Outage Probability for Two-Way Solar-Powered Relay Networks with Stochastic Scheduling

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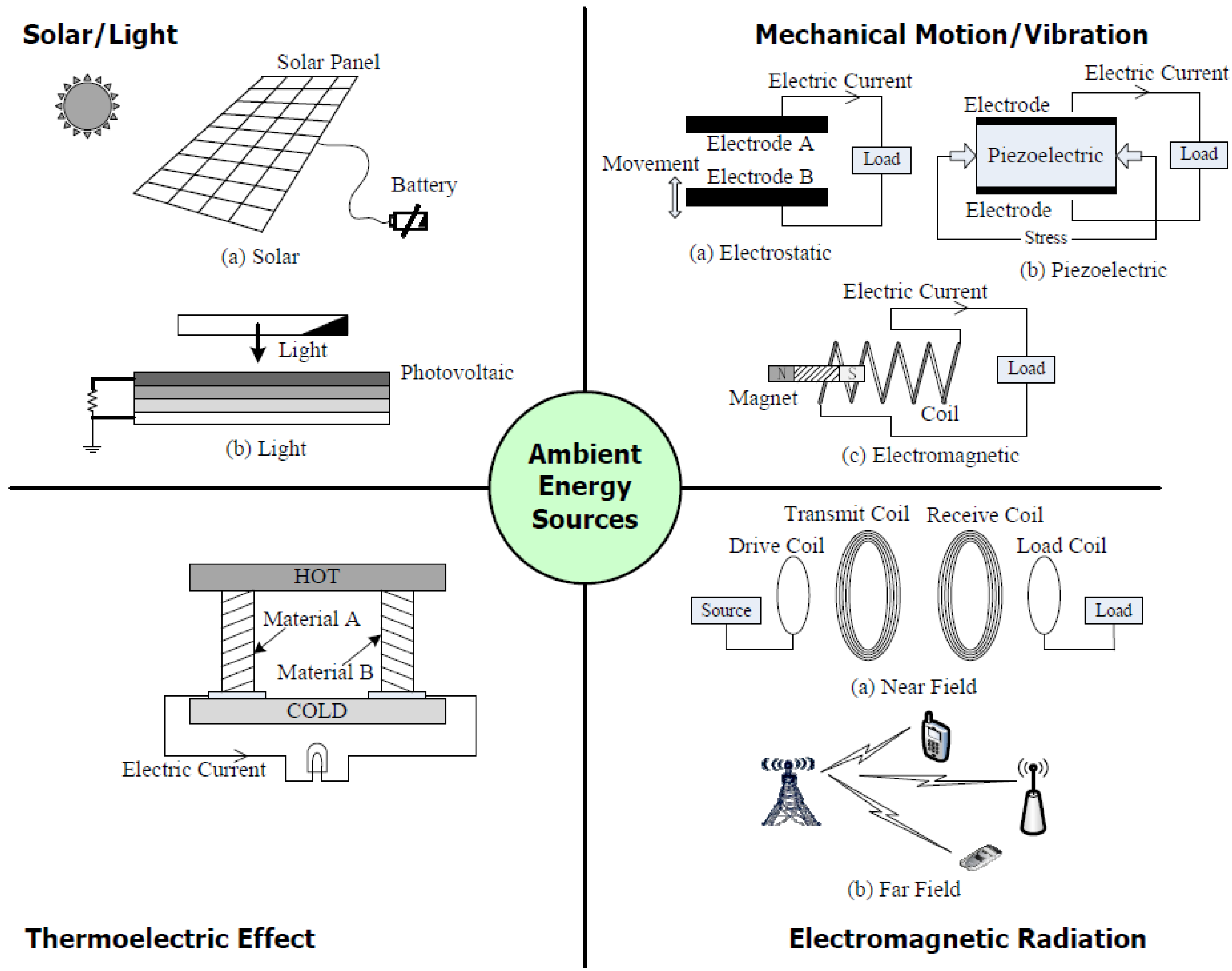
Classification of Energy Sources



Challenges and Motivation

- Key points(problems):
  - Randomness and uncertainty of harvested energy, how to model
  - Optimization of transmission policy, e.g., power allocation, time scheduling, modulation, etc.

