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Pilot-aided Direction of Arrival Estimation for mmWave Cellular Systems

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Outline

- Motivation
- Prior work
- Pilot assisted, sub-sample based MUSIC algorithm
- Simulation results

Motivation

Problem formulation:

Concurrent DoA estimation of mmWave primary and secondary beams.

- Dynamic mmWave channel is susceptible to blockage
- 5G requires ultra low latency

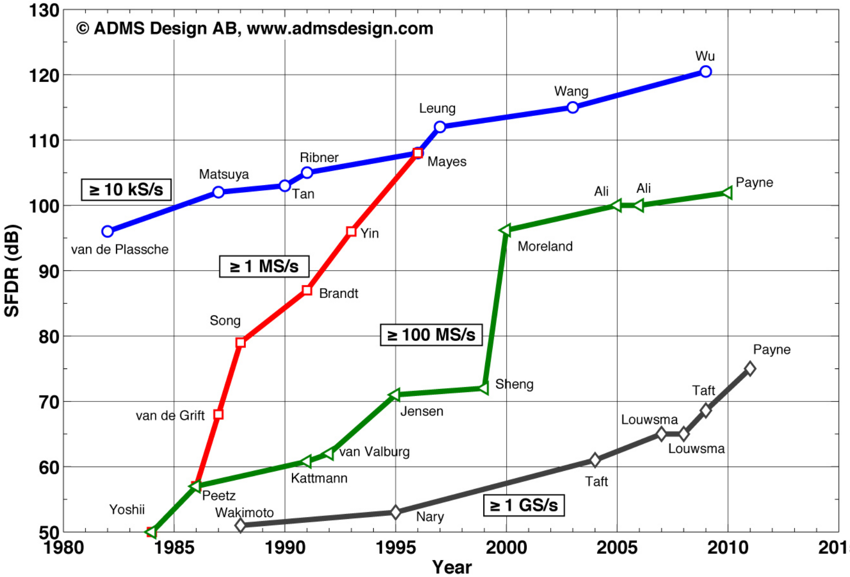
5G Requirements:

- Bandwidth
- Latency
- Energy efficiency
- Reliability

Solution:

