Multicasting with Untrusted Relays: A Noncoherent Secure Network Coding Approach

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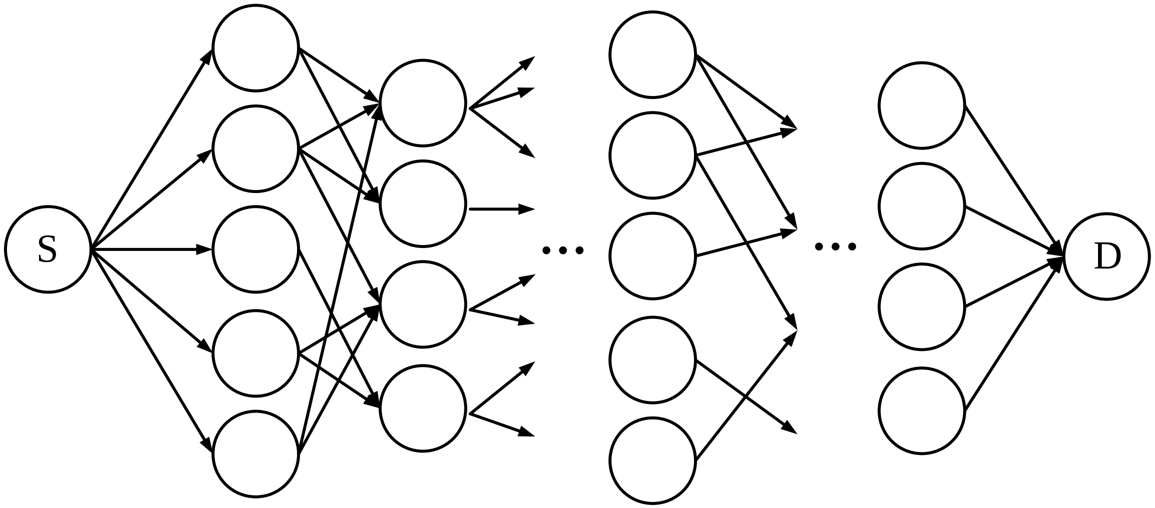
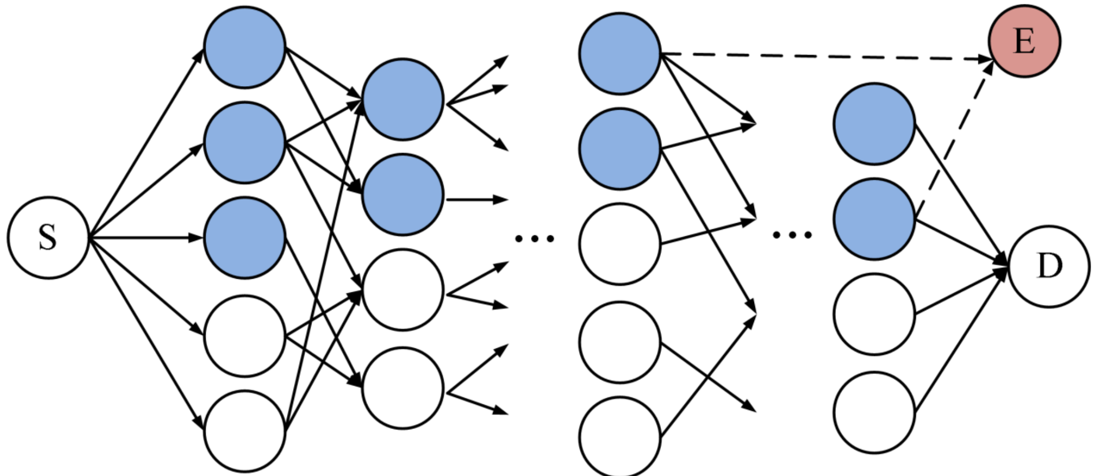
Multihop Network



- Network coding in general improves throughput and reliability.
- It is common to assume that all the relays are trustworthy.
- However, in practice, some of them may be provided by a third party which cannot be fully trusted.

