Ergodic Rate Analysis for Massive MIMO Relay Systems with Multi-Pair Users under Imperfect CSI

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Presentation Outline

- **Background**
- **System Model**
- **Analysis of Ergodic Rate and Power Scaling**
- **Numerical Results**

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- **Numerical Simulation**

Background

- **What's Large-Scale Antenna System?**
  - **Hundreds** or more antennas are employed
  - The number of antennas is **much more than** that of users
  - **Centralized** deployment, e.g., **Base Station/Relay**

    Distributed deployment, e.g., C-RAN
  - …

E. Bjornson, E. Larsson, and T. Marzetta, "Massive MIMO: 10 Myths and One Grand Question," submitted to *IEEE Commun. Mag.*, Mar. 2015.

Background

- **Potential Benefits of Large-Scale MIMO?**
  - Huge gain in **spectral and energy efficiency**
    - asymptotically **orthogonal channels** for perfect **interference elimination**, $\frac{\mathbf{h}_i^H \mathbf{h}_j}{N} \xrightarrow{N\to\infty} \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$
  - **Simple linear signal processing**, MRC/MRT, near optimal
  - **Power scaling** law: $1/N$ perfect CSI, and $1/\sqrt{N}$ imperfect CSI
  - Extensive use of **inexpensive low-power components**
  - …

H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral effciency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. , pp. 1436–1449, Apr. 2013.

Background

- **Multi-Pair Multi-antenna Relay Systems**
  - Expand cellular **coverage area**
  - Enhance link **reliability, spectral efficiency** and **capacity**
  - Improve the throughput for **the users at the cell edge**

However, the **inter-user interference (IUI)** is a serious bottleneck that limits multi-user MIMO relay system performance.

- Allocate **orthogonal resource** for multi-users to avoid IUI, spectral inefficiency.
- Design **complicated precoder/decoder**, high computational complexity.

M. Tao and R. Wang, "Linear precoding for multi-pair two-way MIMO relay systems with max-min fairness," *I EEE Trans. Signal Process.*, vol. 60, no. 10, pp. 5361–5370, Oct. 2012.

Background

- **Introduce Large-Scale MIMO to Relay**
  - Encouraged by the characteristics in massive MIMO systems, that **low-complexity linear precoding could perfectly alleviate the IUI**.
  - **No extra orthogonal resource**, e.g., time or frequency, is required.

However, the existing research of massive MIMO relaying system mainly concentrated ideal CSI at the relay, which is usually difficult to acquire due to several factors, e.g.,

- channel estimation error
- pilot contamination

H. A. Suraweera, H. Q. Ngo, T. Q. Duong, C. Yuen, and E. G. Larsson, "Multi-pair amplify-and-forward relaying with very large antenna arrays," in *Proc. of IEEE ICC*, Budapest, Jun. 2013, pp. 3228–3233.

Background

Thus, it is of great significance to investigate the system performance for large-scale MIMO relaying system when taking the imperfect CSI into account.

Our Focus:

- Investigate the ergodic rate and power scaling law with respect to the system parameters, i.e., the number of relay antennas, the transmit power at users and relay, and the number of user-pairs.

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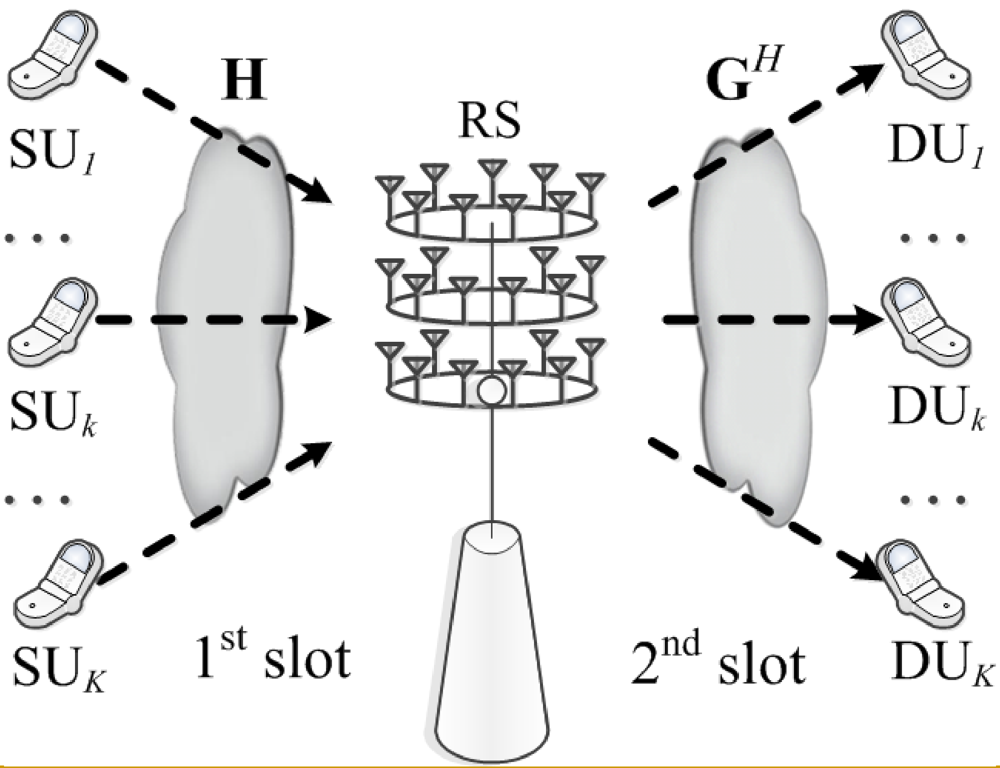
System Model

