GREEDY APPROACHES TO FINDING A SPARSE COVER IN A SENSOR NETWORK WITHOUT LOCATION INFORMATION

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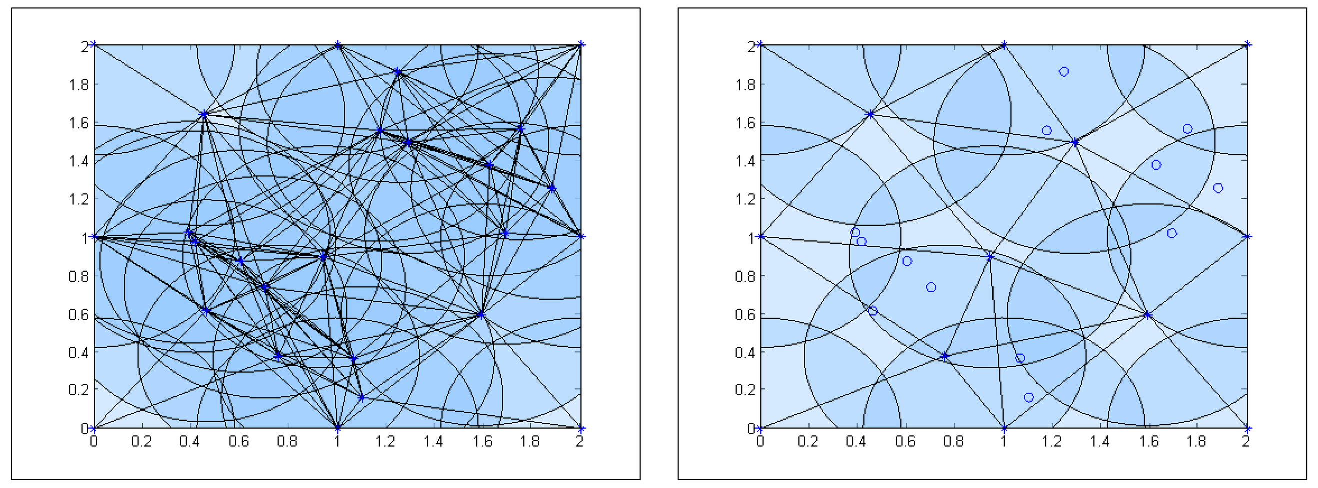
Objective & Main Result

Given a fenced sensor network covering a compact connected region, find a "sparse" cover without location information, i.e., find a small subset of nodes that still covers the region.



A realization of a sensor network with $p = 18$ interior nodes and 8 fence nodes circumscribing the square domain (left). Only $p = 5$ interior nodes ('*' = active, '○' = inactive) are required for coverage of the domain (right).

Main Result

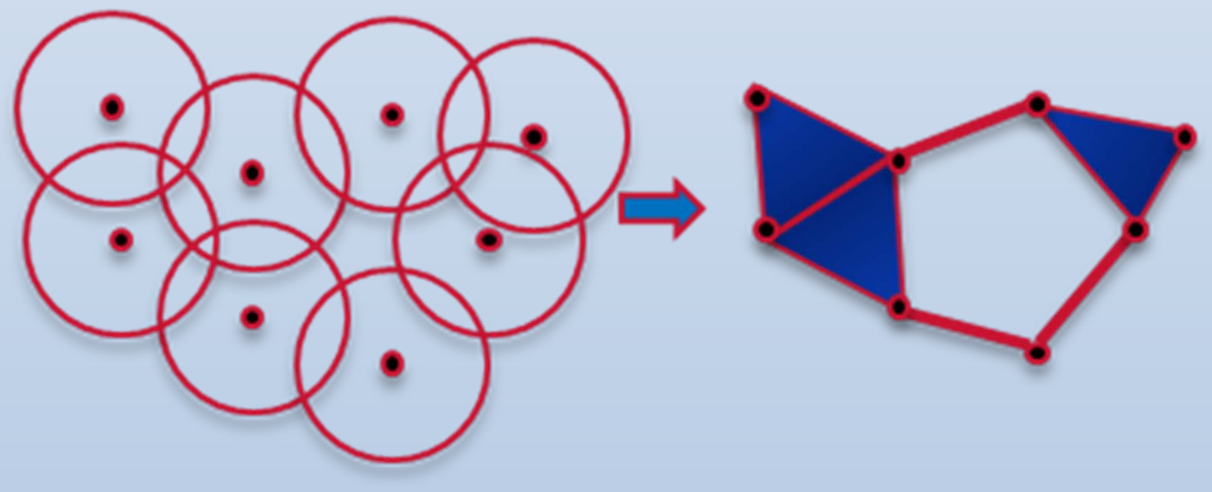
Using approaches from algebraic topology, we develop three greedy distributed approaches (calculating homology changes locally, strong collapsing, and Euler characteristic collapsing) to determine which nodes are redundant to the sensing cover.

