A Dual Block Coordinate Proximal Algorithm with Application to Deconvolution of Interlaced Video Sequences

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Problem formulation

★ **Fast** computation of the proximity operator of a **sum** of **convex functions** involving **linear operators**.

⇒ Design new **dual block-coordinate** forward backward algorithms:

- ✔ Ability of dealing with **large scale problems**.
- ✔ **No need** for inversion of linear operator.
- ✔ **Acceleration** through the introduction of preconditioning matrices.
- ✔ Application to restoration of **interlaced video sequences**.

Minimization problem

Computing the proximity operator of $g = f + \sum_{j=1}^{J} h_j \circ A_j$:

$$\underset{x\in\mathbb{R}^N}{\text{minimize}} \ \ g(x) + \tfrac{1}{2}\|x - \widetilde{x}\|_B^2$$

- $f : \mathbb{R}^N \to]-\infty, +\infty]$ **convex** function.
- $\forall j \in \{1,\dots,J\}$ $h_j : \mathbb{R}^{M_j} \to]-\infty, +\infty]$ **convex** function.
- $\forall j \in \{1,\dots,J\}$ $A_j \in \mathbb{R}^{M_j\times N}$ real valued matrix.
- $B \in \mathbb{R}^{N\times N}$ **preconditioning** matrix.

Dual formulation

$$\underset{(y^j)_{1\leqslant j\leqslant J}\in\mathbb{R}^M}{\text{minimize}} \ \ \varphi\Big(-\sum_{j=1}^{J} A_j^\top y^j + \widetilde{x}\Big) + \sum_{j=1}^{J} h_j^*(y^j)$$

- **Conjugate** of a lower-semicontinuous convex function h_j:

$$h_j^* : \mathbb{R}^{M_j} \to [-\infty, +\infty] : x \to \sup_{\nu\in\mathbb{R}^{M_j}} \left(\langle \nu | x\rangle - h_j(\nu)\right).$$

- φ the **Moreau envelope** with parameter 1 of f^*.

Proposed algorithms

Algorithm 1

Initialization

$$\begin{array}{l} B_j \in \mathbb{R}^{M_j\times M_j} \text{ with } B_j \succeq A_j A_j^\top, \quad \forall j \in \{1,\dots,J\} \\ \epsilon \in]0,1], \ (y_0^j)_{1\leqslant j\leqslant J} \in \mathbb{R}^M, \ z_0 = -\sum_{j=1}^{J} A_j^\top y_0^j. \end{array}$$

For $n = 0, 1, \dots$

$$\begin{array}{l} \gamma_n \in [\epsilon, 2-\epsilon] \\ j_n \in \{1,\dots,J\} \\ x_n = \text{prox}_f(\widetilde{x} + z_n) \\ \widetilde{y}_n^{j_n} = y_n^{j_n} + \gamma_n B_{j_n}^{-1} A_{j_n} x_n \\ y_{n+1}^{j_n} = \widetilde{y}_n^{j_n} - \gamma_n B_{j_n}^{-1}\, \text{prox}_{\gamma_n B_{j_n}^{-1}, h_{j_n}}\left(\gamma_n^{-1} B_{j_n} \widetilde{y}_n^{j_n}\right) \\ y_{n+1}^{j} = y_n^{j}, \quad \forall j \in \{1,\dots,J\}\setminus\{j_n\} \\ z_{n+1} = z_n - A_{j_n}^\top (y_{n+1}^{j_n} - y_n^{j_n}). \end{array}$$

- $(B_j)_{(1\leqslant j\leqslant J)}$: preconditioning matrices.

Special case: Non preconditioned algorithm obtained when setting:

$$\forall j \in \{1,\dots,J\} \ B_j = \|A_j\|^2 I_{M_j}.$$

Variants

Variants of the proposed algorithm can be obtained when:

- → Setting $f = 0$ ⇝ **Algorithm 2**.
- → Updating the dual variables in **parallel without preconditioning** [Combettes *et al.*, 2011] ⇝ **Algorithm 3**.

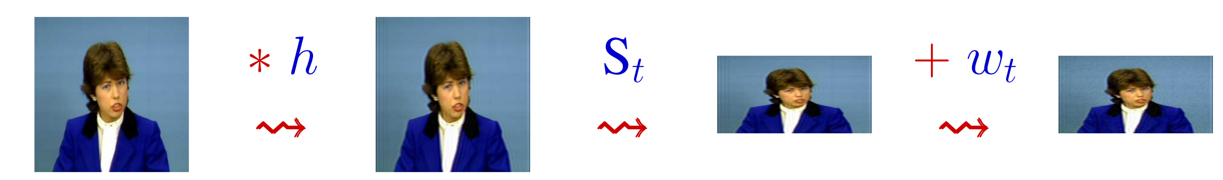
Convergence results

Under appropriate **technical assumptions**:

- ✔ If the sequence $(y_n)_{n\in\mathbb{N}}$ is bounded, then it converges to a solution of the **dual problem**.
- ✔ The sequence $(x_n)_{n\in\mathbb{N}}$ converges to the **proximity operator** of g.

