Introducing Complex Functional Link Polynomial Filters

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Abstract

- We introduce a novel class of complex nonlinear filters, the **complex functional link polynomial (CFLiP)** filters.
- CFLiP filters are a sub-class of **linear-in-the-parameter** (LIP) nonlinear filters.
- They satisfy all conditions of **Stone-Weirstrass theorem** and thus are **universal approximators**.
- The CFLiP basis functions **separate** the **magnitude** and **phase** of the input signal.
- Moreover, CFLiP filters include many families of nonlinear filters with **orthogonal basis functions**.
- They are capable of modeling the nonlinearities of high power amplifiers (HPAs) of telecommunication systems with **better accuracy** than most of the filters currently used for this purpose.

Nonlinear system expansion

- Let us consider a causal, time-invariant, finite-memory, continuous nonlinear systems:

$$y(n) = f[x(n), x(n-1), \ldots, x(n-N+1)].$$

  with $x(n) \in \mathbb{C}_1 = \{x \in \mathbb{C},\ \text{and } |x| \leq 1\}$, $y(n) \in \mathbb{C}$.

- f is a **continuous function** from $\mathbb{C}_1^N$ to $\mathbb{C}$.
- We can expand f with a series of basis functions $f_i(n)$:

$$y(n) = \sum_{i=1}^{+\infty} c_i f_i[x(n), \ldots, x(n-N+1)],$$

  with $c_i \in \mathbb{C}$, and f_i a continuous function from $\mathbb{C}_1^N$ to $\mathbb{C}$, $\forall i$.

- Any choice of the basis functions f_i defines a different class of filters.

The Stone-Weierstrass theorem

- We want to develop a class of complex nonlinear filters that can **arbitrarily well approximate** the system on the compact $\mathbb{C}_1$ according to the Stone-Weirstrass (S-W) theorem:

  "Suppose $\mathcal{A}$ is a self-adjoint algebra of complex continuous functions on the compact set K, $\mathcal{A}$ separates points on K, and $\mathcal{A}$ vanishes at no point on K, then the uniform closure $\mathcal{B}$ of $\mathcal{A}$ consists of all complex continuous functions on K".

- A family $\mathcal{A}$ of complex functions is said to be an **algebra** if $\mathcal{A}$ is closed under addition, multiplication, and scalar multiplication.
- An algebra $\mathcal{A}$ is **self-adjoint** if for any complex function $f \in \mathcal{A}$ also the conjugate f^* belongs to $\mathcal{A}$.

CFLiP basis functions

- We separate module and phase of $x(n)$ using the polar form
  $x(n) = r(n)e^{j\phi(n)}$, $r(n) \in [0, 1]$ and $\phi(n) \in [0, 2\pi]$.
- Let $g_0[r(n)]$, $g_1[r(n)]$, ..., be a set of **real** basis functions satisfying S-W for the continuous real functions defined in $[0, 1]$; $g_0[r(n)] = 1$, $g_i[r(n)]$ a polynomial of order i.
- The N-dimensional **basis functions** of CFLiP filters are defined as follows:

$$\boxed{g_{k_0}[r(n)]e^{jp_0\phi(n)} \cdot \ldots \cdot g_{k_{N-1}}[r(n-N+1)]e^{jp_{N-1}\phi(n-N+1)}}$$

  with $k_0, ..., k_{N-1} \in \mathbb{N}$ and $p_0, ..., p_{N-1} \in \mathbb{Z}$.
- We define the order of a basis function as $K = k_0 + \ldots + k_{N-1}$, and the phase as $P = p_0 + \ldots + p_{N-1}$.
- The CFLiP basis functions and their linear combinations form a self-adjoint algebra that satisfies all conditions of S-W theorem.

Basis functions of order 3

For a CFIiP filter with memory N, order $K = 3$, and phase $P = 1$.

$$\forall \quad n_1 = 0, ..., N-1, \quad n_2 = n_1 + 1, ..., N-1, \quad n_3 = n_2 + 1, ..., N-1:$$

$$g_3[r(n-n_1)]e^{j\phi(n-n_1)}$$

$$g_2[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]$$

$$g_2[r(n-n_1)]g_1[r(n-n_2)]e^{j\phi(n-n_2)}$$

$$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_2[r(n-n_2)]$$

$$g_1[r(n-n_1)]g_2[r(n-n_2)]e^{j\phi(n-n_2)}$$

$$g_2[r(n-n_1)]e^{2j\phi(n-n_1)}g_1[r(n-n_2)]e^{-j\phi(n-n_2)}$$

$$g_1[r(n-n_1)]e^{-j\phi(n-n_1)}g_2[r(n-n_2)]e^{2j\phi(n-n_2)}$$

$$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]g_1[r(n-n_3)]$$

$$g_1[r(n-n_1)]g_1[r(n-n_2)]e^{j\phi(n-n_2)}g_1[r(n-n_3)]$$

$$g_1[r(n-n_1)]g_1[r(n-n_2)]g_1[r(n-n_3)]e^{j\phi(n-n_3)}$$

$$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]e^{j\phi(n-n_2)}g_1[r(n-n_3)]e^{-j\phi(n-n_3)}$$

$$g_1[r(n-n_1)]e^{j\phi(n-n_1)}g_1[r(n-n_2)]e^{-j\phi(n-n_2)}g_1[r(n-n_3)]e^{j\phi(n-n_3)}$$

$$g_1[r(n-n_1)]e^{-j\phi(n-n_1)}g_1[r(n-n_2)]e^{j\phi(n-n_2)}g_1[r(n-n_3)]e^{j\phi(n-n_3)}$$

