

# Channel Estimation Using Joint Dictionary Learning in FDD Massive MIMO Systems

Yacong Ding and Bhaskar Rao

Department of Electrical and Computer Engineering  
University of California, San Diego

December 14, 2015

- 1 Introduction
  - Motivation
  - Previous Work
- 2 Joint Uplink/Downlink Dictionary Learning
  - Joint Sparse Representation
  - Joint Compressed Channel Estimation
- 3 Simulation
  - Simulation Setting
  - Low Dimension Representation
  - Compressed Channel Estimation
- 4 Conclusion

# Introduction

# Motivation

- Acquiring downlink channel information  $h^d$  at base station (CSIT):
  - TDD: easy, FDD: difficult.
  - BS has  $N$  antennas, user has 1 antennas:  $h^d, h^u \in \mathbb{C}^{N \times 1}$

	Training	Feedback
TDD: $h^d = h^u$ , uplink training (UE: $\phi \in \mathbb{C}^{1 \times T^u}$ , BS: $Y^u = h^u \phi + n^u$ )	$T^u \geq 1$	No
FDD: $h^d \neq h^u$ , downlink training (BS: $\Phi \in \mathbb{C}^{T^d \times N}$ , UE: $Y^d = \Phi h^d + n^d$ )	$T^d \geq N$	$\propto N$

# Motivation

- Acquiring downlink channel information  $h^d$  at base station (CSIT):
  - TDD: easy, FDD: difficult.
  - BS has  $N$  antennas, user has 1 antennas:  $h^d, h^u \in \mathbb{C}^{N \times 1}$

	Training	Feedback
TDD: $h^d = h^u$ , uplink training (UE: $\phi \in \mathbb{C}^{1 \times T^u}$ , BS: $Y^u = h^u \phi + n^u$ )	$T^u \geq 1$	No
FDD: $h^d \neq h^u$ , downlink training (BS: $\Phi \in \mathbb{C}^{T^d \times N}$ , UE: $Y^d = \Phi h^d + n^d$ )	$T^d \geq N$	$\propto N$

- FDD is widely employed in existing communication systems:
  - Beneficial if directly adopt Massive MIMO to FDD.

# Motivation

- Acquiring downlink channel information  $h^d$  at base station (CSIT):
  - TDD: easy, FDD: difficult.
  - BS has  $N$  antennas, user has 1 antennas:  $h^d, h^u \in \mathbb{C}^{N \times 1}$

	Training	Feedback
TDD: $h^d = h^u$ , uplink training (UE: $\phi \in \mathbb{C}^{1 \times T^u}$ , BS: $Y^u = h^u \phi + n^u$ )	$T^u \geq 1$	No
FDD: $h^d \neq h^u$ , downlink training (BS: $\Phi \in \mathbb{C}^{T^d \times N}$ , UE: $Y^d = \Phi h^d + n^d$ )	$T^d \geq N$	$\propto N$

- FDD is widely employed in existing communication systems:
  - Beneficial if directly adopt Massive MIMO to FDD.
- FDD Downlink training:  $Y^d = \Phi h^d + n^d, \Phi \in \mathbb{C}^{T^d \times N}$ :
  - To be practical:  $T^d$  small.

# Motivation

- Acquiring downlink channel information  $h^d$  at base station (CSIT):
  - TDD: easy, FDD: difficult.
  - BS has  $N$  antennas, user has 1 antennas:  $h^d, h^u \in \mathbb{C}^{N \times 1}$

	Training	Feedback
TDD: $h^d = h^u$ , uplink training (UE: $\phi \in \mathbb{C}^{1 \times T^u}$ , BS: $Y^u = h^u \phi + n^u$ )	$T^u \geq 1$	No
FDD: $h^d \neq h^u$ , downlink training (BS: $\Phi \in \mathbb{C}^{T^d \times N}$ , UE: $Y^d = \Phi h^d + n^d$ )	$T^d \geq N$	$\propto N$

- FDD is widely employed in existing communication systems:
  - Beneficial if directly adopt Massive MIMO to FDD.
- FDD Downlink training:  $Y^d = \Phi h^d + n^d, \Phi \in \mathbb{C}^{T^d \times N}$ :
  - To be practical:  $T^d$  small.
  - $T^d < N$ : underdetermined inverse problem, infinite solutions.