Consider a large-scale MIMO relaying system, consisting of K pairs of single-antenna users and a N-antenna relay station (RS), where N is pretty large and K is far less than N.



System Model

Communication Procedures:

1. In the 1st slot, source users send signals to RS and the received signal at RS is **r** and $\mathbf{H}=[\mathbf{h}_1,\mathbf{h}_2,\ldots,\mathbf{h}_K]$ with $\mathbf{h}_k \sim \mathcal{CN}\left(0,\sigma_{h_k}^2\mathbf{I}_N\right)$

$$\mathbf{r}=\sqrt{\rho_s}\mathbf{H}\mathbf{x}+\mathbf{n}_r$$

2. In the 2nd slot, RS retransmits the signals to destination users with MRC/MRT precoder **W** and $\mathbf{G}=[\mathbf{g}_1,\mathbf{g}_2,\ldots,\mathbf{g}_K]$ with $\mathbf{g}_k \sim \mathcal{CN}\left(0,\sigma_{g_k}^2\mathbf{I}_N\right)$

$$\mathbf{t}=\mathbf{W}\mathbf{r}$$

Remark: Herein, there is an assumption implying that the RS has the ideal CSI, namely, **H** and **G**.

System Model

Communication Procedures:

3. At the end of the 2nd slot, the *k*-the destination user receives signals as follows

$$y_k = \sqrt{\rho_s}\mathbf{g}_k^H \mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{g}_k^H \mathbf{W}\mathbf{n}_r + n_k$$

System Model

Relay Processing under Imperfect CSI:

In practice, when the source terminals and relay are fixed and the destination terminals serves as mobile terminals, the first-hop cha nnel **H** can be estimated accurately at relay. Whereas the second-hop channel **G** is imperfect.

Invoking the orthogonally property of MMSE estimation, the cha nnel vector $\boldsymbol{g}_k$ can be expressed as

$$\mathbf{g}_k = \widehat{\mathbf{g}}_k + \widetilde{\mathbf{g}}_k$$

$$\widehat{\mathbf{g}}_k \sim \mathcal{CN}\left(0, a_k^2 \mathbf{I}_N\right) \text{ with } a_k^2 \triangleq \frac{\sigma_{g_k}^4 \rho_p}{\sigma_{g_k}^2 \rho_p + \sigma_p^2}$$

$$\widetilde{\mathbf{g}}_k \sim \mathcal{CN}\left(0, b_k^2 \mathbf{I}_N\right) \text{ with } b_k^2 \triangleq \frac{\sigma_{g_k}^2 \sigma_p^2}{\sigma_{g_k}^2 \rho_p + \sigma_p^2}$$

Y. Fu, W.-P. Zhu and C. Liu, “Rate optimization for relay precoding design with imperfect CSI in two-hop MIM O relay networks,” in *Proc. of IEEE VTC Fall*, San Francisco, CA, Sept. 2011, pp. 1–5.