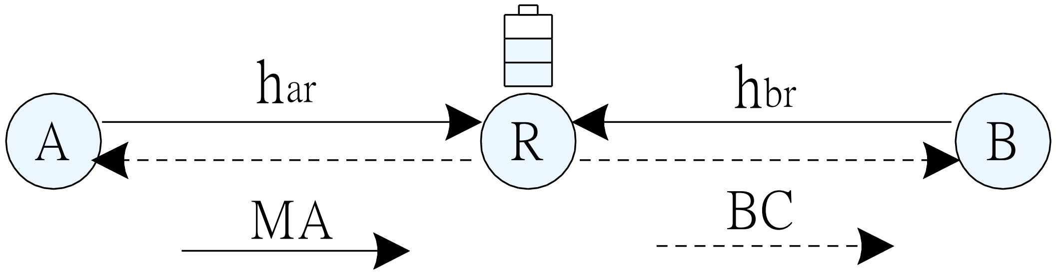
Energy model	Algorithms	Math tools
Deterministic models (Non-causal)	Offline	Convex optimization
Stochastic models (Causal)	Online	Dynamic programming, optimal control

Ref: M.-L. Ku, W. Li, Y. Chen, and K. J. Ray Liu, "Advances in Energy Harvesting Communications: Past, Present, and Future Challenges", to appear in *IEEE Communications Surveys & Tutorials*.

Outline

- Introduction
- Two-way energy harvesting relay networks
- Markov decision process with stochastic models
- Optimization of relay policy
- Structure of optimal relay transmission policy
- Performance analysis of outage probability

Two-way Energy Harvesting Relay Networks



- **Node types:**

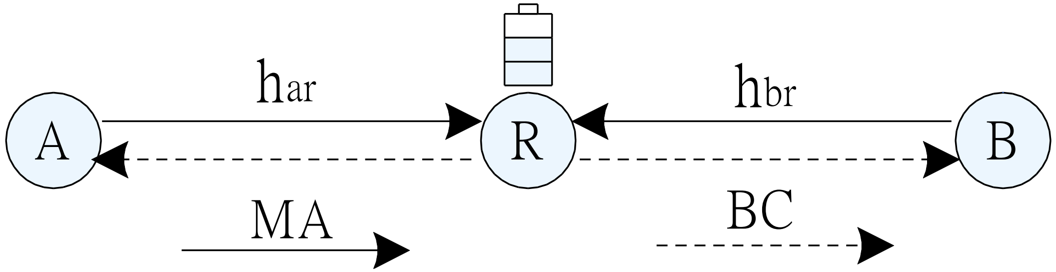
A and B are traditional wireless nodes; R is a solar EH wireless nodes

- **Relay cooperation protocol:**

Amplify-and-Forward (AF)

- **Channel assumptions:**
  - Wireless channels are quasi-static and Rayleigh flat fading;
  - Channels are reciprocal
  - RS has the channel state information (CSI) of all source-relay links;
  - All nodes are half-duplex.

Two-way Energy Harvesting Relay Networks



- Some important variables

$\gamma_1 = |h_{ar}|^2$ the random variable with exponential distribution,

$\gamma_2 = |h_{br}|^2$ the random variable with exponential distribution

R_1 : achievable data rate of link from A to B via R

R_{th1} : target rate of link from A to B via R

R_2 : achievable data rate of link from B to A via R

R_{th2} : target rate of link from B to A via R

$P(= P_a = P_b)$: the transmission power of A and B

P_r : the transmission power of R

Outage Probability in TWR Networks

Link A-R-B: $R_1 = \frac{1}{2}\log\left(1 + \frac{\gamma_1\gamma_2 P_a P_r}{N_0\left(\gamma_1 P_a + \gamma_2 P_b + \gamma_2 P_r + N_0\right)}\right),$

Link B-R-A: $R_2 = \frac{1}{2}\log\left(1 + \frac{\gamma_1\gamma_2 P_b P_r}{N_0\left(\gamma_1 P_a + \gamma_2 P_b + \gamma_1 P_r + N_0\right)}\right),$

Outage Probability:
$$P_{out,AF} = \Pr\left\{\mathcal{E}_{out,AF}^{1} \bigcup \mathcal{E}_{out,AF}^{2}\right\}$$
$$= \Pr\left\{\left(R_1 < R_{th1}\right) \bigcup \left(R_2 < R_{th2}\right)\right\}$$

