A Tucker Decomposition Based Approach for Topographic Functional Connectivity State Summarization

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Outline

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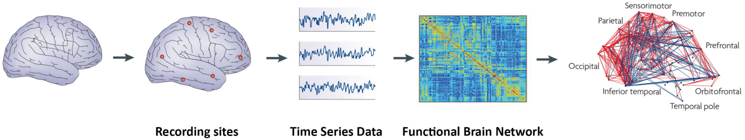
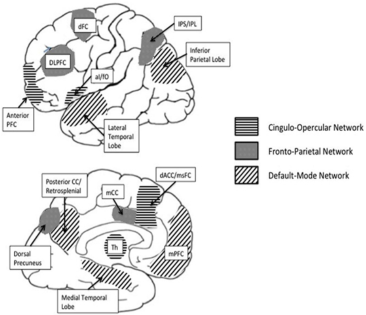
Introduction

- Higher brain functions depend on the balance between local specialization (functional segregation) and global integration (functional integration) of brain processes (Friston, 2011; Friston, 2001; Le Van Quyen, 2003; Stam, 2005; Tononi et al., 1998).
- Imaging neuroscience (EEG, MEG, fMRI) has firmly established functional segregation as a principle of brain organization in humans.
- The integration of segregated areas has proven more difficult to assess.
- Therefore, there is a need to identify task-related interactions between neuronal populations.



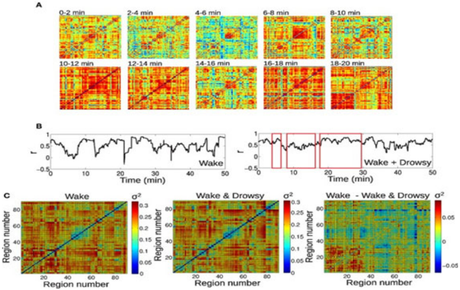
Functional Connectivity

- Cognitive control processes are responsible for goal or context representation and maintenance, attention allocation and stimulus-response mapping.
- In particular, for cognitive control:
  - Medial prefrontal cortex (mPFC) and lateral prefrontal cortex (lPFC) play an important role.
  - Synchronization connects anterior cingulate cortex (ACC) and lPFC (Womelsdorf et al. 2014, Current Biology).
- Impaired cognitive control plays a role in schizophrenia, impulse control and anxiety disorders.



Dynamic Functional Connectivity Networks

- Functional connectivity networks transition through quasi-stationary microstates over time (Lehmann et al. 1997).
- Current Approaches to network state representations:
  - Sliding window FC analysis (Chang and Glover, 2010)
  - k-means clustering (Allen et al. 2012)
  - Principal Component Analysis (Leonardi et al. 2013)
- Shortcomings: The intrinsic network structure is not preserved: Averaging, Vectorizing.
- Our solution: Tensors are used to represent and summarize functional connectivity networks.



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Functional Connectivity: Phase Synchrony

- Reduced-interference Rihaczek distribution (RID-Rihaczek):

$$C_i(t,\omega) = \int\int \underbrace{\exp\left(-\frac{(\theta\tau)^2}{\sigma}\right)}_{\text{Choi-Williams kernel}} \underbrace{\exp(j\frac{\theta\tau}{2})}_{\text{Rihaczek kernel}} A_i(\theta,\tau) e^{-j(\theta t+\tau\omega)} d\tau d\theta. \quad (1)$$

  - Ambiguity function: $A_i(\theta,\tau) = \int s_i(u+\frac{\tau}{2}) s_i^*(u-\frac{\tau}{2}) e^{j\theta u} du$.

- The phase distribution: $\Phi_i(t,\omega) = \arg\left[\frac{C_i(t,\omega)}{|C_i(t,\omega)|}\right]$.
- The phase difference between the two signals can be defined as: $\Phi^k_{(i,j)}(t,\omega) = |\Phi^k_i(t,\omega) - \Phi^k_j(t,\omega)|$.
- Phase locking value (PLV) quantifies the functional integration, as:

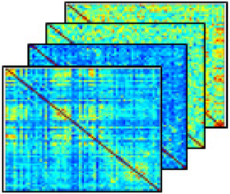
$$PLV_{(i,j)}(t,\omega) = \frac{1}{L}\left|\sum_{k=1}^{L} \exp\left(j\Phi^k_{(i,j)}(t,\omega)\right)\right|, \quad 0 \le PLV \le 1. \quad (2)$$

Construction of d-FCNs

- Functional connectivity matrix:

$$G_{s,(i,j)}(t) = \frac{1}{\Omega} \sum_{\omega=\omega_a}^{\omega_b} PLV_{s,(i,j)}(t,\omega), \tag{3}$$

  - $G_{(i,j)}(t) \in [0,1]$, $[\omega_a, \omega_b]$: frequency band of interest, Ω: the number of frequency bins, s: the subject.