System Model

Ergodic Achievable Rate:

When taking the channel estimation error into account, the receiv ed signal at the k-the destination user can be expressed as

$$y_k = \underbrace{\sqrt{\rho_s}\widehat{\mathbf{g}}_k^H \mathbf{W}\mathbf{h}_k x_k}_{\text{desired signal}} + \underbrace{\sqrt{\rho_s}\sum_{i=1, i\neq k}^{K} \left(\widehat{\mathbf{g}}_k^H + \widetilde{\mathbf{g}}_k^H\right)\mathbf{W}\mathbf{h}_i x_i}_{\text{inter-pair interference}}$$
$$+ \underbrace{\sqrt{\rho_s}\widetilde{\mathbf{g}}_k^H \mathbf{W}\mathbf{h}_k x_k}_{\text{channel estimation error}} + \underbrace{\left(\widehat{\mathbf{g}}_k^H + \widetilde{\mathbf{g}}_k^H\right)\mathbf{W}\mathbf{n}_r + n_k}_{\text{noise}}$$

System Model

Ergodic Achievable Rate:

The received SINR at the *k*-th destination user is

$$\gamma_k = \frac{S_k}{A_k + B_k + C_k + \eta\sigma_{n_k}^2/\rho_r\rho_s}$$

$S_k \triangleq |\hat{\mathbf{g}}_k^H \widehat{\mathbf{G}}\mathbf{H}^H \mathbf{h}_k|^2$: power of useful signal

$A_k \triangleq b_k^2 \|\widehat{\mathbf{G}}\mathbf{H}^H \mathbf{h}_k\|^2$: power of channel estimation error

$B_k \triangleq \sum_{i=1, i\neq k}^{K} \left(|\hat{\mathbf{g}}_k^H \widehat{\mathbf{G}}\mathbf{H}^H \mathbf{h}_i|^2 + b_k^2 \|\widehat{\mathbf{G}}\mathbf{H}^H \mathbf{h}_i\|^2 \right)$: power of inter-user interference

$C_k \triangleq \frac{\sigma_r^2}{\rho_s} \left(\|\hat{\mathbf{g}}_k^H \widehat{\mathbf{G}}\mathbf{H}^H\|^2 + b_k^2 \|\widehat{\mathbf{G}}\mathbf{H}^H\|^2 \right)$: power of addictive noise

System Model

Ergodic Achievable Rate

The rate for the *k*-th destination user is given by

$$R_k = \mathbb{E}\left\{\frac{1}{2}\log_2\left(1+\gamma_k\right)\right\}$$

Accordingly, the total rate achieved by all users is given by

$$R_\Sigma = \sum_{k=1}^{K} R_k = \sum_{k=1}^{K} \mathbb{E}\left\{\frac{1}{2}\log_2\left(1+\gamma_k\right)\right\}$$

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- **Analysis of Ergodic Rate and Power Scaling**
- Numerical Results

Analysis of Ergodic Rate and Power Scaling

Analytical Expression of Ergodic Rate (1)

By using the law of large numbers and new-proved lemma*, we have following theorem about ergodic rate

Theorem 1: Using MRC/MRT precoding at the large-scale

MIMO relay station with imperfect CSI, the ergodic rate of the k-th destination terminal can be approximated as

$$R_k \approx \overline{R}_k = \frac{1}{2}\log_2\left(1 + \frac{\overline{S}_k}{\overline{A}_k + \overline{B}_k + \overline{C}_k + \overline{F}_k}\right)$$

*Q. Zhang, S. Jin, K. K. Wong, and H. B. Zhu, "Power scaling of uplink massive MIMO systems with arbitrary-rank channel means," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 966–981, Oct. 2014.

Analysis of Ergodic Rate and Power Scaling

Analytical Expression of Ergodic Rate (2)

$$\overline{S}_k = \left(N^2 + 2N + 1\right) a_k^4 \sigma_{h_k}^4 + \sum^{K} a_k^2 a_j^2 \sigma_{h_j}^2 \sigma_{h_k}^2$$

$$\overline{A}_k = b_k^2 \left[\left(N + 1\right) \sigma_{h_k}^4 a_k^2 + \sum_{j=1, j \neq k}^{K} \sigma_{h_k}^2 \sigma_{h_j}^2 a_j^2 \right]$$

$$\overline{B}_k = \sum_{i=1, i \neq k}^{K} \left\{ (N+1) \left(a_k^4 \sigma_{h_k}^2 \sigma_{h_i}^2 + a_k^2 a_i^2 \sigma_{h_i}^4 \right) + \sum^{K} a_k^2 a_j^2 \sigma_{h_j}^2 \sigma_{h_i}^2 + (N+1) \sigma_{h_i}^4 a_i^2 b_k^2 + \sum_{j=1, j \neq i}^{K} \sigma_{h_i}^2 \sigma_{h_j}^2 a_j^2 b_k^2 \right\}$$