CFLiP filters

- A CFLiP filter of **memory** N, **order** K, and **phase** P, is a linear combination of basis functions with orders from 0 to K and phase P, satisfying the following **constraints**:
  - All basis functions of order K have phases p_i with $|p_i| \leq k_i$ for $i = 0, ..., N - 1$.
  - All basis functions of order lower than K have phases $p_0, ..., p_{N-1}$ equal to one of the basis functions of order K and orders $k_0, ..., k_{N-1}$, lower than or equal to those of this basis function.
- A CFLiP filter of **memory** N, **order** K, and **phases** P_0,...,P_R is the parallel of R filters having memory N, order K, phase P_i, with $i = 1, ..., R$.
- A **single phase is often sufficient** to model the nonlinear systems of some applications.

Orthogonal CFLiP filters

- Some families of orthogonal FLiP filters have been proposed for the approximation of real nonlinear systems.
- For example, **even mirror Fourier nonlinear** (EMFN) [1] and **Legendre nonlinear** (LN) [2] filters are orthogonal for a white uniform distribution in $[-1, +1]$.
- We can define the corresponding CFLiP filters, whose basis functions are **orthogonal** for a white uniform distribution in $\mathbb{C}_1$.
- The basis functions $\widetilde{g}_k(\widetilde{r})$ of EMFN and LN filters are defined in $[-1, +1]$, but can be shifted on $[0, 1]$ with $g_k(r) = \widetilde{g}_k(2r - 1)$.
- In **Complex EMFN** (CEMFN) filters, we have $g_k(r) = \cos(k\pi r)$.
- In **Complex LN** (CLN) filters, the functions $g_k(r)$ are the Shifted Legendre polynomials,

$$g_k(r) = (-1)^k \sum_{s=0}^{k} \binom{k}{s} \binom{k+1}{s} (-r)^s.$$

Modelling RF High Power Amplifiers (HPAs)

- HPAs used in modern wireless systems are often operated close to **saturation** level to maximize the energy efficiency.
- While the HPA is typically a memory-less device, the filters that precede and follow the HPA introduce **memory effects**.
- If the HPA in the RF passband can be modeled as a Volterra filter, then its complex baseband representation is a **complex Volterra filter** composed only by **odd kernels** [3].
- Each kernel of order $2Q+1$ is composed by products of $Q+1$ direct samples and Q conjugate samples. E.g.,

$$x(n)x(n-n_1)x^*(n-n_2),$$
$$x(n)x^*(n-n_1)x(n-n_2),$$
$$x^*(n)x(n-n_1)x(n-n_2).$$

- Replacing the input samples with their polar form, the model is composed by basis functions with **phase** $P=1$.

Classic models of HPAs

- The memory polynomial filters

$$y(n) = \sum_{k=1}^{K} \sum_{i=0}^{N-1} a_{ki} x(n-i) |x(n-i)|^{k-1}$$

- The generalized memory polynomial filters [3]

$$y(n) = \sum_{k=1}^{K} \sum_{i=1}^{N-1} a_{ki} x(n-i) |x(n-i)|^{k-1} + \sum_{k=1}^{K} \sum_{i=0}^{N-1} \sum_{l=1}^{D_\mathrm{N}} b_{kil} x(n-i) |x(n-i-l)|^{k} + \sum_{k=1}^{K} \sum_{i=0}^{N-1} \sum_{l=1}^{D_\mathrm{N}} c_{kil} x(n-i) |x(n-i+l)|^{k}$$

- The orthogonal memory polynomial filters [4]

$$y(n) = \sum_{k=1}^{K} \sum_{i=0}^{N-1} a_{ki} \psi_k [x(n-i)]$$

(ψ_k shifted Legendre polynomials)

- These filters are special cases of the CFLiP filters, but they are **not universal approximators** since they do not satisfy S-W.