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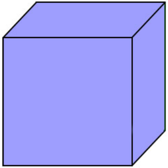
Overview of tensors

- The extension of vectors and matrices to higher dimension is called multiway array, or tensor.
- $\mathcal{X} \in \mathbb{R}^{m_1 \times m_2 \times \ldots \times m_d}$ is a d-way tensor, where $x_{i_1, i_2, i_3, \ldots, i_d}$ is its $(i_1, i_2, i_3, \ldots, i_d)$th element.
- Collection of the FC matrices of all subjects, $\mathbf{G}_s(t)$, forms $\mathcal{G}(t) \in \mathbb{R}^{N \times N \times S}$.



Tucker Decomposition

- Tucker Decomposition is flexible in representing higher order data, and has orthogonal component matrices.
- Tucker decomposition is calculated using alternative least square (ALS) method.

Tucker decomposition of $\mathcal{X} \in \mathbb{R}^{m_1 \times m_2 \times \ldots \times m_d}$:

$$\mathcal{X} = \mathcal{C} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times_3 \mathbf{U}^{(3)} \ldots \times_d \mathbf{U}^{(d)} + \mathcal{E},$$
$$\mathcal{X} = \sum_{i_1,i_2,i_3,\ldots,i_d} \mathcal{C}_{i_1,i_2,i_3,\ldots,i_d} \left(\mathbf{u}_{i_1}^{(1)} \circ \mathbf{u}_{i_2}^{(2)} \circ \mathbf{u}_{i_3}^{(3)} \circ \ldots \circ \mathbf{u}_{i_d}^{(d)} \right) + \mathcal{E}_{i_1,i_2,i_3,\ldots,i_d}, \tag{4}$$

- $\mathcal{C} \in \mathbb{R}^{r_1 \times r_2 \times \ldots \times r_d}$ is the core tensor.
- $\mathbf{U}^{(1)} \in \mathbb{R}^{m_1 \times r_1}$, $\mathbf{U}^{(2)} \in \mathbb{R}^{m_2 \times r_2}$, $\ldots \mathbf{U}^{(d)} \in \mathbb{R}^{m_d \times r_d}$.
- $\mathcal{E} \in \mathbb{R}^{m_1 \times m_2 \times \ldots \times m_d}$ is the residual.

Tucker Decomposition
continued

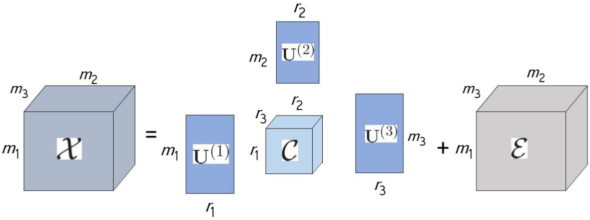


Figure: Tucker decomposition for a 3-way tensor.

n–mode product

n–mode product is multiplying the tensor unfolded along the nth mode by a matrix.

$$\mathcal{X} \times_n \mathbf{U} = \mathbf{U}^{\dagger}\mathbf{X}_{(n)} = \sum_{i_n} x_{i_1,i_2,\ldots,i_n,\ldots,i_d} U_{j_n,i_n} \tag{5}$$

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Overview

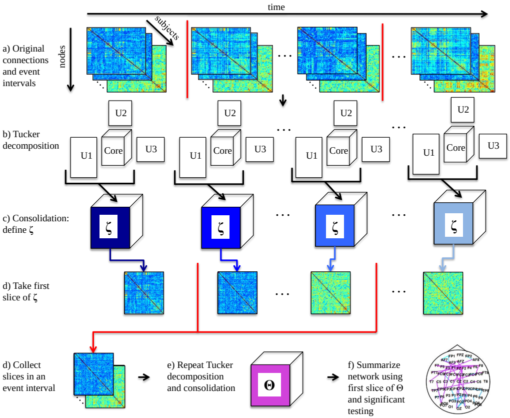

Figure: Functional connectivity state summarization algorithm flowchart.