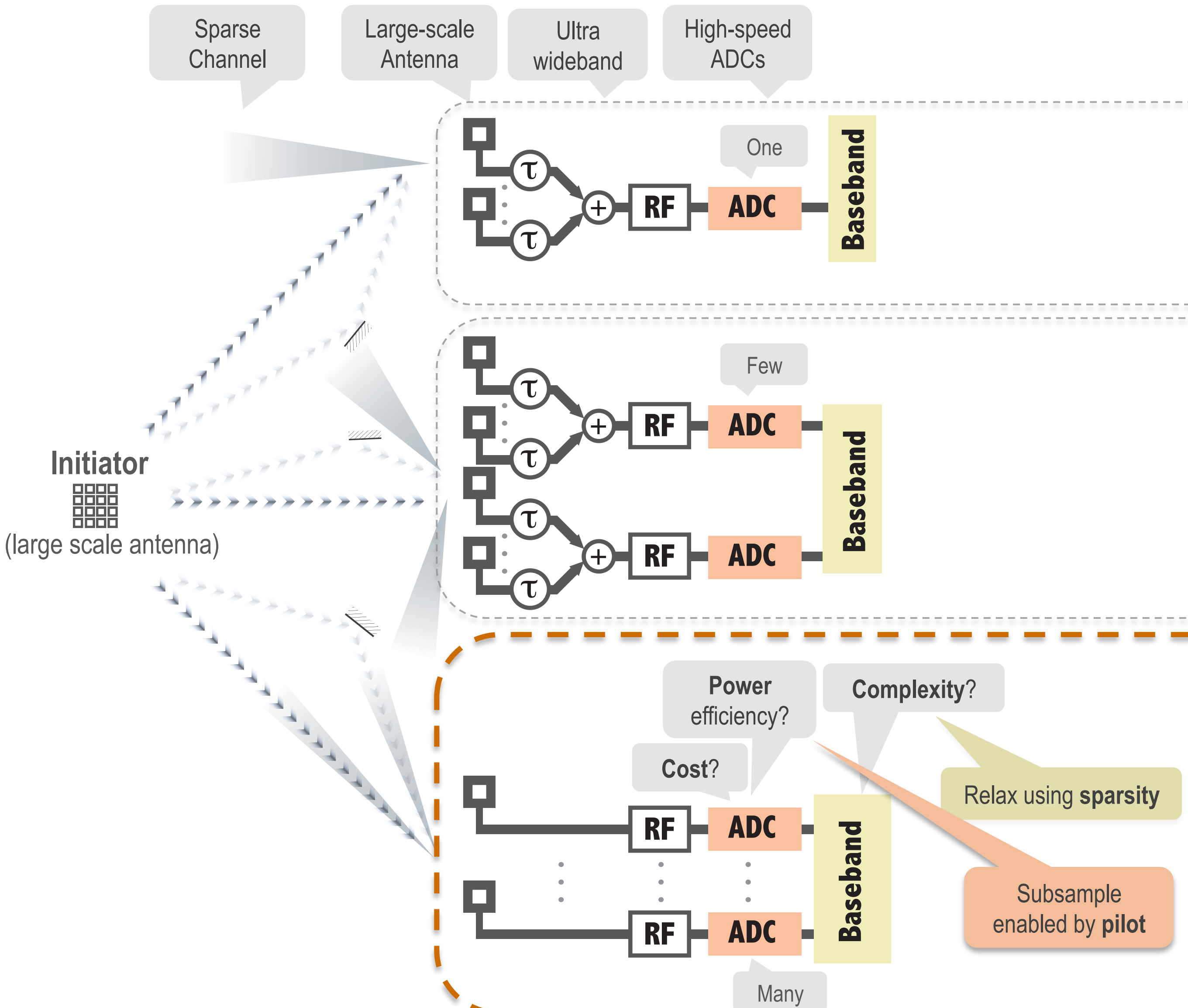
All-digital

Challenges:



Evolution of peak Spurious Free Dynamic Range (SFDR) for ADCs

Prior work & challenges



Analog Responder

- low cost,
- low resolution,
- high latency
  - sweep search

Hybrid Responder

- reduced cost,
- low resolution,
- moderate latency
  - sweep search

[Ayach'14][Desai'14]

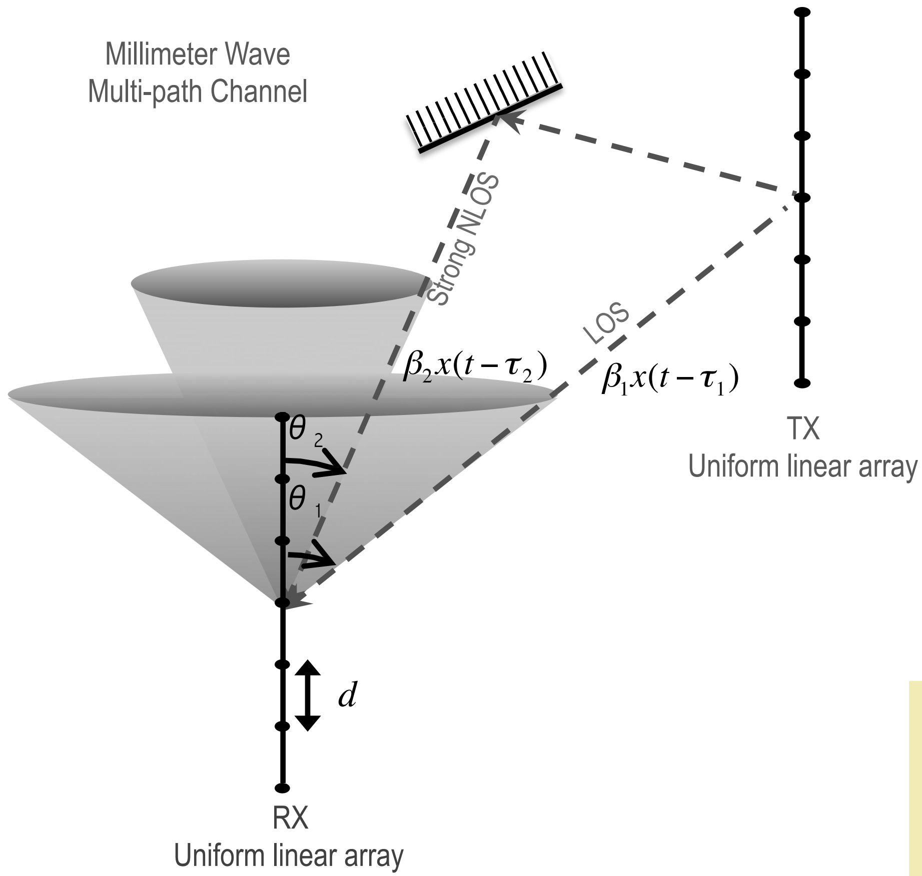
Digital Responder

- **flexible**,
- **performance**,
- **low latency**
  - concurrent search

[Barati'14]