# Motivation

- Acquiring downlink channel information  $h^d$  at base station (CSIT):
  - TDD: easy, FDD: difficult.
  - BS has  $N$  antennas, user has 1 antennas:  $h^d, h^u \in \mathbb{C}^{N \times 1}$

	Training	Feedback
TDD: $h^d = h^u$ , uplink training (UE: $\phi \in \mathbb{C}^{1 \times T^u}$ , BS: $Y^u = h^u \phi + n^u$ )	$T^u \geq 1$	No
FDD: $h^d \neq h^u$ , downlink training (BS: $\Phi \in \mathbb{C}^{T^d \times N}$ , UE: $Y^d = \Phi h^d + n^d$ )	$T^d \geq N$	$\propto N$

- FDD is widely employed in existing communication systems:
  - Beneficial if directly adopt Massive MIMO to FDD.
- FDD Downlink training:  $Y^d = \Phi h^d + n^d, \Phi \in \mathbb{C}^{T^d \times N}$ :
  - To be practical:  $T^d$  small.
  - $T^d < N$ : underdetermined inverse problem, infinite solutions.
- Explore channel structure to regularize the problem?



# Motivation

- Acquiring downlink channel information  $h^d$  at base station (CSIT):
  - TDD: easy, FDD: difficult.
  - BS has  $N$  antennas, user has 1 antennas:  $h^d, h^u \in \mathbb{C}^{N \times 1}$

	Training	Feedback
TDD: $h^d = h^u$ , uplink training (UE: $\phi \in \mathbb{C}^{1 \times T^u}$ , BS: $Y^u = h^u \phi + n^u$ )	$T^u \geq 1$	No
FDD: $h^d \neq h^u$ , downlink training (BS: $\Phi \in \mathbb{C}^{T^d \times N}$ , UE: $Y^d = \Phi h^d + n^d$ )	$T^d \geq N$	$\propto N$

- FDD is widely employed in existing communication systems:
  - Beneficial if directly adopt Massive MIMO to FDD.
- FDD Downlink training:  $Y^d = \Phi h^d + n^d, \Phi \in \mathbb{C}^{T^d \times N}$ :
  - To be practical:  $T^d$  small.
  - $T^d < N$ : underdetermined inverse problem, infinite solutions.
- Explore channel structure to regularize the problem?
  - **Sparse** channel structure: **compressive sensing**.

# Compressed Channel Estimation

- Low dimensional representation of high dimensional signal:
  - Find a  $\Psi$  such that  $h^d = \Psi\beta^d$ ,  $\|\beta^d\|_0 < N$ .

# Compressed Channel Estimation

- Low dimensional representation of high dimensional signal:
  - Find a  $\Psi$  such that  $h^d = \Psi\beta^d$ ,  $\|\beta^d\|_0 < N$ .
  - Downlink training:  $Y^d = \Phi h^d + n^d = \Phi\Psi\beta^d + n^d$

# Compressed Channel Estimation

- Low dimensional representation of high dimensional signal:
  - Find a  $\Psi$  such that  $h^d = \Psi\beta^d$ ,  $\|\beta^d\|_0 < N$ .
  - Downlink training:  $Y^d = \Phi h^d + n^d = \Phi\Psi\beta^d + n^d$
- Apply **compressive sensing** algorithm to estimate the sparse coefficient  $\beta^d$ .

## Compressed Channel Estimation :

$$\hat{\beta}^d = \arg \min_{\beta^d} \|\beta^d\|_0 \quad \text{subject to} \quad \|Y^d - \Phi\Psi\beta^d\|_2^2 \leq \sigma^2 \quad (1)$$

$$\hat{h}^d = \Psi\hat{\beta}^d$$

# Compressed Channel Estimation

- Low dimensional representation of high dimensional signal:
  - Find a  $\Psi$  such that  $h^d = \Psi\beta^d$ ,  $\|\beta^d\|_0 < N$ .
  - Downlink training:  $Y^d = \Phi h^d + n^d = \Phi\Psi\beta^d + n^d$
- Apply **compressive sensing** algorithm to estimate the sparse coefficient  $\beta^d$ .