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Markov Decision Process with Stochastic Models

- **State space** $\mathcal{S} = \mathcal{Q}_e \times \mathcal{H}_{ar} \times \mathcal{H}_{br} \times \mathcal{Q}_b$

  solar energy harvesting state subspace: $\mathcal{Q}_e = \{0, 1, \cdots, N_e - 1\}$

  channel state subspace: $\mathcal{H}_{br} = \{0, 1, \cdots, N_c - 1\}$ $\mathcal{H}_{ar} = \{0, 1, \cdots, N_c - 1\}$

  battery state subspace: $\mathcal{Q}_b = \{0, 1, \cdots, N_b - 1\}$

- **Relay action space** $\mathcal{W}$

  one energy quantum $E_u = P_u T$

  relay transmission power subspace $\mathcal{W} = \{0, 1, \cdots, N_p - 1\} \left(N_p \leq N_b \right)$

  $P_r = w P_u$

- **Reward function**

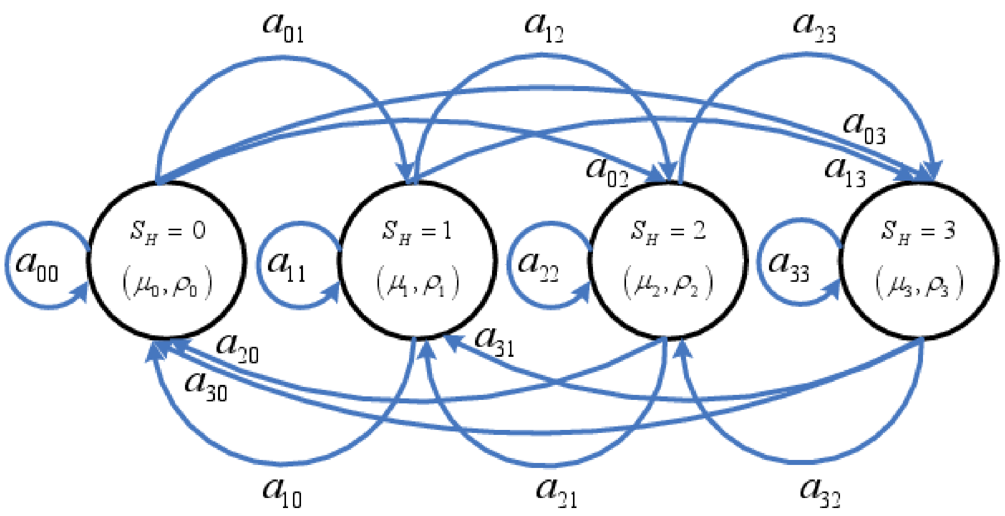
  condition outage probability, i.e., the outage probability conditioned on a fixed system state and relay action, especially the channel states

Stochastic Solar Power Model

N_e-state Gaussian mixture hidden Markov model

solar power per unit area: $P_H \sim \mathcal{N}(\mu_e, \rho_e), \quad e \in \mathcal{S}_E = \{0, 1, \cdots, N_e - 1\}$

solar state transition probability: $P(S_E = j \mid S_E = i) = a_{ij}$



Ref: M.-L. Ku, Y. Chen, and K. J. R. Liu, "Data-Driven Stochastic Transmission Policies for Energy Harvesting Sensor Communications," *IEEE J. Select. Areas Commun.*, vol. 33, no. 8, pp. 1505-1520, Aug. 2015.

Harvested Energy Storage

- **Harvesting-store-and-use (HSU) protocol**
- **Quantization model**

basic transmission power: P_U

one basic energy quantum: $E_U = P_U \cdot \frac{T}{2}$

harvested energy during one policy period *T:* $E_H = P_H T s \eta.$

EH probability in terms of the number of harvested energy quanta:

$$P\left(Q = q \mid S_E = e\right) \quad for \;\; q \in \left\{0, 1, \cdots, \infty\right\}$$