Sparse channel model

mmWave sparse channel model can reduce complexity.



$$\mathbf{y}(t) = \mathbf{A}\mathbf{B}\mathbf{x}(t) + \mathbf{n}(t)$$

Received signal at antenna array

$$\mathbf{A} = \left[\; \mathbf{a}(\theta_1) \;\cdots\; \mathbf{a}(\theta_p) \;\right]$$

$$\mathbf{a}(\theta) = e^{i\frac{(m-1)}{2}\psi} \begin{bmatrix} 1 \\ e^{i\psi} \\ \vdots \\ e^{i(m-1)\psi} \end{bmatrix}$$

ULA direction vector

$$\psi = -\frac{2\pi d}{\lambda}\cos(\theta)$$

$$\mathbf{B} = \begin{bmatrix} \beta_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \beta_p \end{bmatrix}$$

$$\mathbf{x}(t) = \begin{bmatrix} x(t-\tau_1) \\ \vdots \\ x(t-\tau_p) \end{bmatrix}$$

Delayed pilot signals

Assumptions

- Number of multipath components, p is small.
- All p multi-paths have distinct delays.
- Maximum delay-spread is a known parameter and is small for mmWave propagation channel.

Subspace based DoA estimation

Channel Model: $\mathbf{y}(t) = \mathbf{ABx}(t) + \mathbf{n}(t)$

Covariance matrices:

$$\mathbf{R}_{\mathbf{yy}} = E\{\mathbf{y}(t)\mathbf{y}(t)^H\} = \mathbf{ABPB}^H\mathbf{A}^H + \sigma^2\mathbf{I}$$

$$\mathbf{P} = E\{\mathbf{x}(t)\mathbf{x}(t)^H\}$$

Decomposed into p dimensional **signal-subspace** and $(m\text{-}p)$ dimensional **noise subspace**.

Accurate estimation requires $(p+1)$ **high speed ADCs**

The covariance matrix $\mathbf{P}$ is non-singular if:

- The propagation delays are distinct.
- Pilot signals have good autocorrelation properties.

Large number $(p+1)$ of **RF chains with high speed ADCs** are impractical to implement in terms of cost and power consumption.

Pilot assisted sub-sample based MUSIC-like algorithms

Pilot design can assist an all digital solution.

Cyclic Prefix (**CP**)

Subsequences maintain good circular correlation properties

Reduced complexity
frequency domain algorithms

Sub-Nyquist rate sampling

Proposed pilot design

Energy Efficient (constant amplitude)
Zero circular correlation

(N,D) : positive integers
D : decimation factor
ND > delay_spread

Zadoff Chu (ZC) sequence ($L=ND^2$)

Decimated by D, **subsequence**'s properties:

I. **Zero circular cross-correlations**.
II. **Zero circular auto-correlation** within N lags.

Proof outline of property I & II

ZC sequence of length L
The root parameter u is relatively prime to L.

$$s[n] = \begin{cases} e^{-\frac{i\pi u n^2}{L}}, & \text{for } L \text{ even} \\ e^{-\frac{i\pi u n(n+1)}{L}}, & \text{for } L \text{ odd} \end{cases}$$

Decimated subsequence
with the j^{th} phase offset.
$j = 0,..,D-1$

$$s_j[k] = s[j + Dk], k = 0, \cdots, ND - 1$$

Decompose

First term		*Second term*		*Third term*
Constant phase term	✖	Linear phase term	✖	Linear frequency term