$$\overline{C}_k = \frac{\sigma_r^2}{\rho_s} \left[\left(N a_k^2 + \sigma_{g_k}^2 \right) a_k^2 \sigma_{h_k}^2 + \sigma_{g_k}^2 \sum_{j=1, j \neq k}^{K} a_j^2 \sigma_{h_j}^2 \right]$$

$$\overline{F}_k = \frac{\sigma_{n_k}^2}{\rho_r \rho_s} \sum_{i=1}^{K} \left[(N+1) \rho_s \sigma_{h_i}^4 + \rho_s \sum_{j=1, j \neq i}^{K} \sigma_{h_j}^2 \sigma_{h_i}^2 + \sigma_r^2 \sigma_{h_i}^2 \right] a_i^2$$

Analysis of Ergodic Rate and Power Scaling

Analytical Expression of Ergodic Rate (3)

The analytical form of rate relies on the statistical characteristics of all the channel vectors, i.e., the pass loss factors. It is still difficult for subsequent analysis.

For analysis simplicity, we consider the case of $\sigma_{h_k}^2 = \sigma_{g_k}^2 = 1$ and $\sigma_p^2 = \sigma_{n_k}^2 = \sigma_n^2, k = 1, \ldots, K$ such that

$$\overline{R}_k = \frac{1}{2}\log_2\left(1 + \frac{N^2 + 2N + K}{(2N+K)(K-1) + K(N+K)b^2/a^2 + N/\rho_s + K/\rho_s a^2 + K(N+K)/\rho_r a^2 + K/\rho_r\rho_s a^2}\right)$$

Analysis of Ergodic Rate and Power Scaling

Analytical Expression of Ergodic Rate (4)

Due to $N >> K$, the achievable rate can be further approximated, from $N + K \approx N$ and $K/N << 1$

$$\overline{R}_k \approx \frac{1}{2}\log_2\left(1 + \frac{N+2}{2(K-1) + \frac{K}{\rho_p} + \frac{1}{\rho_s} + \frac{K}{\rho_r a^2}}\right)$$

Remark 2: Obviously, the rate has a nearly logarithmic increase with the number of relay antennas N increasing. In addition, although the rate decreases as K increases, the sum rate, i.e., $R_\Sigma = \sum_{k=1}^{K} \overline{R}_k$, monotonically increases over K, whose proof can be referred to [17, Lemma 2].

Analysis of Ergodic Rate and Power Scaling

Power Scaling Law (1)

We quantify how much power can be saved at the terminals and the relay as N grows large. For this purpose, we rewrite the powers as $\rho_p = \rho_s = Es/N^{\alpha}$ and $\rho_r = Er/N^{\beta}$, where Es and Er are fixed constants and $\alpha, \beta \geq 0$ indicate the power scaling laws

As $N \to \infty$, the ergodic rate can be simplified as

$$\overline{R}_k \approx \frac{1}{2}\log_2\left(1 + \frac{N}{2\,(K-1) + \frac{(K+1)N^{\alpha}}{E_s} + \frac{KN^{\alpha+\beta}}{E_r E_s}}\right)$$

Analysis of Ergodic Rate and Power Scaling

Power Scaling Law (2)

In particular, when $(\alpha, \beta) = (v, 1 - v)$, R_k will converge to different constants for $0 \leq v \leq 1$ with $N \rightarrow \infty$ as follows

$$\overline{R}_k = \begin{cases} \frac{1}{2}\log_2\left(1 + E_r E_s / K\right), & 0 \leq v < 1 \\ \frac{1}{2}\log_2\left(1 + \frac{E_r E_s}{(K+1) E_r + K}\right), & v = 1 \end{cases}$$

Analysis of Ergodic Rate and Power Scaling

Power Scaling Law (3)

In a special case of perfect CSI, the ergodic rate becomes

$$\overline{R}_k \approx \frac{1}{2}\log_2\left(1+\frac{N}{2(K-1)+N^{\alpha}/E_s+KN^{\beta}/E_r}\right)$$

As $N\to\infty$, the ergodic rate will converge to different constants

$$\overline{R}_k \overset{N\to\infty}{\to} \begin{cases} \frac{1}{2}\log_2\left(1+\frac{E_sE_r}{E_r+KE_s}\right),(\alpha,\beta)=(1,1) \\ \frac{1}{2}\log_2\left(1+E_s\right),(\alpha,\beta)=(1,v),0\le v<1 \\ \frac{1}{2}\log_2\left(1+E_r/K\right),(\alpha,\beta)=(v,1),0\le v<1 \end{cases}$$

Analysis of Ergodic Rate and Power Scaling

Power Scaling Law (4)

The results indicate that the power at the terminals and relay can be scaled down simultaneously maximally by $\frac{1}{\sqrt{N}}$ with imperfect CSI and by $\frac{1}{N}$ with perfect CSI.