Assumptions & Theoretical Background

The sensor nodes:

- have no location information (e.g., no GPS)
- have unique IDs
- have communication radius r_c and can exchange ID information with other nodes within this radius
- have sensing radius $r_s \geq r_c/\sqrt{3}$
- reside in a compact connected domain $\mathcal{D} \subset \mathbb{R}^2$ with a connected boundary $\partial\mathcal{D}$ covered by "fence nodes" connected in a simple loop

Giving these assumptions, each node can learn the connections among its neighbors and each clique it resides in the communication graph. These sets of cliques form the simplices in a ***simplicial complex***, the Rips complex, where each set is a subset of the true coverage representation given by the Čech complex.



3 Greedy Approaches

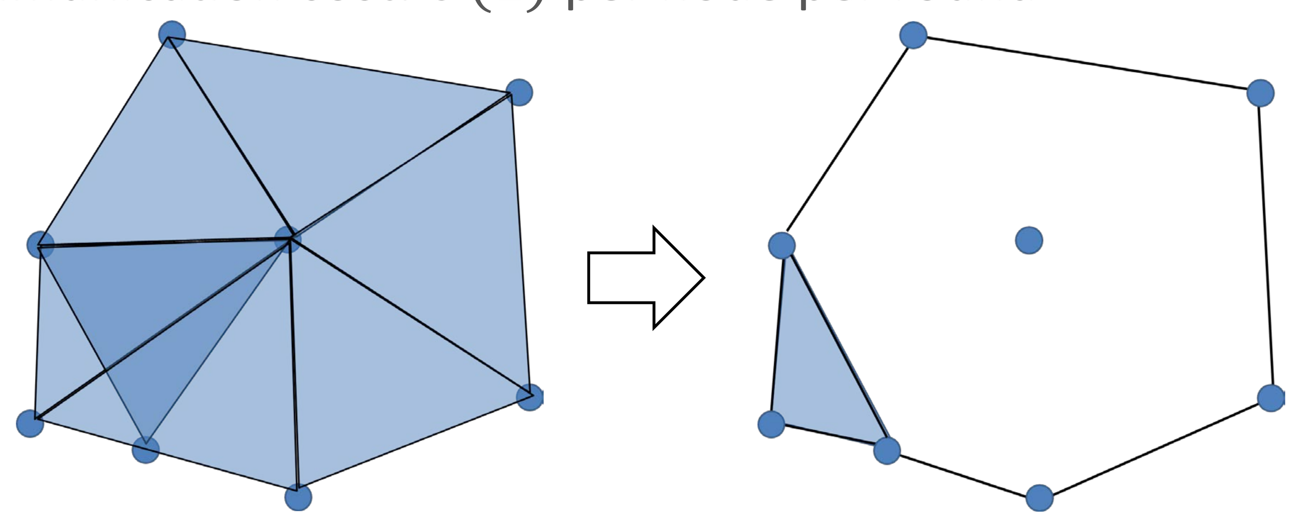
Calculating Homology Changes Locally

Observe that if a node is removed, the only change in the simplicial complex is that every simplex containing that node is removed. So if a homology class in H_1 is created, then a cycle that does not bound exists in the link (neighborhood) of the node.

Calculation: Test if $rank\big(H_1(\mathcal{L}(v))\big) = 0$.