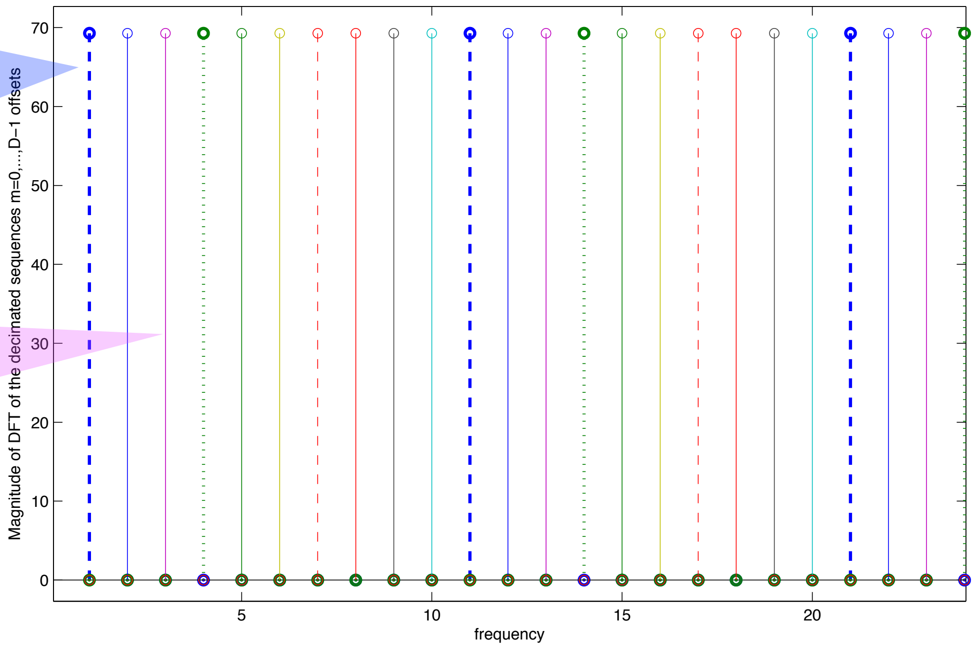
- Constant phase term: Does not affect the circular correlation properties
- Linear phase term: Adds circular shift to the ND-point DFT of the third term. Each subsequence's circular shift amount is **distinct** and from the set $\{0,..,D-1\}$
- Linear frequency term: A ZC sequence with length N and root u repeated D times

Proof outline

An Example of the *ND*-point DFT of the subsequences with phase offsets, *j=0,…,D-1*

The *ND*-point DFT of the subsequence with phase offsets *j=0.*
(Also the DFT of the third term)

The *ND*-point DFT of another subsequence which is a circular shifted version the DFT of the third term.



Example shows:

I. Subsequences have **zero circular cross-correlations**.

II. Each subsequence have **zero circular auto-correlations** within *N* lags.

Algorithm description

$$\mathbf{y}(t) = \mathbf{ABx}(t) + \mathbf{n}(t)$$

Ultra wideband signal at the antenna array

Subsample by a factor D

Enabled by pilot design

$$\mathbf{y}(Dt) = \mathbf{ABx}(Dt) + \mathbf{n}(Dt)$$

ADC working at sub-Nyquist rate

ND-point DFT of the received samples at each antenna

$$\mathbf{Y}$$

Few due to sparsity

The phases of the **dominant multipaths** can be identified from the DFT of the received vector:

$$\mathbf{Y}$$

Circular correlation by the pilot subsequence with the j^{th} phase

$$\tilde{\mathbf{Y}}$$

Circular correlation decouples the contribution of each subsequence of the pilot due to property II

Apply MUSIC algorithm on frequency domain signal

Circular autocorrelation of the correlation output will be zero within maximum delay spread, as long as:

$\tau_{max} < ND$ (property I)

DOA estimates for the pilot signal with the $(j+kD)^{th}$ phase offsets.