## Compressed Channel Estimation :

$$\hat{\beta}^d = \arg \min_{\beta^d} \|\beta^d\|_0 \text{ subject to } \|Y^d - \Phi\Psi\beta^d\|_2^2 \leq \sigma^2 \quad (1)$$

$$\hat{h}^d = \Psi\hat{\beta}^d$$

- Many practical algorithms. **Measurements:**  $T^d \propto \|\beta^d\|_0 < N$

# Compressed Channel Estimation

- Low dimensional representation of high dimensional signal:
  - Find a  $\Psi$  such that  $h^d = \Psi\beta^d$ ,  $\|\beta^d\|_0 < N$ .
  - Downlink training:  $Y^d = \Phi h^d + n^d = \Phi\Psi\beta^d + n^d$
- Apply **compressive sensing** algorithm to estimate the sparse coefficient  $\beta^d$ .

## Compressed Channel Estimation :

$$\hat{\beta}^d = \arg \min_{\beta^d} \|\beta^d\|_0 \text{ subject to } \|Y^d - \Phi\Psi\beta^d\|_2^2 \leq \sigma^2 \quad (1)$$

$$\hat{h}^d = \Psi\hat{\beta}^d$$

- Many practical algorithms. **Measurements:**  $T^d \propto \|\beta^d\|_0 < N$
- Core requirement: find  $\Psi$ .

# Sparse Channel Representation

# Drawbacks of Existing Work

The existing work that applies compressed channel estimation use **orthogonal** DFT basis as  $\Psi$ :

- Agree with array manifold using ULA.
- **Infinite** number of antennas, **limited** scattering environment.



# Drawbacks of Existing Work

The existing work that applies compressed channel estimation use **orthogonal** DFT basis as  $\Psi$ :

- Agree with array manifold using ULA.
- **Infinite** number of antennas, **limited** scattering environment.

For common channels models, such as 3GPP SCM channels:

- High  $\|\beta^d\|_0$ :  $h^d = \Psi_{DFT}\beta^d$
- $T^d \propto \|\beta^d\|_0$ : lose benefits of compressive sensing

# Drawbacks of Existing Work

The existing work that applies compressed channel estimation use **orthogonal** DFT basis as  $\Psi$ :

- Agree with array manifold using ULA.
- **Infinite** number of antennas, **limited** scattering environment.

For common channels models, such as 3GPP SCM channels:

- High  $\|\beta^d\|_0$ :  $h^d = \Psi_{DFT}\beta^d$
- $T^d \propto \|\beta^d\|_0$ : lose benefits of compressive sensing

One easy better choice is **overcomplete** DFT matrix: redundancy in basis

- Only applicable to **ULA**.
- Can not adapt to **specific** cell characteristics: urban, rural, hills, etc.

# Drawbacks of Existing Work

The existing work that applies compressed channel estimation use **orthogonal** DFT basis as  $\Psi$ :

- Agree with array manifold using ULA.
- **Infinite** number of antennas, **limited** scattering environment.

For common channels models, such as 3GPP SCM channels:

- High  $\|\beta^d\|_0$ :  $h^d = \Psi_{DFT}\beta^d$
- $T^d \propto \|\beta^d\|_0$ : lose benefits of compressive sensing

One easy better choice is **overcomplete** DFT matrix: redundancy in basis

- Only applicable to **ULA**.
- Can not adapt to **specific** cell characteristics: urban, rural, hills, etc.

How can we design a dictionary/basis such that:

# Drawbacks of Existing Work

The existing work that applies compressed channel estimation use **orthogonal** DFT basis as  $\Psi$ :

- Agree with array manifold using ULA.
- **Infinite** number of antennas, **limited** scattering environment.

For common channels models, such as 3GPP SCM channels:

- High  $\|\beta^d\|_0$ :  $h^d = \Psi_{DFT}\beta^d$
- $T^d \propto \|\beta^d\|_0$ : lose benefits of compressive sensing

One easy better choice is **overcomplete** DFT matrix: redundancy in basis

- Only applicable to **ULA**.
- Can not adapt to **specific** cell characteristics: urban, rural, hills, etc.

How can we design a dictionary/basis such that:

- Adapt to **specific** cell properties (antenna, environment).