Battery State

- Available energy quanta in the relay battery:

$$b \cdot E_U, \ b \in \mathcal{S}_B = \{0, 1, \cdots, N_b - 1\}$$

- Battery transition model:

$$b' = b - w + q, \ w \in \{0, 1, \cdots, \min(b, N_p - 1)\}$$

- Battery state transition probability under the solar state and relay action

$$P_w(S_B = b' \mid S_B = b, S_E = e) = \begin{cases} P(Q = b' - b + w \mid S_E = e), \ b' = (b - w), \cdots, N_b - 2 \\ 1 - \sum_{q=0}^{N_b - 2 - b + w} P(Q = q \mid S_E = e), b' = N_b - 1 \end{cases}$$

Ref: M.-L. Ku, Y. Chen, and K. J. R. Liu, "Data-Driven Stochastic Transmission Policies for Energy Harvesting Sensor Communications," *IEEE J. Select. Areas Commun.*, vol. 33, no. 8, pp. 1505-1520, Aug. 2015.

Channel State

- N_c-state Markov chain

$$\Gamma = \{0 = \Gamma_0, \Gamma_1, \cdots, \Gamma_{N_c} = \infty\} \qquad S_{AR} = i \Leftrightarrow \gamma_{AR} \in [\Gamma_i, \Gamma_{i+1})$$

- Channel state stationary probability

$$P(H = i) = \int_{\Gamma_i}^{\Gamma_{i+1}} \frac{1}{\lambda} \exp\left(-\frac{\gamma}{\lambda}\right) d\gamma = \exp\left(-\frac{\Gamma_i}{\lambda}\right) - \exp\left(-\frac{\Gamma_{i+1}}{\lambda}\right).$$

- Channel state transition probability $h(\gamma) = f_D \sqrt{2\pi\gamma/\lambda} \exp(-\gamma/\lambda)$

$$P(H = j \mid H = i) = \begin{cases} \dfrac{h(\Gamma_{i+1})}{P(H = i)}, \ j = i+1, i = 0, 1, \cdots, N_c - 2 \\ \dfrac{h(\Gamma_i)}{P(H = i)}, \ j = i-1, i = 1, 2, \cdots, N_c - 1 \\ 1 - \dfrac{h(\Gamma_i)}{P(H = i)} - \dfrac{h(\Gamma_{i+1})}{P(H = i)}, \ j = i, i = 1, \cdots, N_c - 2 \end{cases}$$

Ref: H. S. Wang and N. Moayeri, “Finite-State Markov Channel-A Useful Model for Radio Communication Channels,” *IEEE Trans. Wireless Commun.*, vol. 44, no. 1, pp. 163–171, Feb. 1995.

System States

- System state transition probability

$$S = (Q_e, H_{ar}, H_{br}, Q_b) \in \mathcal{S}$$

$$P_w\{S = (e', h', g', b') \mid S = (e, h, g, b)\}$$
$$= P(Q_e = e' \mid Q_e = e) \cdot P(H_{ar} = h' \mid H_{ar} = h) \cdot$$
$$P(H_{br} = g' \mid H_{br} = g) \cdot P_a(Q_b = b' \mid Q_b = b, Q_e = e),$$

- Relay action space $\mathcal{W}$

one energy quantum $E_u = P_u T$

relay transmission power subspace $\mathcal{W} = \{0, 1, \cdots, N_p - 1\} (N_p \le N_b)$

$$P_r = w P_u$$

Reward Function

Condition Outage Probability:

the outage probability conditioned on a fixed system state and relay action, especially the channel states.

$$When \ \ s = (e, h, g, b)$$

$$R_w(s) = \Pr\{\text{outage}|w, s\} = P_{out}(w, f, h, g).$$

$$P_{out}(w, f = 0, h, g) = \Pr\left\{\mathcal{E}^1_{out,AF} \cup \mathcal{E}^2_{out,AF} | P_r = wP_u, H_{ar} = h, H_{br} = g\right\},$$

Conditional Outage Probability in AF Mode

Theorem 1: Define four channel power thresholds as follows:

$$\gamma_{th1} = \frac{(P+wP_u)N_0}{P \cdot wP_u}\left(2^{2R_{th1}}-1\right), \gamma_{th2} = \frac{N_0}{wP_u}\left(2^{2R_{th2}}-1\right),$$

$$\gamma_{th3} = \frac{N_0}{wP_u}\left(2^{2R_{th1}}-1\right), \gamma_{th4} = \frac{(P+wP_u)N_0}{P \cdot wP_u}\left(2^{2R_{th2}}-1\right).$$

