A Finite Field Cosine Transform-Based Image Processing Scheme for Color Image Encryption

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Outline

Introduction

- Applications for finite field and number-theoretic tranforms:
  - Signal processing: computation of error-free fast convolutions
  - Error-correcting codes: decoding in frequency domain
  - Information security: image encryption and watermarking
- In this paper, we introduce a new finite field transform:

Cosine Transform of Fields of Characteristic Two (FFCT)

- The FFCT can be applied to color images: each pixel of a 24-bit RGB image is treated as an element of $\mathrm{GF}(2^{24})$ and a 32-point 2-D FFCT is performed:
  - A transform-based scheme useful for application in image encryption is proposed.

Definition

Let $\zeta \in \mathrm{GF}(2^r)$ be an element of multiplicative order denoted by $\mathrm{ord}(\zeta)$. **The finite field cosine function** related to ζ is defined, for $x = 0, 1, \ldots, \mathrm{ord}(\zeta)$, as

$$\cos_\zeta(x) := \zeta^x + \zeta^{-x}.$$

Definition

Let $\zeta \in \mathrm{GF}(2^r)$ be an element such that $\mathrm{ord}(\zeta) = 2N - 1$. **The finite field cosine transform** of the vector $\mathbf{x} = (x_i)$, $x_i \in \mathrm{GF}(2^r)$, $i = 0, 1, \ldots, N-2$, is the vector $\mathbf{X} = (X_k)$, $X_k \in \mathrm{GF}(2^r)$, $k = 1, 2, \ldots, N-1$, whose components are

$$X_k := \sum_{i=0}^{N-2} x_i \cos_\zeta(k(i + 1/2)).$$

The components of the **inverse finite field cosine transform** are computed by

$$x_i = \sum_{k=1}^{N-1} X_k \cos_\zeta(k(i+1/2)).$$

The relationship between $\mathbf{x}$ and $\mathbf{X}$ can be expressed as

$$\mathbf{X} = \mathbf{C} \cdot \mathbf{x},$$

where $C_{k,i} = \cos_\zeta(k(i+1/2))$.

Important remarks:

- The FFCT can be related to the finite field Fourier transform (FFFT).
- Differently from the FFFT, the FFCT allows to define even-point transforms.
    - 2^r-point FFCT can be defined, which makes easier designing and implementing fast algorithms.
- While the *period* of $\mathbf{F}$ is 4, i.e., $\mathbf{F}^4 = \mathbf{I}$ (the identity matrix), matrix $\mathbf{C}$ has periods significantly larger and dependent of its dimension.
    - The FFCT can be considered as a potential candidate to be part of cryptographic schemes based on iterative transform computations.

The FFCT-Based Processing Scheme

We construct a 32-point FFCT over $\mathrm{GF}(2^{24})$:

- The field $\mathrm{GF}(2^{24})$ is generated using the element α, which is a root of the primitive polynomial $f(x) = x^{24} + x^7 + x^2 + x + 1$.
- We obtain the element $\zeta = \alpha^{\frac{2^{24}-1}{65}} = \alpha^{258111}$, such that $\mathrm{ord}(\zeta) = 65 = 2N - 1$ and $N - 1 = 32$.
- The elements of the corresponding transform matrix $\mathbf{C}$ are

$$C_{k,i} = \cos_\zeta(k(i+1/2)) = \left(\zeta^{\frac{1}{2}}\right)^{k(2i+1)} + \left(\zeta^{-\frac{1}{2}}\right)^{k(2i+1)}.$$

- The two-dimensional transform $\mathbf{M}$ of a 32×32 matrix $\mathbf{m}$ over $\mathrm{GF}(2^{24})$ is computed as

$$\mathbf{M} = \mathbf{C} \cdot \mathbf{m} \cdot \mathbf{C}^T. \qquad (1)$$

The FFCT-Based Processing Scheme

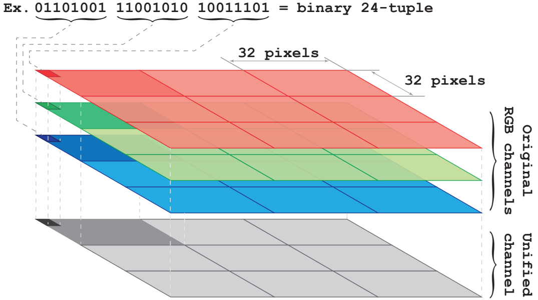


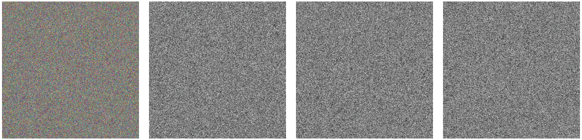
Figure: Procedure for representing an RGB image as a *unified channel* (matrix) of 24-bit numbers.