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Numerical Results

Simulation parameters

1. Generate 10^4 independent channel realization to be averaged to produce the numerical results.

2. The number of relay antennas ranges from 20 to 1000.

3. The path loss factor is set to be unit for simplicity,

4. The power of AWGN is set to be unit without loss of generality.

Numerical Results

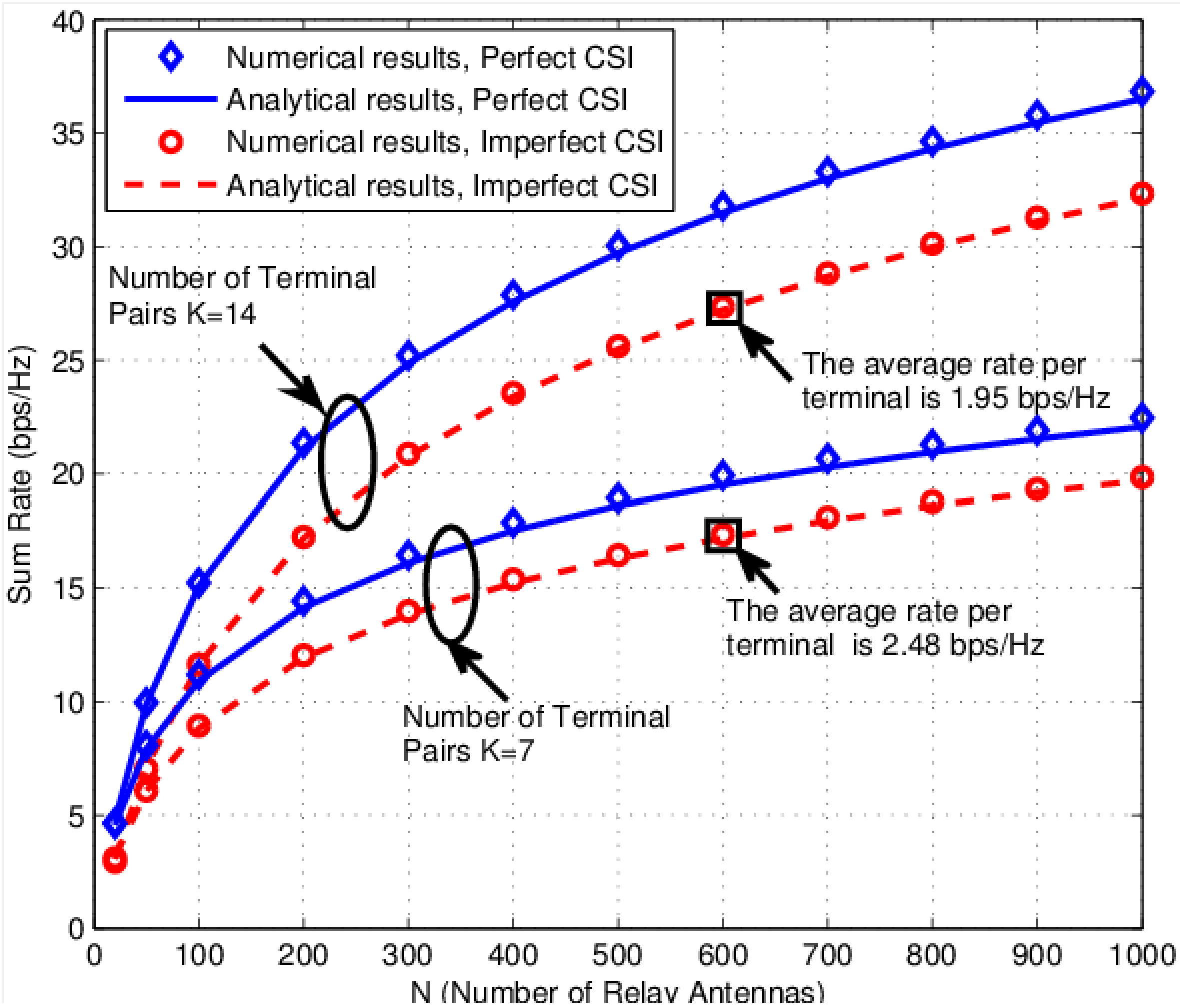

Fig. 1. The sum rate versus the number of relay antennas, where the transmit SNR $\frac{\rho_s}{\sigma_r} = \frac{\rho_r}{\sigma_{n_k}} = 10\text{dB}$ and training SNR $\frac{\rho_p}{\sigma_p} = 0\text{dB}$.