# Drawbacks of Existing Work

The existing work that applies compressed channel estimation use **orthogonal** DFT basis as  $\Psi$ :

- Agree with array manifold using ULA.
- **Infinite** number of antennas, **limited** scattering environment.

For common channels models, such as 3GPP SCM channels:

- High  $\|\beta^d\|_0$ :  $h^d = \Psi_{DFT}\beta^d$
- $T^d \propto \|\beta^d\|_0$ : lose benefits of compressive sensing

One easy better choice is **overcomplete** DFT matrix: redundancy in basis

- Only applicable to **ULA**.
- Can not adapt to **specific** cell characteristics: urban, rural, hills, etc.

How can we design a dictionary/basis such that:

- Adapt to **specific** cell properties (antenna, environment).
- Lead to **sparse** representation.

# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:

# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:
  - **Overcomplete:**  $D^d \in \mathbb{C}^{N \times M}, N < M$

# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:
  - **Overcomplete:**  $D^d \in \mathbb{C}^{N \times M}, N < M$
  - **Fit** model to data:  $h_i^d \approx D^d \beta_i^d, i = 1, \dots, L$



# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:
  - **Overcomplete:**  $D^d \in \mathbb{C}^{N \times M}$ ,  $N < M$
  - **Fit** model to data:  $h_i^d \approx D^d \beta_i^d$ ,  $i = 1, \dots, L$
  - **Encourage** sparsity:  $\|\beta_i^d\|_0 \ll M, \forall i$ .

# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:
  - **Overcomplete:**  $D^d \in \mathbb{C}^{N \times M}$ ,  $N < M$
  - **Fit** model to data:  $h_i^d \approx D^d \beta_i^d$ ,  $i = 1, \dots, L$
  - **Encourage** sparsity:  $\|\beta_i^d\|_0 \ll M, \forall i$ .
- What data can be utilized?

# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:
  - **Overcomplete:**  $D^d \in \mathbb{C}^{N \times M}$ ,  $N < M$
  - **Fit** model to data:  $h_i^d \approx D^d \beta_i^d$ ,  $i = 1, \dots, L$
  - **Encourage** sparsity:  $\|\beta_i^d\|_0 \ll M, \forall i$ .
- What data can be utilized?
  - Channel measurements: collected within a **specific** cell.

# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:
  - **Overcomplete:**  $D^d \in \mathbb{C}^{N \times M}$ ,  $N < M$
  - **Fit** model to data:  $h_i^d \approx D^d \beta_i^d$ ,  $i = 1, \dots, L$
  - **Encourage** sparsity:  $\|\beta_i^d\|_0 \ll M, \forall i$ .
- What data can be utilized?
  - Channel measurements: collected within a **specific** cell.
  - Effect of **environment** on the transmitted electromagnetic waves represented at **antennas**.

# Our Previous Work: Learning Good Representation

- Rather than using **predefined** dictionary/basis, **learn** *cell specific*  $D^d$  from data:
  - **Overcomplete:**  $D^d \in \mathbb{C}^{N \times M}$ ,  $N < M$
  - **Fit** model to data:  $h_i^d \approx D^d \beta_i^d$ ,  $i = 1, \dots, L$
  - **Encourage** sparsity:  $\|\beta_i^d\|_0 \ll M$ ,  $\forall i$ .
- What data can be utilized?
  - Channel measurements: collected within a **specific** cell.
  - Effect of **environment** on the transmitted electromagnetic waves represented at **antennas**.
  - **Big data** paradigm in wireless communication.

- Combine data fitting and sparsity encouragement, dictionary learning can be formulated:

$$\min_{D^d, \beta_1^d, \dots, \beta_L^d} \lambda \|H^d - D^d B^d\|_F^2 + \sum_{i=1}^L \|\beta_i^d\|_0 \quad (2)$$

where  $H^d = [h_1^d, \dots, h_L^d]$   $B^d = [\beta_1^d, \dots, \beta_L^d]$ .