The condition outage probability of TWR networks using AF cooperation protocol with respect to the system state $s = (e, h, g, b)$ and relay transmission power w can be expressed as follows:

Case 1: $\gamma_{th1} \geq \Gamma_{h+1}$, or $\gamma_{th2} \geq \Gamma_{h+1}$, or $\gamma_{th3} \geq \Gamma_{g+1}$ or $\gamma_{th4} \geq \Gamma_{g+1}$,

$$P_{out}(w, f = 0, h, g) = 1;$$

Case 2: $\gamma_{th1} \leq \Gamma_h$, and $\gamma_{th2} \leq \Gamma_h$, and $\gamma_{th3} \leq \Gamma_g$ and $\gamma_{th4} \leq \Gamma_g$,

$$P_{out}(w, f = 0, h, g) = 0;$$

Case 3: otherwise,

$$P_{out}(w, f = 0, h, g) \approx 1 - \frac{e^{-\max(\gamma_{th1},\gamma_{th2})/\lambda} - e^{-\Gamma_{h+1}/\lambda}}{e^{-\Gamma_h/\lambda} - e^{-\Gamma_{h+1}/\lambda}} \cdot \frac{e^{-\max(\gamma_{th3},\gamma_{th4})/\lambda} - e^{-\Gamma_{g+1}/\lambda}}{e^{-\Gamma_g/\lambda} - e^{-\Gamma_{g+1}/\lambda}}.$$

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Optimization of Relay Policy

Define the policy $\pi(s)$: $\mathcal{S} \to \mathcal{W}$ as the relay action in the state s

the expected discount long-term reward

$$V_{\pi}(s_0) = E_{\pi}\left[\sum_{k=0}^{\infty} \lambda^k R_{\pi(s_k)}(s_k)\right], \quad s_k \in \mathcal{S}, \quad \pi(s_k) \in \mathcal{W}.$$

the optimal policy can be found through the Bellman equation

$$V_{\pi^*}(s) = \min_{w \in W}\left(R_w(s) + \lambda \sum_{s' \in S} P_w(s' \mid s) V_{\pi^*}(s')\right), \quad s \in \mathcal{S}.$$

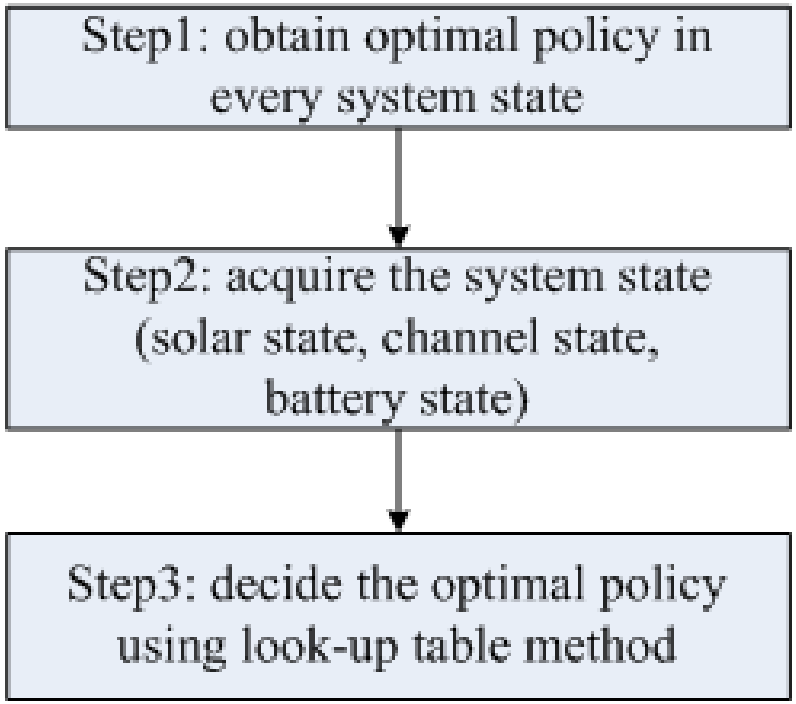
the well-known value iteration approach can be applied to find the optimal policy

$$V_w^{i+1}(s) = R_w(s) + \lambda \sum_{s' \in S} P_w(s' \mid s) V^{(i)}(s'), \quad s \in \mathcal{S}, \quad w \in \mathcal{W};$$

$$V^{i+1}(s) = \min_{w \in W}\left(V_w^{i+1}(s)\right), \quad s \in \mathcal{S}.$$

$$\left|V^{i+1}(s) - V^{i}(s)\right| \le \varepsilon$$

Online Algorithm



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Structure of Optimal Relay Transmission Policy