Complexity: $O(d^9)$ per node per round, where d is node degree

Communication cost: $O(1)$ per node per round



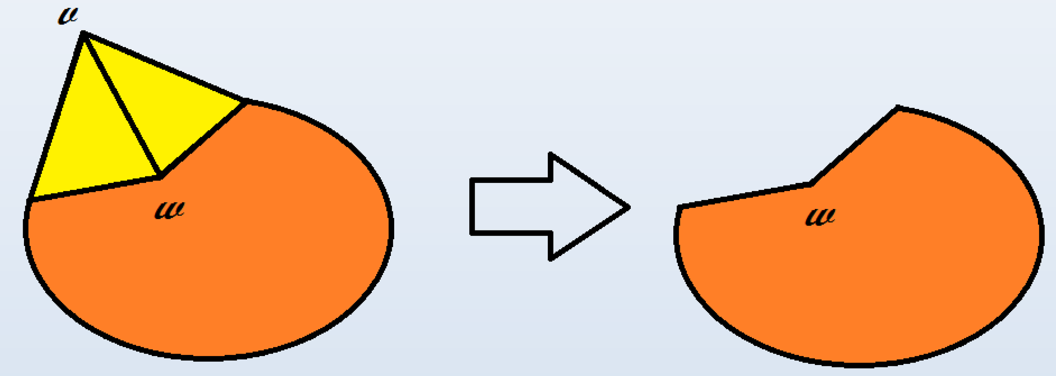
Strong Collapsing

If every maximal simplex (clique) containing v also contains w, then v is redundant.

Calculation: Test if $\bigcap\{f_i | f_i \; maximal \; simplex \ni v\}/\{v\} \neq 0$.

Complexity: $O(d + pm)$ per node per round, where d is node degree, m is node maximal simplex degree, and p is size of the maximal simplex

Communication cost: $O(1)$ per node per round



Euler Characteristic Collapsing

Observe that if the homology of an object hasn't changed, then its Euler characteristic hasn't changed either. (Note: the homology can change and keep the same characteristic, but this probability is hopefully low.)

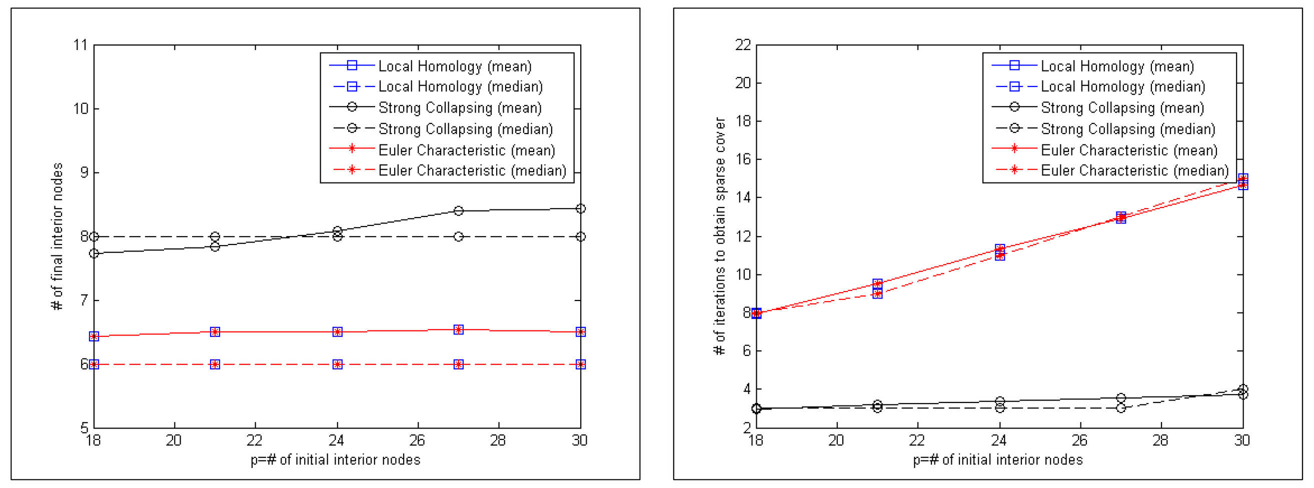
Calculation: Test if $\sum_k (-1)^k \left|\{\sigma^{(k)} | v \in \sigma^{(k)}\}\right| = 0$.

Complexity: $O(q2^{\min(d,m)})$ per node per round, where d is node degree, m is node maximal simplex degree, and q is a polynomial