- Combine data fitting and sparsity encouragement, dictionary learning can be formulated:

$$\min_{D^d, \beta_1^d, \dots, \beta_L^d} \lambda \|H^d - D^d B^d\|_F^2 + \sum_{i=1}^L \|\beta_i^d\|_0 \quad (2)$$

where  $H^d = [h_1^d, \dots, h_L^d]$   $B^d = [\beta_1^d, \dots, \beta_L^d]$ .

- Benefits of dictionary learning and compressed channel estimation:
  - Applicable to **any antenna configurations**: no assumed structure.

- Combine data fitting and sparsity encouragement, dictionary learning can be formulated:

$$\min_{D^d, \beta_1^d, \dots, \beta_L^d} \lambda \|H^d - D^d B^d\|_F^2 + \sum_{i=1}^L \|\beta_i^d\|_0 \quad (2)$$

where  $H^d = [h_1^d, \dots, h_L^d]$   $B^d = [\beta_1^d, \dots, \beta_L^d]$ .

- Benefits of dictionary learning and compressed channel estimation:
  - Applicable to **any antenna configurations**: no assumed structure.
  - Robust to any **irregularities**: mismatched antennas, non-plane wave.



- Combine data fitting and sparsity encouragement, dictionary learning can be formulated:

$$\min_{D^d, \beta_1^d, \dots, \beta_L^d} \lambda \|H^d - D^d B^d\|_F^2 + \sum_{i=1}^L \|\beta_i^d\|_0 \quad (2)$$

where  $H^d = [h_1^d, \dots, h_L^d]$   $B^d = [\beta_1^d, \dots, \beta_L^d]$ .

- Benefits of dictionary learning and compressed channel estimation:
  - Applicable to **any antenna configurations**: no assumed structure.
  - Robust to any **irregularities**: mismatched antennas, non-plane wave.
  - Training and feedback overhead: proportional to **channel sparsity**  $S$ .

# Joint Uplink/Downlink Dictionary Learning and Compressed Channel Estimation

# Utilizing Uplink Channel Information

In compressive sensing, more measurements are always better:

- More information about the underlying sparse coefficients.
- Better recovery performance.

# Utilizing Uplink Channel Information

In compressive sensing, more measurements are always better:

- More information about the underlying sparse coefficients.
- Better recovery performance.

In Massive MIMO, it implies  $T^d$  to be larger:

- Larger training and feedback overhead.
- Waste of resources.

# Utilizing Uplink Channel Information

In compressive sensing, more measurements are always better:

- More information about the underlying sparse coefficients.
- Better recovery performance.

In Massive MIMO, it implies  $T^d$  to be larger:

- Larger training and feedback overhead.
- Waste of resources.

Is it possible to have more information about the underlying sparse coefficient, but without need of larger  $T^d$ ?

# Utilizing Uplink Channel Information

In compressive sensing, more measurements are always better:

- More information about the underlying sparse coefficients.
- Better recovery performance.

In Massive MIMO, it implies  $T^d$  to be larger:

- Larger training and feedback overhead.
- Waste of resources.

Is it possible to have more information about the underlying sparse coefficient, but without need of larger  $T^d$ ?

From uplink channel  $h^u$ :

# Utilizing Uplink Channel Information

In compressive sensing, more measurements are always better:

- More information about the underlying sparse coefficients.
- Better recovery performance.

In Massive MIMO, it implies  $T^d$  to be larger:

- Larger training and feedback overhead.
- Waste of resources.

Is it possible to have more information about the underlying sparse coefficient, but without need of larger  $T^d$ ?

From uplink channel  $h^u$ :

- Easy to obtain:  $T^u \geq 1$ .
- Common sparse channel structure between  $h^d$  and  $h^u$ .

# Joint Uplink/Downlink Channel Representation

- Similar to  $h^d = D^d \beta^d$  :  $h^u = D^u \beta^u$ .



# Joint Uplink/Downlink Channel Representation

- Similar to  $h^d = D^d \beta^d$  :  $h^u = D^u \beta^u$ .
- Duplex distance not large: **similar scattering effect** for uplink and downlink transmission.

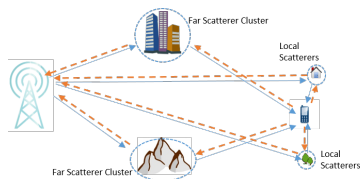


Figure 1: Uplink/Downlink Channel Model

# Joint Uplink/Downlink Channel Representation

- Similar to  $h^d = D^d \beta^d$  :  $h^u = D^u \beta^u$ .
- Duplex distance not large: **similar scattering effect** for uplink and downlink transmission.