Multihop Network with Untrusted Relays



- Untrusted (or third party) relays may potentially be compromised by an outside adversary (or an eavesdropper).
- More relays (trusted or not) provides more paths for simultaneous information transfer, but yields higher risk of being eavesdropped.
- Intuitively, one should recruit untrusted relays ONLY when the secrecy capacity can be improved by doing so.
  - Secrecy capacity: Maximum transmission rate without information leakage

Main Contributions

- Exam the impact of untrusted relays in the multihop network system and determine the optimal input signal that maximizes secrecy capacity when untrusted relays are recruited.

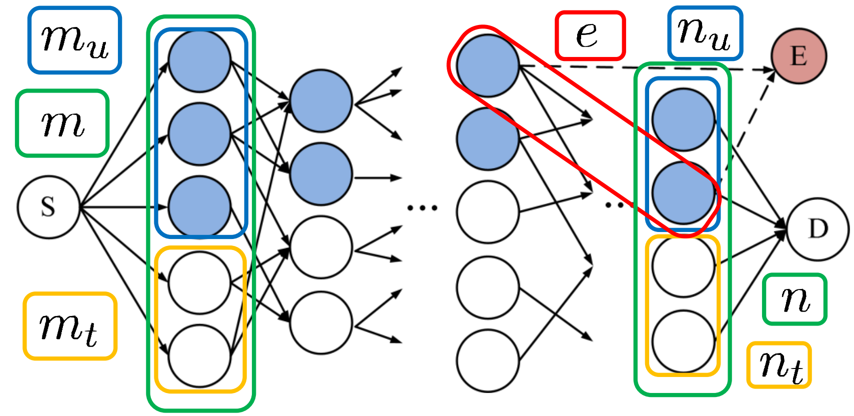
- Discuss the untrusted relays recruitment problem based on the secrecy capacity in two different cases:
  - Case1: All untrusted relays ***near the destination*** are compromised ***with probability*** 1.
  - Case 2: ***Each*** untrusted relay is compromised ***with probability*** p.

System Model: Random Linear Coding

- The signal transmitted from the source to the first hop of relays is

$$X \in \mathcal{F}_q^{m \times T}$$

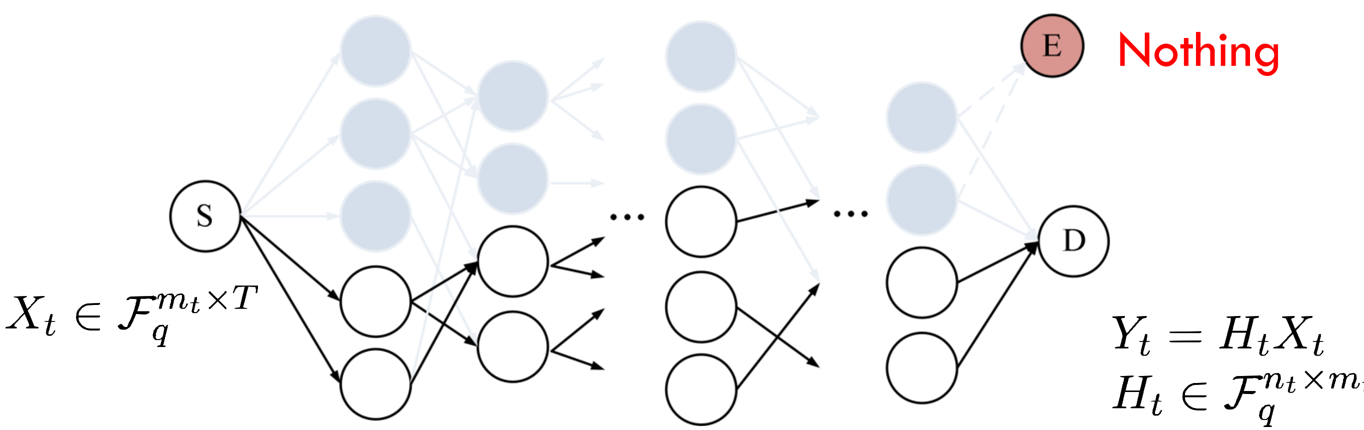
  - m is the # of relays in the first layer, T is packet length, and q is field size.