- Lemma 1: For any fixed system state $s = (e, h, g, b > 0)$ in the i^{th} value iteration, the expected long-term reward is non-increasing in the battery state, and the differential value of the expected long-term rewards between two adjacent battery states is not larger than one, i.e.,

$$1 \geq V^{(i)}(e,h,g,b-1) - V^{(i)}(e,h,g,b) \geq 0, \quad \forall b \in \mathcal{Q}_b \backslash \{0\}$$

- Lemma 2: For any fixed system state $s = (e, h, g, b > 0)$ with the non-empty battery, in sufficiently high SNRs, i.e., N_0 approaches to zero, the optimal relay power action w^* is equal to one.

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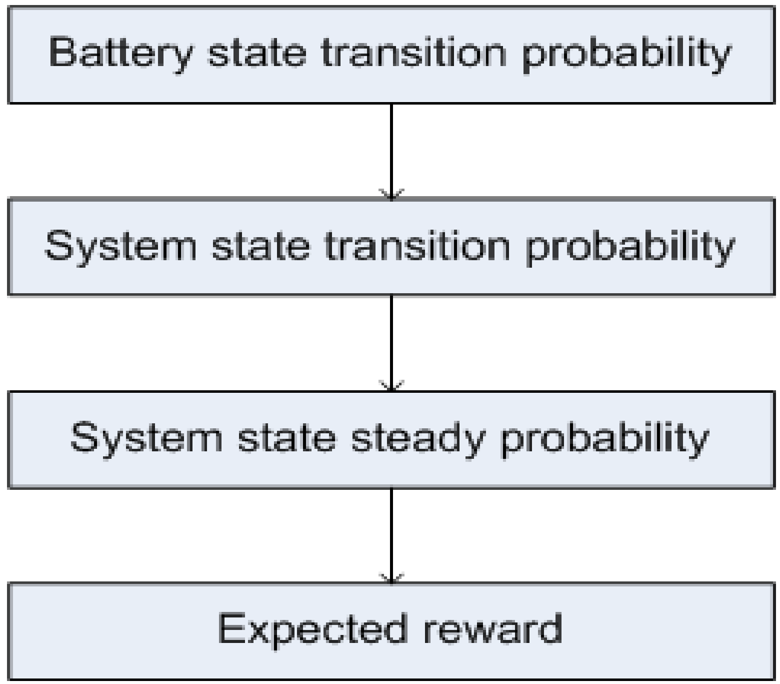
Expected Reward Analysis

Algorithm: Calculate the state steady probability and expected reward



$$\bar{R} = \sum_{s \in S} p_s \times R_{w^*} \left(s = \left(e, h, g, b \right) \right)$$

Saturation Structure of Outage Probability

Theorem: In sufficiently high SNRs, the expected outage probability for the proposed optimal policy π^* is equal to the battery empty probability.

$$\bar{R} = \sum_{s\in\mathcal{S}} p_{\pi^*}(s) \times R_{w^*=\pi^*(s)}(s)$$
$$= \sum_{s\in\mathcal{S}} \left[p_{\pi^*}(e,h,g,b=0) \times R_{w^*}(e,h,g,b=0) + p_{\pi^*}(e,h,g,b\geq 1) \times R_{w^*}(e,h,g,b\geq 1) \right].$$