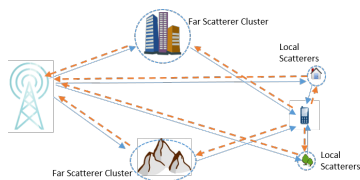


Figure 1: Uplink/Downlink Channel Model

- In our model, equivalently to assume  $\chi(\beta^u) = \chi(\beta^d)$ , where  $\chi(\beta) = \{i | \beta(i) \neq 0\}$  denotes the set of locations of nonzero entries in  $\beta$ .

## Joint Uplink/Downlink Dictionary Learning

$$\begin{aligned} & \min_{D^u, B^u, D^d, B^d} \|H^u - D^u B^u\|_F^2 + \|H^d - D^d B^d\|_F^2 \\ & \text{subject to } \|\beta_i^u\|_0 = \|\beta_i^d\|_0 \leq T_0, \chi(\beta_i^u) = \chi(\beta_i^d) \quad \forall i \end{aligned} \quad (3)$$

## Joint Uplink/Downlink Dictionary Learning

$$\begin{aligned} \min_{D^u, B^u, D^d, B^d} \quad & \|H^u - D^u B^u\|_F^2 + \|H^d - D^d B^d\|_F^2 \\ \text{subject to} \quad & \|\beta_i^u\|_0 = \|\beta_i^d\|_0 \leq T_0, \quad \chi(\beta_i^u) = \chi(\beta_i^d) \quad \forall i \end{aligned} \quad (3)$$

## Joint Uplink/Downlink Sparse Representation

$$\begin{aligned} h^u &\approx D^u \beta^u, \quad h^d \approx D^d \beta^d \\ \|\beta_i^u\|_0 &= \|\beta_i^d\|_0 \leq T_0, \quad \chi(\beta_i^u) = \chi(\beta_i^d) \quad \forall i \end{aligned} \quad (4)$$

## Joint Uplink/Downlink Dictionary Learning

$$\begin{aligned} & \min_{D^u, B^u, D^d, B^d} \|H^u - D^u B^u\|_F^2 + \|H^d - D^d B^d\|_F^2 \\ & \text{subject to } \|\beta_i^u\|_0 = \|\beta_i^d\|_0 \leq T_0, \chi(\beta_i^u) = \chi(\beta_i^d) \quad \forall i \end{aligned} \quad (3)$$

## Joint Uplink/Downlink Sparse Representation

$$\begin{aligned} & h^u \approx D^u \beta^u, h^d \approx D^d \beta^d \\ & \|\beta_i^u\|_0 = \|\beta_i^d\|_0 \leq T_0, \chi(\beta_i^u) = \chi(\beta_i^d) \quad \forall i \end{aligned} \quad (4)$$

## Joint Uplink/Downlink Compressed Channel Estimation :

$$\begin{aligned} & \arg \min_{\beta^u, \beta^d} \|Y^u - \Phi^u D^u \beta^u\|_2^2 + \|Y^d - \Phi^d D^d \beta^d\|_2^2 \\ & \text{subject to } \chi(\beta^u) = \chi(\beta^d), \|\beta^u\|_0 = \|\beta^d\|_0 \leq T_0 \end{aligned} \quad (5)$$

# Benefits of Joint Sparse Framework

- Joint dictionary learning:
  - Regularize the learning process.
  - Better performance when underlying generative model satisfies joint sparsity.

# Benefits of Joint Sparse Framework

- Joint dictionary learning:
  - Regularize the learning process.
  - Better performance when underlying generative model satisfies joint sparsity.
- Joint channel estimation:
  - Better recovery: additional measurements from uplink training.

# Benefits of Joint Sparse Framework

- Joint dictionary learning:
  - Regularize the learning process.
  - Better performance when underlying generative model satisfies joint sparsity.
- Joint channel estimation:
  - Better recovery: additional measurements from uplink training.
- In other words, we can further decrease downlink training duration  $T^d$  with the same performance.