Subject Summarization

- $\mathcal{G}(t) \in \mathbb{R}^{N \times N \times S}$ within the time interval $t = 1, 2, \ldots, T$ is fully decomposed using Tucker decomposition:

$$\mathcal{G}(t) = \mathcal{C}(t) \times_1 \mathbf{U}^{(1)}(t) \times_2 \mathbf{U}^{(2)}(t) \times_3 \mathbf{U}^{(3)}(t). \quad (6)$$

- Let's define:
$\zeta(t) = \mathcal{C}(t) \times_1 \mathbf{U}^{(1)}(t) \times_2 \mathbf{U}^{(2)}(t) \rightarrow \mathcal{G}(t) = \zeta(t) \times_3 \mathbf{U}^{(3)}(t)$.
- The subtensor $\theta(t) \in \mathbb{R}^{N \times N}$ captures most of the energy of the activation patterns across subjects at time:

$$\theta(t) = \zeta_{i_3=1}(t) = \sum_{s=1}^{S} U_{s,1}^{(3)}(t)\mathbf{G}_s(t). \quad (7)$$

Time Summarization

- $\theta(t),\ \forall t \in \{1, 2, \cdots, T\}$ are summarized across time mode to derive the state connectome.
- The 3-way tensor $\Theta \in \mathbb{R}^{N \times N \times T}$ is constructed from $\theta(t)$, and fully decomposed using Tucker decomposition:

$$\Theta = \vartheta \times_1 \bar{\mathbf{U}}^{(1)} \times_2 \bar{\mathbf{U}}^{(2)} \times_3 \bar{\mathbf{U}}^{(3)} = \bar{\zeta} \times_3 \bar{\mathbf{U}}^{(3)}. \tag{8}$$

- The subtensor $\eta = \bar{\zeta}_{i_3=1} = \sum_{t=1}^{T} \bar{U}_{t,1}^{(3)} \Theta_{i_3=t}$ captures the largest amount of energy across all time steps.

Significance Testing

- The significant edges of η is determined through hypothesis testing.
- A Gaussian distribution for the edge values in η is assumed.
- This assumption can be validated using Kolmogorov–Smirnov test.
- z-test is used on the edges of η to determine the most significant edges.

$$\mathbf{H_0}: \quad \eta(i,j) \sim \mathcal{N}_{erp}(\mu_{erp}, \sigma_{erp})$$
$$\mathbf{H_1}: \quad \eta(i,j) \sim \mathcal{N}_1(\mu_1 \neq \mu_{erp}, \sigma_1 \neq \sigma_{erp})$$

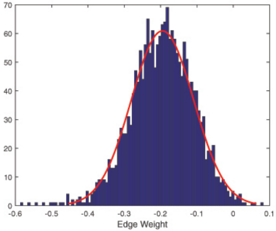

Figure: The histogram of the projected tensor edge values in the matrix η for ERN.

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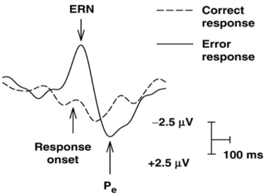
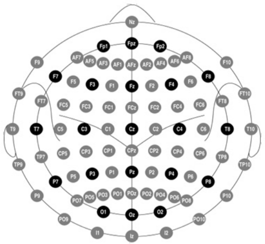
EEG Data

- Error-Related Negativity (ERN) occurs 50-100ms after subjects made errors in response to a speeded motor task.
- Modified Eriksen flanker task for 2 seconds with multiple trials (10-40 error trials per subject).
- 91 subjects, 63 electrodes collected from undergraduates at the University of Minnesota.
- Sampling rate: 128 Hz.
- ERN is dominated by partial phase-locking of intermittent theta band (3-7 Hz) EEG activity between mPFC and lPFC (Cavanagh et al., 2009).





Experimental Results

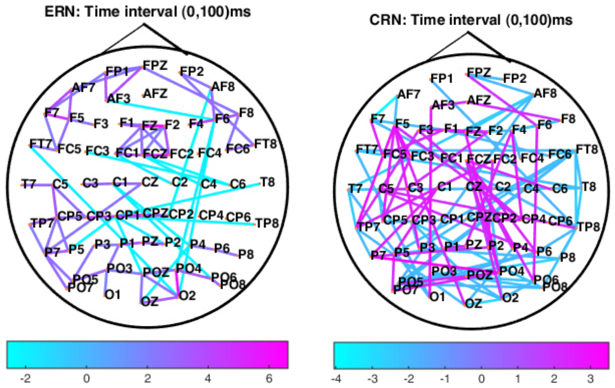


Figure: The most significant edges of the network summarization matrix, η with $p = 0.95$ for: (a) ERN, (b) CRN.

Experimental Results
Discussion

- ERN time interval:
  - Increased connectivity in medial- prefrontal regions, engaging electrodes (F1, Fz, F2, FC1, FCz, FC2) → Engagement of these regions during the ERN.
  - Sparse connections from right lateral frontal to parietal and occipital regions.
- CRN time interval:
  - Connectivity between right lateral frontal and left-temporal regions.
  - Strong connections between left lateral frontal and parietal region.

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Summary

- We proposed a tensor based method for data reduction of dynamic functional connectivity matrices across subjects.
- Tensor-tensor projection along both directions can be used to summarize the connectivity within different time intervals.

Future Work

- Detect the change points instead of using *a priori* information to define time intervals.
- Extend this work to include the frequency information as the $5th$ mode of the tensor.