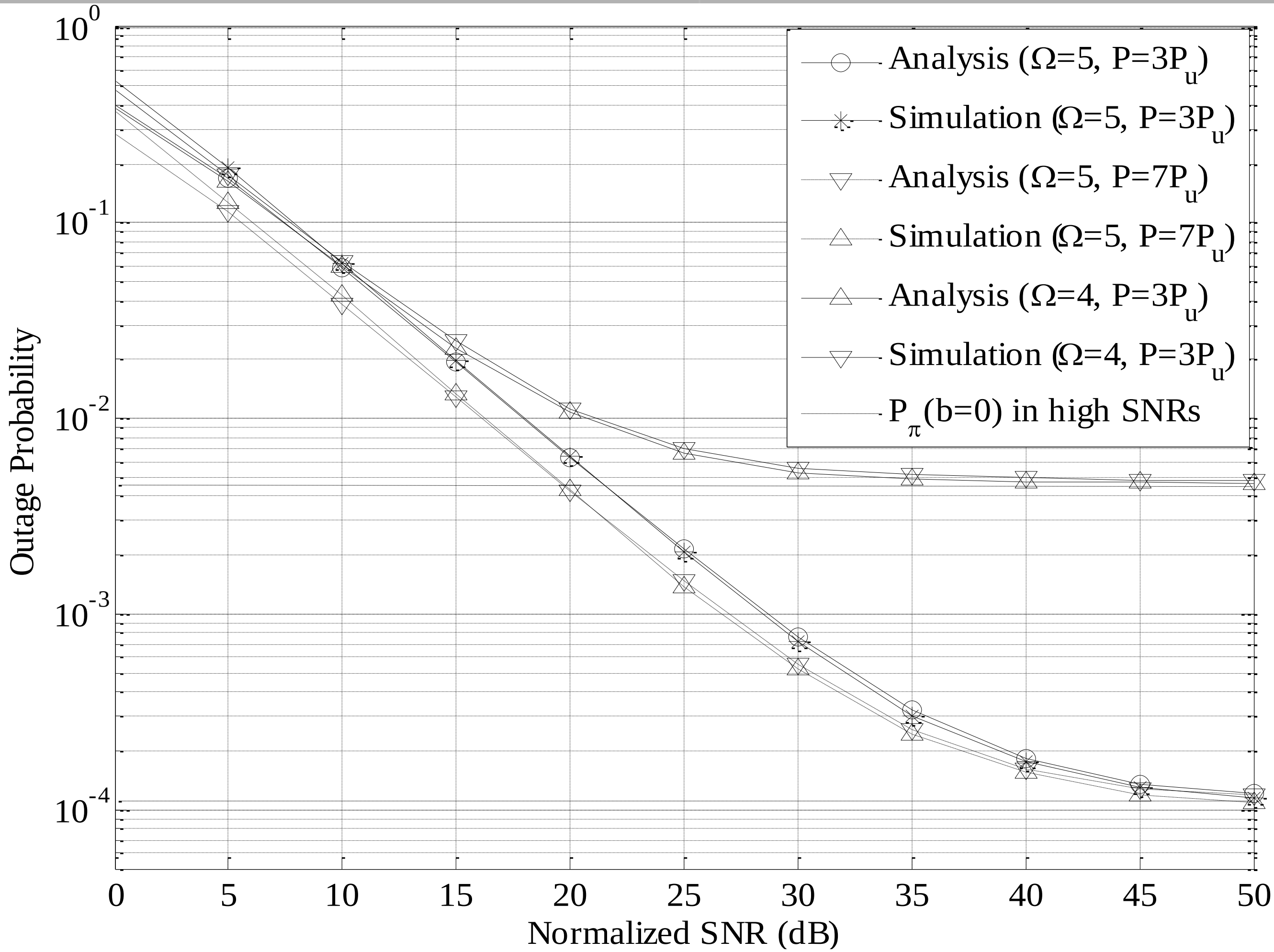
$$\lim_{N_0\to 0} \bar{R} = \sum_{e=0}^{N_e-1} \sum_{h=0}^{N_c-1} \sum_{g=0}^{N_c-1} p_{\pi^*}(e,h,g,b=0) = P_{\pi^*}(b=0)$$

Simulation Parameters

SIMULATION PARAMETERS

Basic transmission power (P_u)	35mW
Policy management period (T_M)	300s
Number of solar states (N_e)	4
Solar panel area (Ω)	5cm^2
Energy conversion efficiency (η)	20% [1]
Average channel power (θ)	1
Channel simulation model	Jakes' model
Number of channel states (N_c)	6
Channel quantization thresholds (Γ)	$\{0, 0.3, 0.6, 1.0, 2.0, 3.0, \infty\}$
Discount factor (λ)	0.99
Stopping criterion parameter (ε)	10^{-5}
Number of battery states (N_b)	12
Target rate proportion (σ)	0.5