# Numerical Results

- Apply 3GPP SCM: far scatterer clusters and local scatterer clusters.

# Simulation Settings

- Apply 3GPP SCM: far scatterer clusters and local scatterer clusters.
- Each channel snapshot:
  - 4 local SC: locations change with user.
  - 2 far SC: fixed locations.

# Simulation Settings

- Apply 3GPP SCM: far scatterer clusters and local scatterer clusters.
- Each channel snapshot:
  - 4 local SC: locations change with user.
  - 2 far SC: fixed locations.
- Training samples: 50000 channel snapshots uniformly sampled in the cell.

# Simulation Settings

- Apply 3GPP SCM: far scatterer clusters and local scatterer clusters.
- Each channel snapshot:
  - 4 local SC: locations change with user.
  - 2 far SC: fixed locations.
- Training samples: 50000 channel snapshots uniformly sampled in the cell.
- 100 antennas at base station and 1 antenna at user. Apply uniform linear array.

# Simulation Settings

- Apply 3GPP SCM: far scatterer clusters and local scatterer clusters.
- Each channel snapshot:
  - 4 local SC: locations change with user.
  - 2 far SC: fixed locations.
- Training samples: 50000 channel snapshots uniformly sampled in the cell.
- 100 antennas at base station and 1 antenna at user. Apply uniform linear array.
- Pair of uplink/downlink channel: same angles, different amplitudes and phases.

# Low Dimension Representation

# Dictionary Learning in Channel Representation

Constrain  $T_0$  atoms to be used. Compare  $\text{MSE}(E\|h^d - \hat{h}^d\|_2^2)$  between  $h^d$  and  $\hat{h}^d = D^d\beta^d$ .  $\|h^d\|_2 = 1$ .

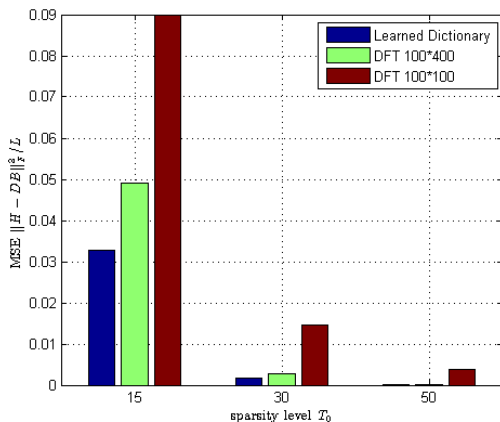


Figure 2: MSE comparison.



# Compressed Channel Estimation

# Dictionary Learning in Joint UL/DL Channel Estimation

Compare MSE between  $h^d$  and  $\hat{h}^d = D^d \hat{\beta}^d$ .  $\hat{\beta}^d = \text{OMP}(Y^d, \Phi, D^d)$ , or  $\hat{\beta}^d = \text{jointOMP}(Y^d, \Phi, D^d; Y^u, \phi, D^u)$ .  $D^d, D^u$ : learned dictionary.

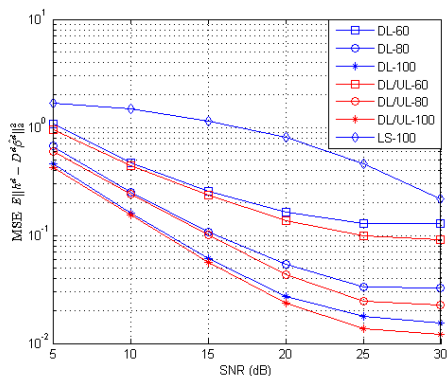


Figure 3: MSE comparison.

- In this work we propose a novel downlink channel estimation algorithm in FDD Massive MIMO systems.

- In this work we propose a novel downlink channel estimation algorithm in FDD Massive MIMO systems.
- Joint uplink/downlink dictionary learning can explore similar scattering effect between the uplink and downlink channel, leading to a joint sparse representation.

- In this work we propose a novel downlink channel estimation algorithm in FDD Massive MIMO systems.
- Joint uplink/downlink dictionary learning can explore similar scattering effect between the uplink and downlink channel, leading to a joint sparse representation.
- Joint compressed channel estimation can further improve the recovery performance by utilizing uplink training information.