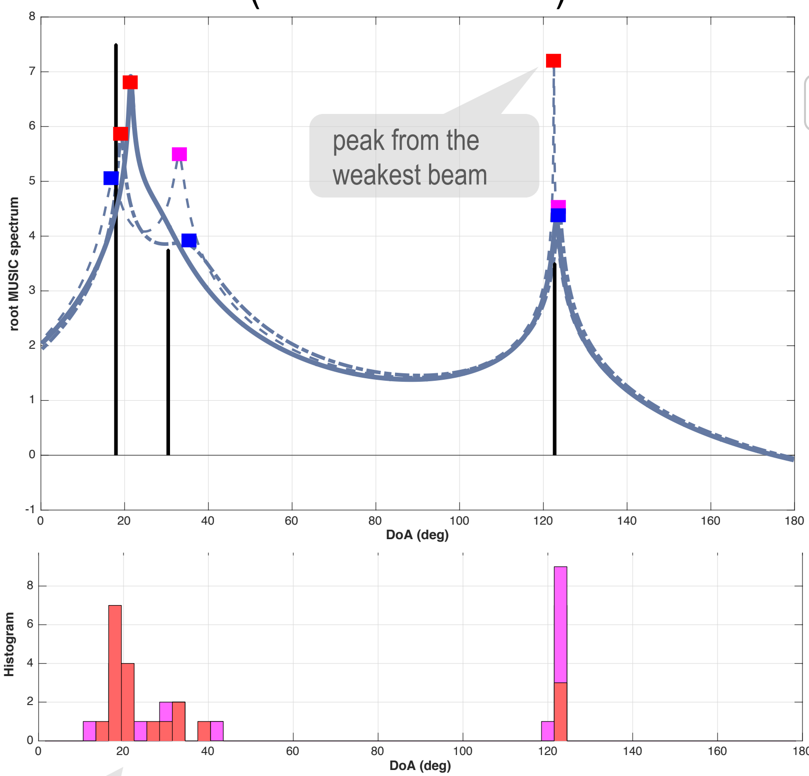
$$k = 0, \cdots, \lceil \tau_{max}/D \rceil$$

Reduces antenna size

Simulation Results

(Pilot = ZC(4096,11), decimation factor = 16)

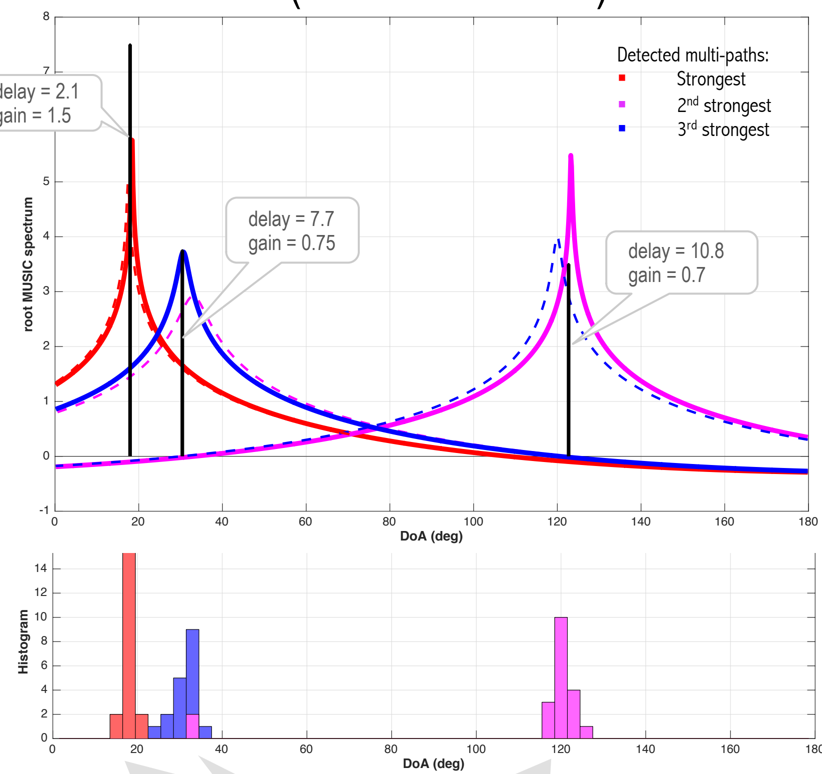
Root MUSIC on all **4096 symbols** (Antenna size = 4)



Unable to resolve

4x4 covariance matrix

Pilot aided root MUSIC on **256 symbols** (Antenna size = 2)



Finds DoA of each multipath (strongest to weakest)

2x2 covariance matrices

Conclusion

- Low complexity all digital solution.
  - Eliminates high speed ADC without performance degradation.
- Sub-Nyquist rate sampling using ZC based pilot design.
- Reduced antenna size requirements.

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