- **Random linear network coding:** Each relay forwards a linear combination of its received signals with coefficients chosen uniformly over the finite field $\mathcal{F}_q$.
- Received signal:
  - Destination: $Y = HX$ where $H \in \mathcal{F}_q^{n \times m}$.
  - Eavesdropper: $Z = GX$ where $G \in \mathcal{F}_q^{e \times m}$.
    - e: the number of untrusted relays compromised by the eavesdropper.

System Model: Without Recruitment

- **Assumptions:**
  - (i) We assume that, after a sufficient number of hops, the effective channel matrices H and G are i.i.d. uniform in $\mathcal{F}_q$. [Siavoshani & Fragouli '12]
  - (ii) H and G are unknown at all nodes (i.e., a noncoherent framework), e.g., when the encoding vector is NOT appended to the network coding packets.
- **Special Case:** When ***NO*** untrusted relays are recruited, the system model can be reduced as



Secrecy Capacity: Equivalent Degraded Channel

- The secrecy capacity

$$\max_{V \to X \to Y,Z} I(V;Y) - I(V;Z)$$ [Csiszar & Korner '78]

  - V is a auxiliary variable.
  - It is difficult to joint optimize V and X.

- **Equivalent degraded channel:**
  - Focus on the case $n > e$ (if $n \le e$, $C_s = 0$)

Original Channel:

$$Y = HX$$
$$Z = GX$$

Equivalent Degraded Channel

$$Y' = \begin{bmatrix} G \\ H' \end{bmatrix} X$$
$$Z' = GX$$

  - Equivalent: Secrecy capacity only depend on $p(\mathbf{Y}|\mathbf{X})$ and $p(\mathbf{Z}|\mathbf{X})$.
  - Degraded: $X \to Y' \to Z'$ forms a Markov chain.

➢ The secrecy capacity of degraded channel is

$$C_s = \max_{p_x} I(X;Y') - I(X;Z'),$$ [Wyner '75]

Secrecy Capacity: Optimal Input Structure

Lemma 1([Siavoshani & Fragouli '12]): The secrecy capacity is given as

$$C_s = \max_{\Pi_X} I(\Pi_X; \Pi_{Y'}) - I(\Pi_X; \Pi_{Z'}).$$

where Π_X is the subspace which spanned by the row vectors of X . Moreover, the distribution of optimal input Π_X^* is given by

$$P_{\Pi_X^*}(\pi_x) = \alpha_{d_x} \begin{bmatrix} T \\ d_x \end{bmatrix}^{-1}$$

where $\alpha_{d_x} \triangleq \Pr[\dim(\Pi_X) = d_x]$ is the probability that Π_X is of dimension d_x .

- Only depend on the subspace spanned by the row vectors of input signal X .
- All subspaces of the same dimension occur with equal probability.

Optimization Problem

- **Input optimization problem:**

$$C_s = \max_{\underline{\alpha}} R(\underline{\alpha}), \quad \text{subject to } \|\underline{\alpha}\|_1 = 1,$$

where $R(\underline{\alpha}) \triangleq I(\Pi_X^*; \Pi_{Y'}) - I(\Pi_X^*; \Pi_{Z'})$ and the subspace-dimension probabilities $\underline{\alpha} \triangleq \left[\alpha_0, \cdots, \alpha_{\min(m,T)}\right]^T$.

