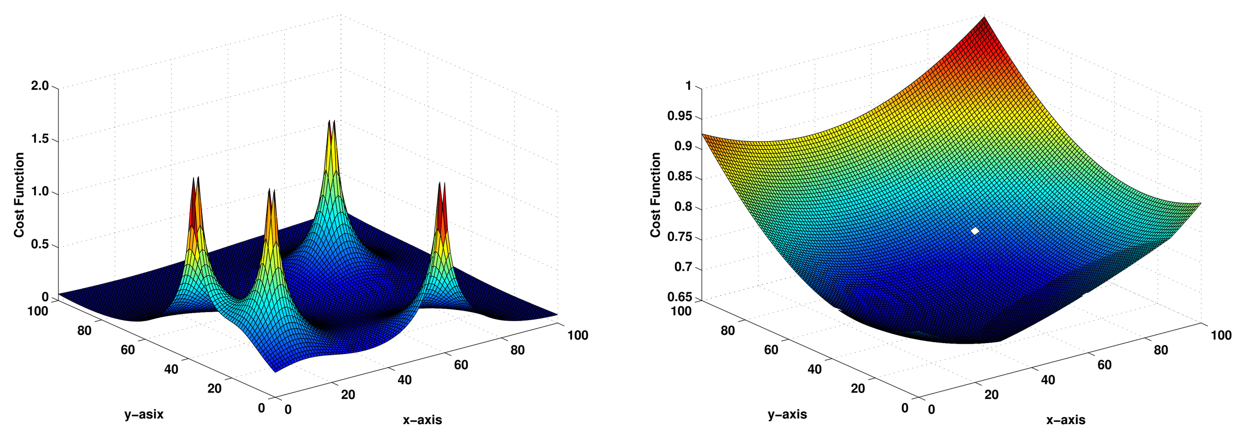
Convex problem:

$$\min_{\mathbf{x}\in\mathcal{R}^M} \sum_i^N \left|\lambda_i^{(f_k)}d_i^2 + \gamma^{(f)}d_i - 1\right|$$

$$\gamma_i^{(f)} = 10^{\frac{P_i^{(f)} - P_0^{(f)} - \alpha^{(f)}}{10\beta}} \text{ and } \lambda_i^{(f)} = \frac{\gamma_i^{(f)}\alpha^{(f)}\ln 10}{10\beta}$$

CRLB

In $\mathbb{R}^2$, 4 anchors located at $(59,26)$, $(35,5)$,$(10,40)$ and $(26,1)$.



Important Result

1. We introduce a new smooth optimization model for RSS-based acoustic localization.
2. We propose a semi-definite programming (SDP) approach using the RSS measurement.
3. Another SDP approach is also developed using the FDRSS measurement.

RSS-based Approach

- First introduce two new variables:
  - $\mathbf{D} \doteq \begin{bmatrix}\mathbf{d}\\1\end{bmatrix}\begin{bmatrix}\mathbf{d}^T & 1\end{bmatrix} = \begin{bmatrix}\mathbf{d}\mathbf{d}^T & \mathbf{d}\\ \mathbf{d}^T & 1\end{bmatrix}$;
  - $\mathbf{X} \doteq \begin{bmatrix}\mathbf{x}\\1\end{bmatrix}\begin{bmatrix}\mathbf{x}^T & 1\end{bmatrix} = \begin{bmatrix}\mathbf{x}\mathbf{x}^T & \mathbf{x}\\ \mathbf{x}^T & 1\end{bmatrix}$;
- $\mathbf{d}$ collects all d_i, $d_i^2 = [\mathbf{D}]_{i,i}$ and $d_i = [\mathbf{D}]_{N+1,i}$;
- $[\mathbf{D}]_{i,i} = \begin{bmatrix}\mathbf{x}^T & 1\end{bmatrix}\begin{bmatrix}\mathbf{I}^T & -\mathbf{s}_i\\ -\mathbf{s}_i^T & \mathbf{s}_i^T\mathbf{s}_i\end{bmatrix}\begin{bmatrix}\mathbf{x}\\1\end{bmatrix} = \text{Trace}\,[\mathbf{X}\mathbf{S}_i]$
- Then the semi-definite relaxation (SDR) dumps the rank constrains $\text{Rank}(\mathbf{X}) = \text{Rank}(\mathbf{D}) = 1$.
- Finally, introduce the slack variables t_i and solve the semi-definite programming (SDP):