A simulation experiment

- CFLiP filters of phase $P = 1$ are interesting candidates for modeling and compensating HPAs.
- We provide some simulation results about the **identification of an HPA model**: the Wiener-Hammerstein model of [4].
- The model is composed by the cascade of a linear time-invariant (LTI) system $H(z)$, followed by a memory-less nonlinearity, in turn followed by a LTI system $G(z)$:

  $$H(z) = \frac{1}{1.5}\frac{1+0.25z^{-2}}{1+0.4z^{-1}}, \qquad G(z) = \frac{1}{0.52}\frac{1-0.25z^{-1}}{1-0.2z^{-1}}$$

  with memoryless nonlinearity:

  $$y(n) = \left(\gamma_1 \tan^{-1}(\zeta_1|z(n)|) + \gamma_2 \tan^{-1}(\zeta_2|z(n)|)\right) e^{j\rho(n)}$$

  with $\rho(n)$ the phase of $z(n)$.
- The input signal $x(n)$ is a white uniform noise in $\mathbb{C}_1$.
- The signal-to-noise ration is 80 dB.

The HPA identification

- The model has been identified using different polynomial filters with input-output relationship

  $$y(n) = [f_1(n), f_2(n), \ldots, f_L(n)] \cdot \mathbf{h}$$

  with $f_i(n)$ the basis functions, and $\mathbf{h}$ the coefficient vector.
- The coefficients can be estimated with the LS approach:

  $$\mathbf{h}_{\mathrm{LS}} = (\mathbf{F}^{\mathsf{H}}\mathbf{F})^{-1}\mathbf{F}^{\mathsf{H}}\mathbf{d}.$$

  with $\mathbf{d}$ a column vector of output samples and $\mathbf{F}$ a matrix whose n-th row is formed by the basis functions $[f_1(n), f_2(n), \ldots, f_L(n)]$.
- CLN and CLN2 filters for $D_{\mathrm{N}} \geq 2$ outperform by several dB most of the filters currently considered in the literature.
- The improved modeling accuracy is provided by the richer set of phase terms, i.e., from the use of the phase terms $e^{2j\phi(n-n_1)}e^{-j\phi(n-n_2)}$, $e^{j\phi(n-n_1)}e^{j\phi(n-n_2)}e^{-j\phi(n-n_3)}$.

Identification results for the HPA model

Filter	N	K	N_D	L	NMSE(dB)	Cond.Num.
CLN	5	3	0	20	−24.7	7.78
CLN	5	3	2	257	−38.5	36.6
CLN	5	3	3	419	**−41.6**	43.3
CLN	5	3	4	530	**−42.3**	47.4
CLN2	5	3	0	15	−24.1	2.54
CLN2	5	3	2	103	−37.8	785.
CLN2	5	3	3	147	**−40.2**	$1.12 \cdot 10^3$
CLN2	5	3	4	175	**−40.9**	$1.32 \cdot 10^3$
CEMFN	5	3	0	20	−24.7	3.38
CEMFN	5	3	2	257	−32.0	10.5
CEMFN	5	3	3	419	−32.4	12.4
CEMFN	5	3	4	530	−32.4	13.6
MP	5	3	0	15	−24.7	$3.30 \cdot 10^3$
MP	5	5	0	25	−24.7	$4.93 \cdot 10^6$
MP	5	7	0	35	−24.7	$6.21 \cdot 10^9$

Filter	N	K	N_D	L	NMSE(dB)	Cond.Num.
GMP	5	3	2	39	−24.7	$4.85 \cdot 10^3$
GMP	5	3	3	49	−24.7	$7.42 \cdot 10^3$
GMP	5	3	4	55	−24.7	$9.33 \cdot 10^3$
OP	5	3	0	15	−24.7	2.53
OP	5	5	0	25	−24.7	3.99
OP	5	7	0	35	−24.7	5.48
GOP	5	3	2	39	−24.7	425.
GOP	5	3	3	49	−24.7	719.
GOP	5	3	4	44	−24.7	$1.14 \cdot 10^3$
Vp1	5	3	0	10	−23.9	30.6
Vp1	5	3	2	47	−29.7	45.5
Vp1	5	3	4	80	−30.4	49.3
Vp1	5	5	0	15	−24.6	916
Vp1	5	5	2	203	−38.1	$1.67 \cdot 10^3$
Vp1	5	5	3	407	**−40.8**	$1.88 \cdot 10^3$
Vp1	5	5	4	605	**−41.4**	$2.09 \cdot 10^3$

- The model has been identified with :
  CLN filter, CLN2 filter (using polynomials ψ_k of [4]), CEMFN filter, memory polynomial filter (MP), generalized MP filter (GMP) [3], orthogonal polynomial filter (OP) [4], generalized OP filters (GOP), complex Volterra filter composed only by phase 1 terms (Vp1).

Concluding Remarks

- CFLiP filters have been introduced in this paper.
- They belong to the class of **linear-in-the-paramenter** nonlinear filters and are **universal approximators** according to the S-W theorem.
- CFLiP filters include many classes of nonlinear filters based on **orthogonal polynomials**.
- The **orthogonality** of the basis functions **improves** the **condition number** of the autocorrelation matrix used in LS identification and **increases** the **convergence speed** of gradient-descent identification algorithms.
- Future work will include the development of **Perfect Periodic Sequences** for CFLiP filters, which guarantee the orthogonality of the basis function on a finite period.

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