- The rate function can be written as

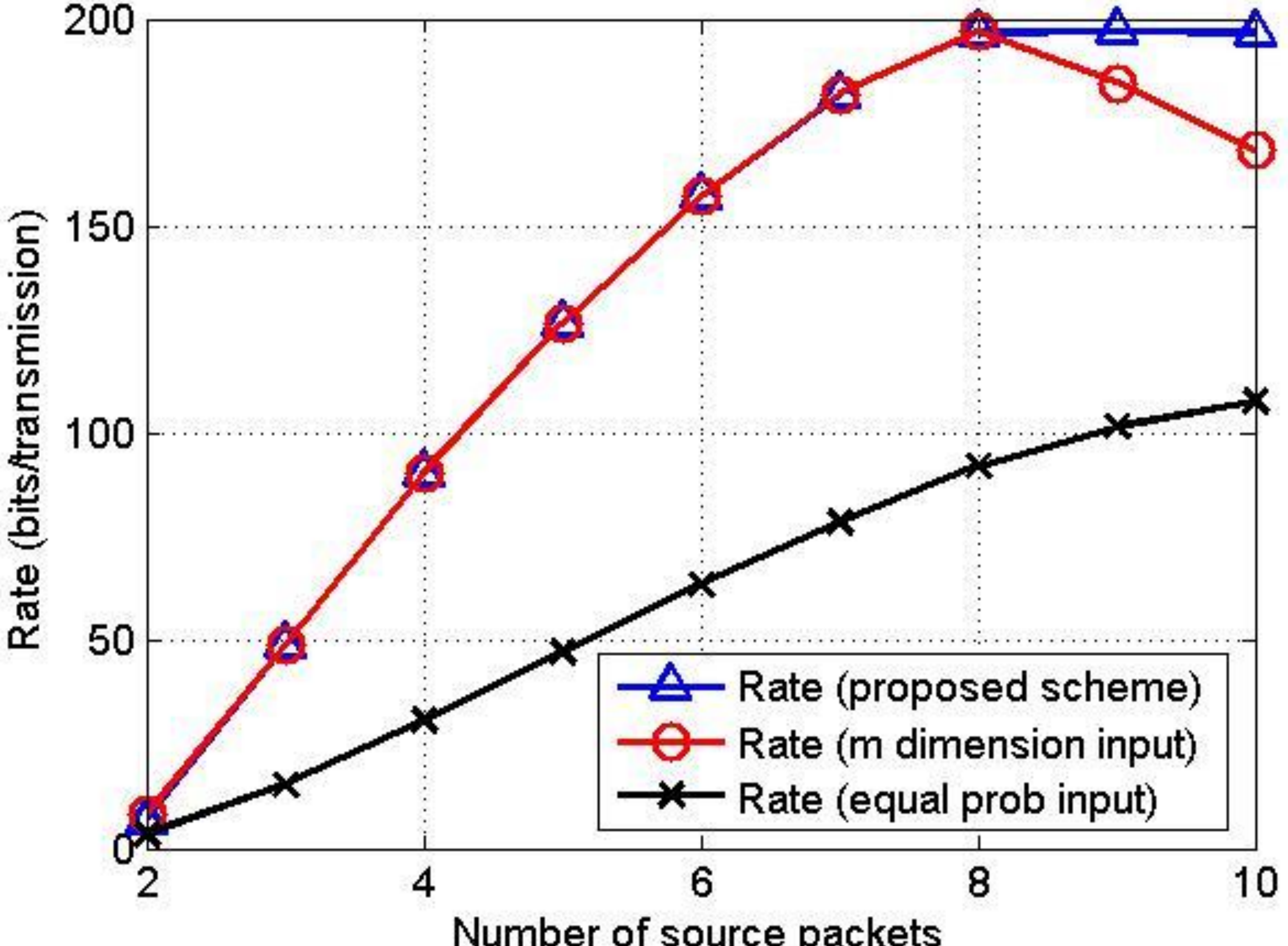
$$R(\underline{\alpha}) = -\sum_{d_x=0}^{\min(m,T)} \alpha_{d_x} n d_x \log_2 q - \sum_{d_x=0}^{\min(m,T)} \alpha_{d_x} q^{-nd_x} \cdot \sum_{d_{y'}=0}^{\min(n,d_x)} \psi(n, d_{y'}) \begin{bmatrix} d_x \\ d_{y'} \end{bmatrix} \log_2(f_{Y'}(d_{y'}, \underline{\alpha}))$$
$$+ \sum_{d_x=0}^{\min(m,T)} \alpha_{d_x} e d_x \log_2 q + \sum_{d_x=0}^{\min(m,T)} \alpha_{d_x} q^{-ed_x} \cdot \sum_{d_{z'}=0}^{\min(e,d_x)} \psi(e, d_{z'}) \begin{bmatrix} d_x \\ d_{z'} \end{bmatrix} \log_2(f_{Z'}(d_{z'}, \underline{\alpha})),$$

  - Too complex to derive analytically.