The FFCT-Based Processing Scheme

- Each binary 24-tuples of the *unified channel* is directly mapped into elements of $\mathrm{GF}(2^{24})$.
- Such a *unified channel* is divided into blocks with dimension 32×32, which are submitted to the 2-D FFCT according to Equation (1).
- The resulting transformed matrix is reconverted into a three channel transformed image denoted by $\mathbf{I}_t$.
- We expect that each channel of $\mathbf{I}_t$ has uniform histogram and low correlation among adjacent pixels.
- The original image $\mathbf{I}$ can be recovered from $\mathbf{I}_t$ using the inverse FFCT.

(a) (b) (c) (d)

Figure: (a) *lena.bmp*, (b) *peppers.bmp*, (c) *mandril.bmp*, (d) *lake.bmp*.



(a) (b) (c) (d)

Figure: Transformed versions of (a) *lena.bmp* and its (b) R, (c) G and (d) B channels.

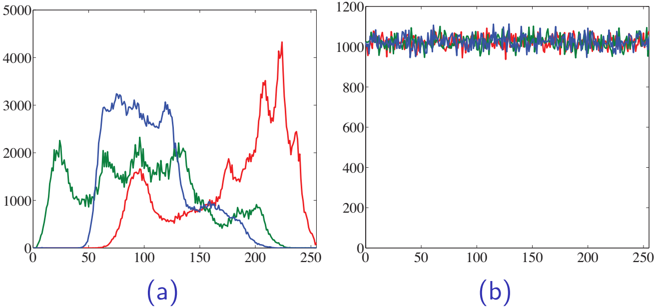
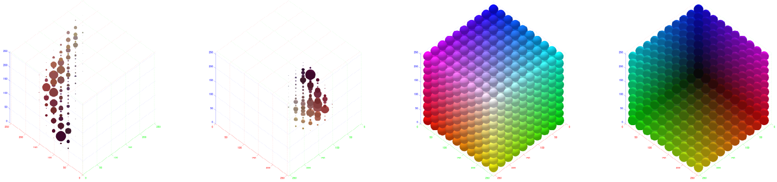


Figure: Histograms of color channels of (a) original and (b) transformed *lena.bmp*.

Computer Experiments and Security Aspects



(a) (b) (c) (d)

Figure: (a), (b) Color distributions of original *lena.bmp* in the RGB space; (c), (d) color distributions of transformed *lena.bmp* in the RGB space.

- The entropy of the color channels of the transformed images has assumed values varying from 7.9992 to 7.9994.
- These values are considerably close to 8, the entropy of a random source emitting 256 equiprobable symbols.

Table: Correlation coefficients of original (r_{xy}) and processed images ($\tilde{r}_{xy}$); (U) is related to unified-channel images; (R), (G) and (B) are related to individual channels.

Metric	*lena*	*peppers*	*house*	*mandril*
$r_{xy}(U)$	0.9671	0.9676	0.9679	0.8818
$\tilde{r}_{xy}(U)$	0.0029	-0.0016	-0.0021	0.0002
$r_{xy}(R)$	0.9892	0.9668	0.9582	0.8683
$\tilde{r}_{xy}(R)$	0.0061	-0.0070	0.0101	-0.0093
$r_{xy}(G)$	0.9825	0.9812	0.9397	0.7674
$\tilde{r}_{xy}(G)$	-0.0010	-0.0009	0.0066	0.0058
$r_{xy}(B)$	0.9571	0.9673	0.9678	0.8815
$\tilde{r}_{xy}(B)$	-0.0048	-0.0016	-0.0088	0.0008

Concluding Remarks

- We have introduced a cosine transform over fields of characteristic two and demonstrated its applicability in color image processing.
- Our approach is immune to rounding-off errors and allows using the same digital encoding scheme in both spatial and transform domains.
- The method we have proposed modifies visual and statistical properties of an image, which makes it adequate to be used as a key-independent portion of an image encryption scheme.
- A key-dependent stage must be included to perform image encryption.

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- Questions?
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