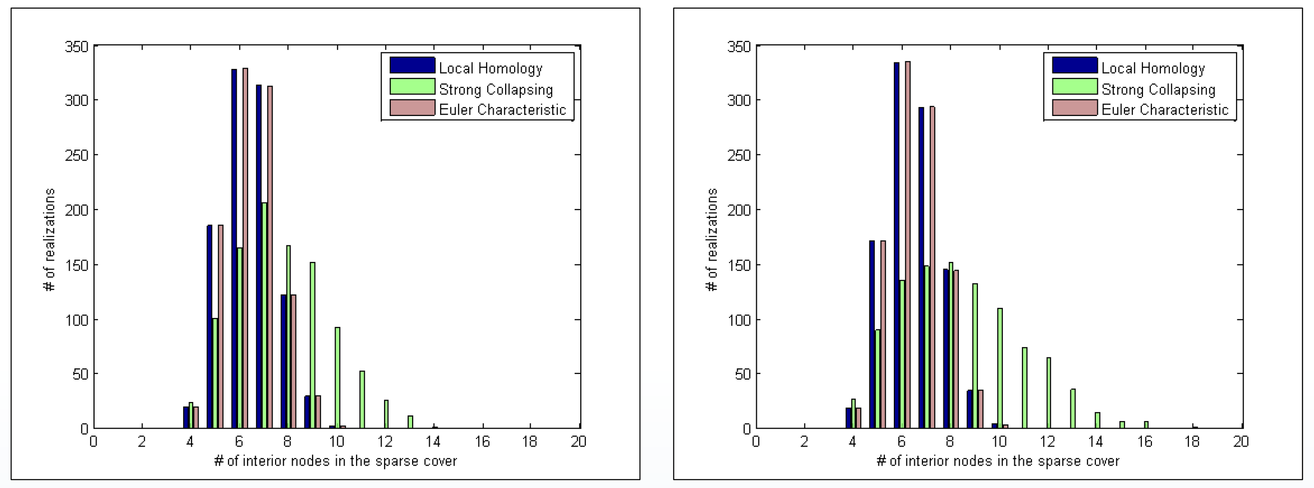
Communication cost: $O(1)$ per node per round

Simulation Results

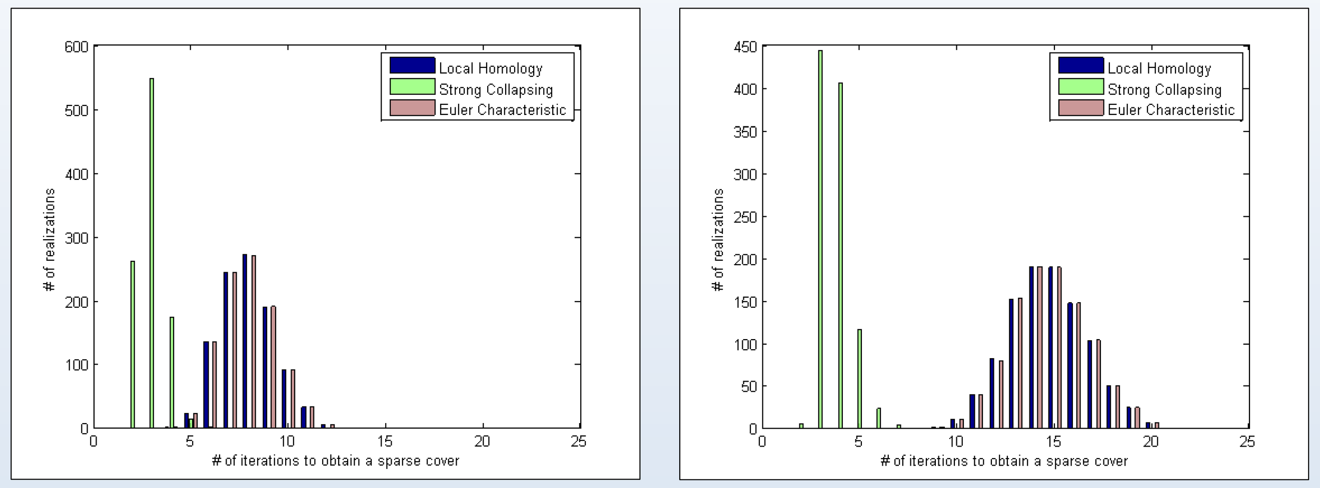
We randomly distribute p (=18, 21, 24, 27, 30) nodes in a 2x2 square region, with $r_c = 1$ and $r_s = 1/\sqrt{3}$ over 1000 realizations.



Mean and median values for the final number of interior nodes in the sparse cover (left) and the number of iterations required to obtain that state (right), for each method.



Distribution of the final number of interior nodes in the sparse cover for each method when the initial number of nodes was p=18 (left) and p=30 (right).



Distribution of the number of iterations required to obtain the sparse cover for each method when the initial number of nodes was p=18 (left) and p=30 (right).

References

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