- Solved using a projection-based gradient descend algorithm.
  - Converge to the optimal solution.

Numerical Result: Secrecy Rate with Different Input Signals

- $T = 20, n = 8, e = 2, q = 7$



Untrusted Relay Recruitment Problem

- **Large field size approximation**: When field size $q \gg 1$, the secrecy capacity can be approximated as

$$C_s \approx (\min(m_t + m_u, n_t + n_u) - e)(T - \min(m_t + m_u, n_t + n_u)) \log q,$$

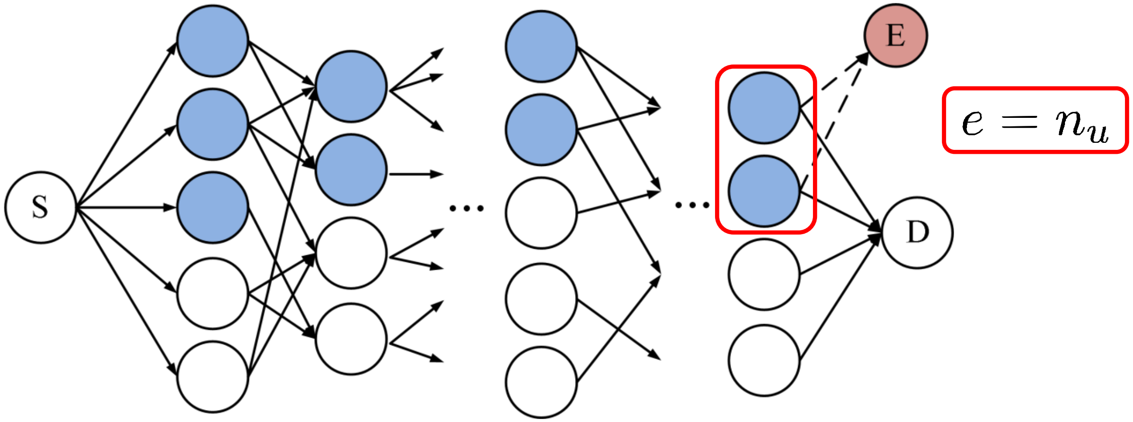
[Siavoshani & Fragouli '12]

➔ Special Case (No Untrusted Relays): $m_u = n_u = e = 0.$

$$C \approx \min(m_t, n_t)(T - \min(m_t, n_t)) \log q.$$

- Question: When should we recruit untrusted relays?
  - Case I: All untrusted relays ***near the destination*** are compromised ***with probability*** 1.
  - Case II: ***Each*** untrusted relay is compromised ***with probability*** p.