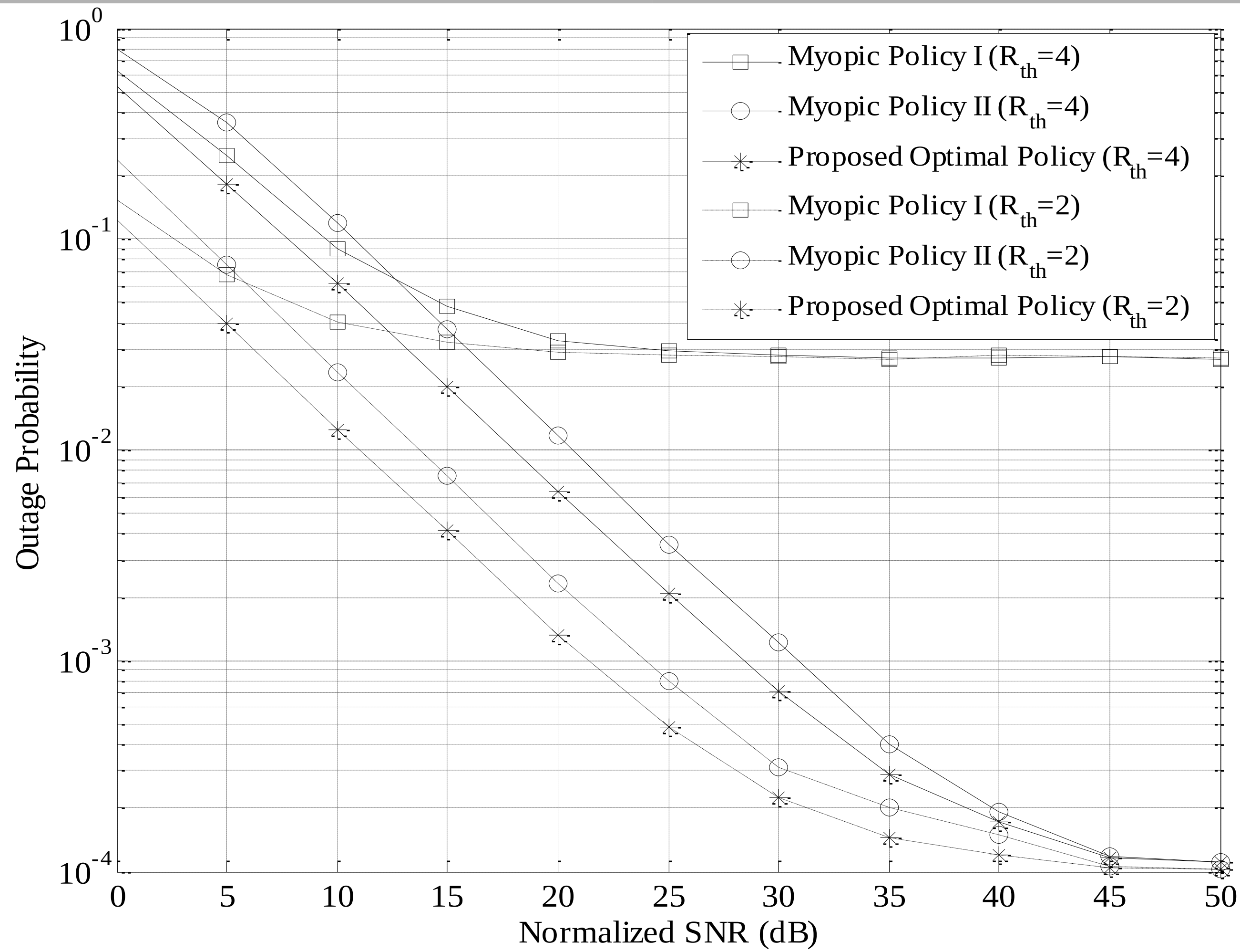
Simulation Results of Optimal Outage Probability



Outage probability for solar panel size Ω and source nodes' transmission power *P*

(R_{th} = 4 bit/s/Hz, Unit of Ω : cm^2)

Simulation Results of Optimal Outage Probability



Outage probabilities of the proposed optimal policy and myopic policies

($P = 3Pu$, Unit of R_{th}: bit/s/Hz)

Thank you!