Application of Compressed Sensing to Wideband Spectrum Sensing in Cognitive Radio Networks

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Wideband Spectrum Sensing Problem

- Fast and accurate wideband spectrum sensing is required for CRN applications
- Traditional techniques require ADCs at excessively high sampling rates for wideband spectrum sensing
  - Not possible
  - Would result in a huge amount of data
- Existing techniques use:
  - Several band pass filters (excessive hardware requirement)
  - Sequentially sweep the spectrum (excessive time requirement)
- Since the spectrum is under-utilized, **compressed sensing** can be used
  - Much lower sampling rate
  - Much smaller resulting data

Compressed Sensing

- **Traditionally**, a signal is sampled at least at the **Nyquist rate** for perfect reconstruction.
  - Nyquist rate of some applications is so high, too expensive or impossible to implement
  - Many applications compress the sampled signal for efficient storage or transmission.
- **Compressed Sensing** simultaneously performs sensing and compression
  - The signal is sensed in a compressed form.
  - Sampling is performed at a rate much less than Nyquist rate.
  - A considerable reduction in the costs of sampling and computation.
- Consider a sparse signal $x \in \mathbb{R}^n$, of sparsity level k, and a measurement (sampling) system that acquires m linear measurements:

$$y = Ax$$

- $y \in \mathbb{R}^m$ is the measurement vector
- $A \in \mathbb{R}^{m \times n}$ is the sensing or measurement matrix
- In compressed sensing $m \ll n$

- The signal to be acquired should be either:
  - Sparse
  - Compressible, with a few significant coefficients in a suitable basis or domain (e.g. Fourier, Wavelets, …, etc.)

Existing Recovery Algorithms

- Select a **fixed** number of elements from the correlation vector per iteration
- Selection is performed from the **whole** set of correlation values
- Mostly perform least square minimization through matrix inversion

Fast Matching Pursuit (FMP)

- **FMP** significantly improves the **reconstruction time and accuracy**
  - Selects **a sufficient number** of coefficients per iteration.
  - Selection is performed using a **reduced set** of the correlation values.
    - A set containing the βk top magnitude elements
    - Elements which magnitudes are larger than a fixed fraction 0 < α < 1 of the maximum element are selected from the reduced set
  - The selected number is **adapted** between iterations according to **the distribution of correlation values**
  - Incorrectly selected indices are excluded in each iteration **(pruning)**
  - Least square minimization is performed **iteratively** avoiding large matrix inversion

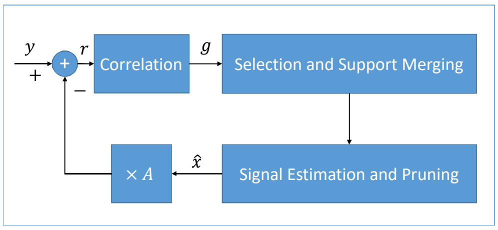
FMP Recovery Algorithm

Initially, signal estimate $\hat{x} = 0$, residual $r = y$. In each iteration:

1. **Signal Proxy Formation.** Signal proxy g is formed by correlating the residual with the sensing matrix columns.
2. **Selection and Support Merging.** g is sorted in a descending order of absolute values. The elements of absolute values $\geq \alpha\, max|g_l|$, where $0 < \alpha < 1$, are selected from a reduced set containing the βk largest magnitude elements. The indices of selected elements are united with already identified support set.
3. **Signal estimation.** An estimate of the signal is formed by least square minimization, avoiding large matrix inversion.
4. **Pruning.** The k largest magnitude components in the signal estimate are retained. The rest are set to zero.
5. **Residual Calculation.** The new residual is calculated from the pruned signal.

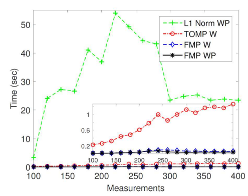
The algorithm terminates if: $|r| < \epsilon_1$, or $|r_i - r_{i-1}| < \epsilon_2$

Otherwise, a maximum of k iterations are performed.

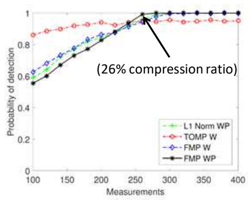


Performance Evaluation

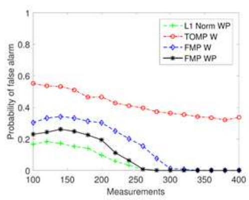
- Data of length *n* = 1000, corresponding to Nyquist-rate samples.



- FMP needs the least reconstruction time.



- FMP gives 99% probability of detection using only 260 measurements.



- At the same compression ratio, FMP gives 1% probability of false alarm.

Conclusions

- FMP achieves significant complexity reduction with high reconstruction accuracy compared to related algorithms
- We have applied FMP to wideband spectrum sensing in cognitive radio networks
- Sampling rate can be cut down by about 75%, significantly faster than other related techniques, at a remarkable accuracy.