Numerical Results

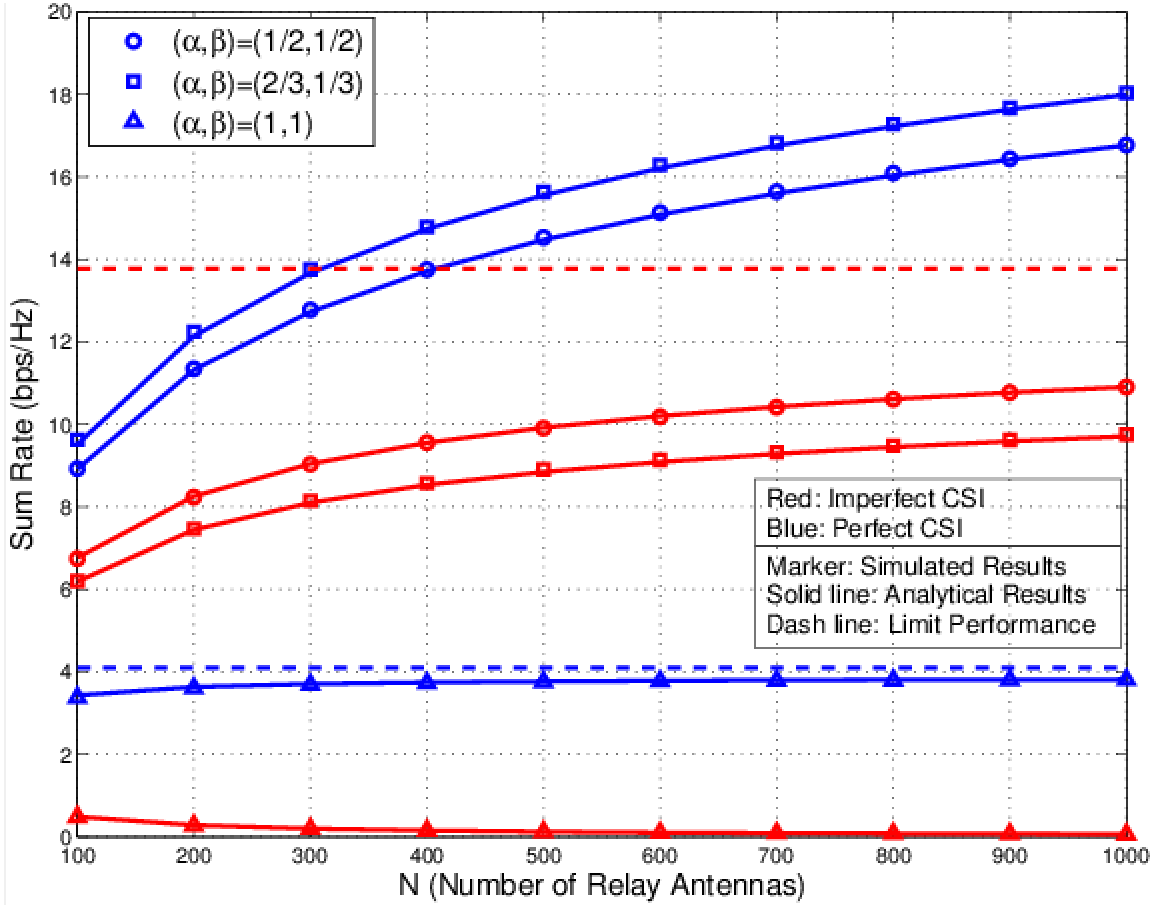

Fig. 2. The sum rate versus the number of relay antennas, where the transmit power are set as $\rho_p = \rho_s = \frac{E_s}{N^\alpha}$ and $\rho_r = \frac{E_r}{N^\beta}$, and $\frac{E_s}{\sigma_r} = \frac{E_r}{\sigma_{n_k}} = 10$dB are fixed.

Any questions?

Thank you!