Case 1: All Untrusted Relays Near the Destination are Compromised

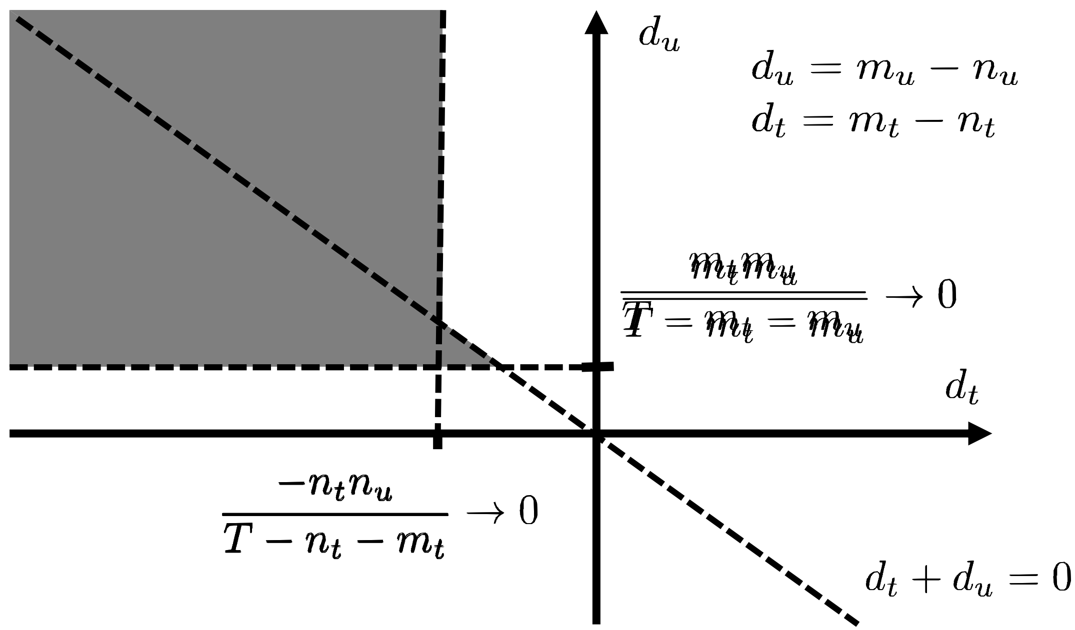


- In this case, we assume that the eavesdropper is near the destination so that all n_u untrusted relays in the last hop are compromised.

Theorem 1: Let $d_t = m_t - n_t$ and $d_u = m_u - n_u$.
When $T > m_t + \max(m_u, n_t)$, untrusted relays should be recruited if (d_t, d_u) satisfies one of the following conditions.

(1) $d_t + d_u \leq 0$ and $d_u > \frac{m_t m_u}{T - m_t - m_u}$ (2) $d_t + d_u > 0$ and $d_t < \frac{-n_t n_u}{T - n_t - m_t}$.

Recruit Region



- Eavesdropper can obtain $e = n_u$ dimension.
- Large d_u : Recruiting untrusted relays provide more Tx dimension than Rx dimension.
- Small d_t : Lack of transmit dimension in the original system.
- When $T \to \infty$, the recruit region is characterized by (d_u, d_t) only.

Case 2: Each Untrusted Relay is Compromised with Probability p

- There is a total of r_u untrusted relays that may be compromised with probability p .
  - The number of compromised relays: $\mathbf{e} \sim \mathcal{B}(r_u, p)$ (*Binomial distribution*)

- Outage probability: (The probability of no improvement)

$$\begin{aligned} P_{out} &\triangleq P_r\left[C_s(\mathbf{e}) - C \leq 0\right] \\ &= P_r\left[\mathbf{e} \geq \frac{(k_1 - k_2)(T - k_1 - k_2)}{(T - k_1)}\right]. \end{aligned}$$

where $k_1 = \min(m, n)$ and $k_2 = \min(m_u, n_u)$.

Asymptotic Outage Probability

- Suppose that $r_u \to \infty$ and that $m_u = \beta_m r_u$ and $n_u = \beta_n r_u$ for some positive ratio β_m, β_n.
- In this case, m_t, n_t are negligible compared to r_u (and also m_u and n_u).

Theorem 2: Let us consider a multihop network with parameters (m_u, n_u, r_u). If $m_u = \beta_m r_u$ and $n_u = \beta_n r_u$ and $\mathrm{T} \geq \min(m_u, n_u)$, then

$$P_{out} \to \begin{cases} 0 & \text{if } \; p < \beta \\ 1 & \text{if } \; p \geq \beta \end{cases}$$

as $r_u \to \infty$, where $\beta = \min(\beta_m, \beta_n)$.

  - $\beta \cdot r_u$: Dimension provided for the legitimate parts.
  - $p \cdot r_u$: Dimension eavesdropped by the eavesdropper.

Conclusions

- Consider a non-coherent multihop network system with the help of untrusted relays which are potentially eavesdropped.

- Determine the optimal input signal when untrusted relays are recruited by a gradient descend algorithm.

- Recruiting untrusted relays problem:
  - Case 1: Determine the recruiting region when all untrusted relays near the destination are compromised.
  - Case 2: Derive the outage probability when each untrusted relay is compromised with probability p, and show that when p is less than a threshold, one should recruit.

Thank You for Listening~!