$$\min_{\mathbf{D},\mathbf{X},t_i} \sum_i^N t_i,$$

$$\text{s.t.} \quad -t_i < \lambda_i^{(f)}[\mathbf{D}]_{i,i} + \gamma^{(f)}[\mathbf{D}]_{N+1,i} - 1 < t_i$$
$$[\mathbf{D}]_{i,i} = \text{Trace}\,[\mathbf{X}\mathbf{S}_i]$$
$$\mathbf{X} \succeq \mathbf{0},\ \mathbf{D} \succeq \mathbf{0}$$
$$[\mathbf{X}]_{M+1,M+1} = [\mathbf{D}]_{N+1,N+1} = 1$$

FDRSS-based Approach

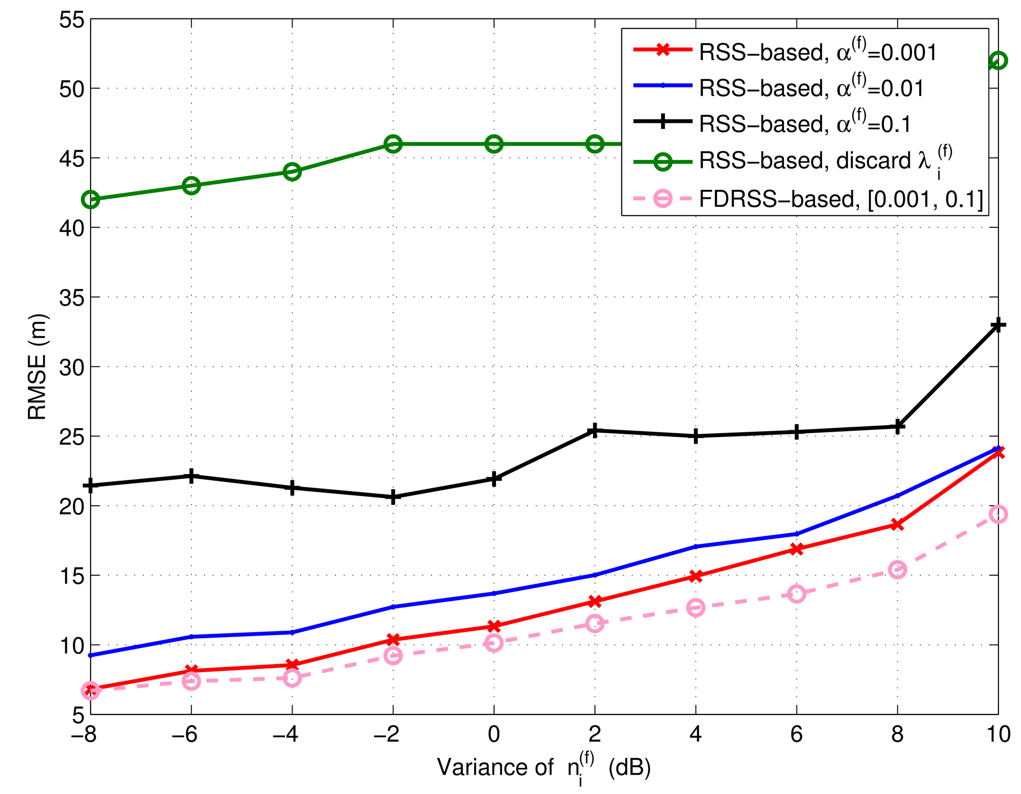
- **The frequency differential RSS (FDRSS) measurement**: $P_i^{(\Delta)} = P_0^{(\Delta)} - \alpha^{(\Delta)}(d_i - d_0) + n_i^{(\Delta)}, k \neq p$
- $P_i^{(\Delta)} = P_i^{(f_k)} - P_i^{(f_p)}$ and $P_0^{(\Delta)} = P_0^{(f_k)} - P_0^{(f_p)}$;
- $\alpha^{(\Delta)} = \alpha^{(f_k)} - \alpha^{(f_p)}$ and $n_i^{(\Delta)} = n_i^{(f_k)} - n_i^{(f_p)}$
- Apply least squares (LS) criterion on $\alpha^{(\Delta)}d_i - \eta_i^{(\Delta)} = n_i^{(\Delta)}$ with $\eta_i^{(\Delta)} = P_0^{(\Delta)} + \alpha^{(\Delta)} - P_i^{(\Delta)}$
- Similarly, solve the SDP:

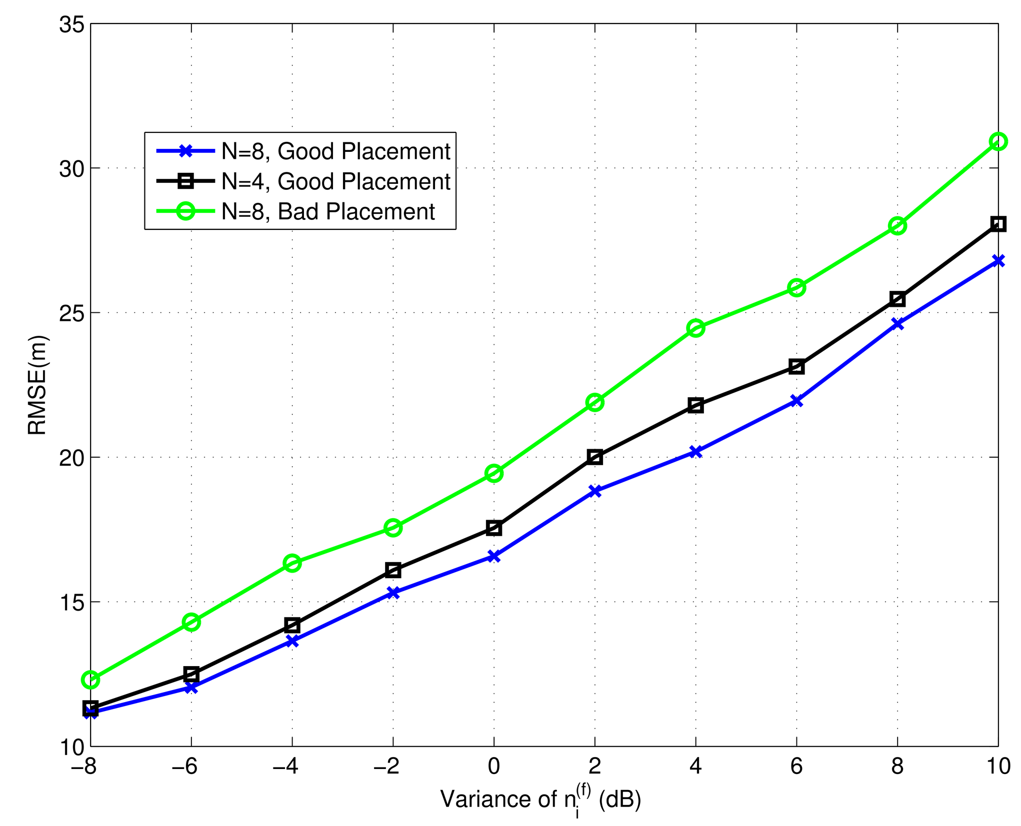
$$\min_{\mathbf{D},\mathbf{X}} \sum_i^N \left(\alpha^{(\Delta)^2}[\mathbf{D}]_{i,i} - 2\alpha^{(\Delta)}\eta_i^{(\Delta)}[\mathbf{D}]_{N+1,i} + \eta_i^{(\Delta)^2}\right),$$

$$\text{s.t.} \quad [\mathbf{D}]_{i,i} = \text{Trace}\,[\mathbf{X}\mathbf{S}_i]$$
$$\mathbf{X} \succeq \mathbf{0},\ \mathbf{D} \succeq \mathbf{0}$$
$$[\mathbf{X}]_{M+1,M+1} = [\mathbf{D}]_{N+1,N+1} = 1$$

Simulation Results

10 anchors is randomly deployed; $\beta = 2$; use *CVX* to solve SDPs.





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