Application to video restoration

Observation model



- $(\overline{x}_t)_{1\leqslant t\leqslant T} \in \mathbb{R}^{TN}$ ⇝ original progressive video sequence.
- $(\overline{y}_t)_{1\leqslant t\leqslant T} \in \mathbb{R}^{TL}$ ⇝ interlaced blurred video sequence ($N = 2L$).
- $h \in \mathbb{R}^P$ ⇝ convolution kernel.
- $(w_t)_{1\leqslant t\leqslant T} \in \mathbb{R}^{TL}$ ⇝ unknown additive noise.
- $S_t \in \mathbb{R}^{L\times N}$ ⇝ row decimation operator with $S_t = S_o$ for odd values of t and $S_t = S_e$ for even values of t.

Estimate of $(\overline{x}_t)_{1\leqslant t\leqslant T} \in \mathbb{R}^{TN}$ obtained by minimizing:

$$(\forall x \in \mathbb{R}^{TN}) \quad F(x) = \Phi(x) + \mathrm{M}(x) + \sum_{t=1}^{T} \Psi_t(x_t)$$

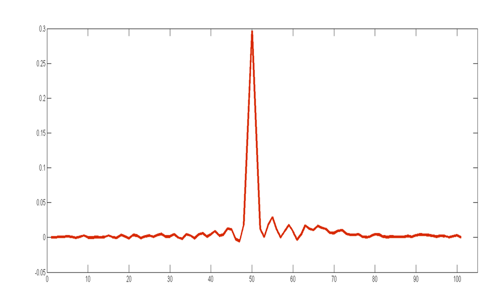
- ∗ Φ ⇝ least squares **data fidelity** term.
- ∗ M ⇝ nonsmooth **temporal regularization** term.
- ∗ Ψ_t ⇝ nonsmooth **spatial regularization** term.

★ Minimization using **block-coordinate forward-backward** algorithm.

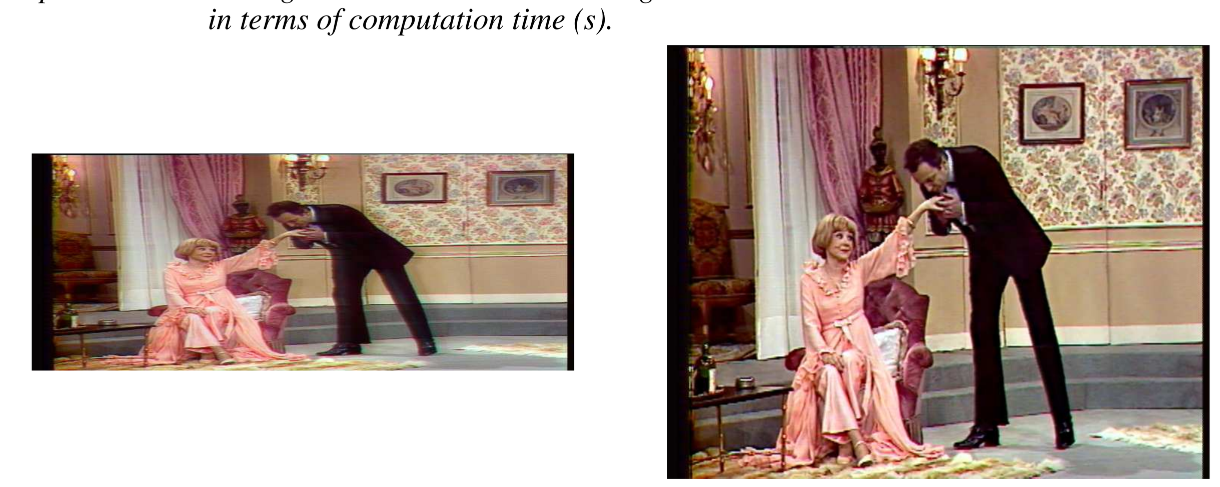
Simulation results

	Algorithm	1	2	3
Claire	Frame 1	2.06	**0.91**	6.06
	Frame 6	2.09	**0.75**	7.99
	Frame 13	2.11	**1.17**	7.38
Au théâtre ce soir	Frame 1	**2.20**	2.41	23.44
	Frame 5	**2.21**	2.45	31.60
	Frame 10	**2.03**	2.59	21.43

Comparison between Algorithm 1 and its variants Algorithms 2 and 3 in terms of computation time (s).



Convolution kernel (P = 101).



Au théâtre ce soir sequence: interlaced degraded field (720 × 288 pixels) (left), restored deinterlaced image (720 × 576 pixels) (right).