# UNMIXING MULTITEMPORAL HYPERSPECTRAL IMAGES WITH VARIABILITY: AN ONLINE ALGORITHM

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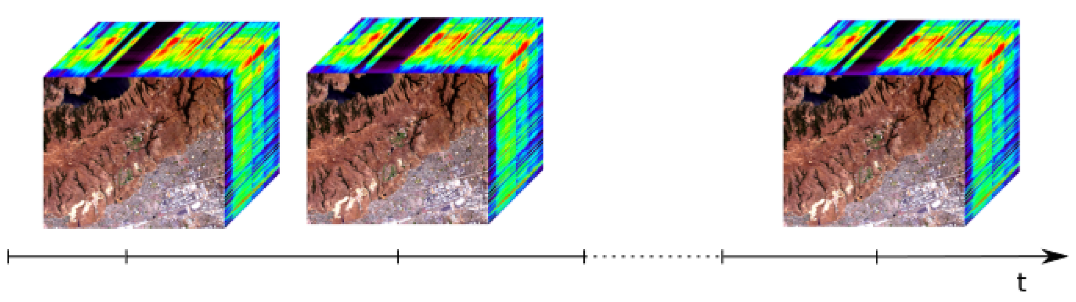
## 1. Introduction

Hyperspectral imagery

- high spectral resolution, low spatial resolution ⇒ hyperspectral unmixing
- hyperspectral unmixing
  - ▷ identifying the reference spectral signatures in the data (*endmembers*)
  - ▷ estimating the endmember relative fraction in each pixel (*abundances*).




Unmixing multi-temporal hyperspectral images

- $T$ hyperspectral images acquired over the same area
- varying acquisition conditions + inherent variability of the imaged scene (natural evolution) ⇒ variability
- increasing number of available images (several large images, significant number of images)
  - ▷ online estimation (sequential analysis)

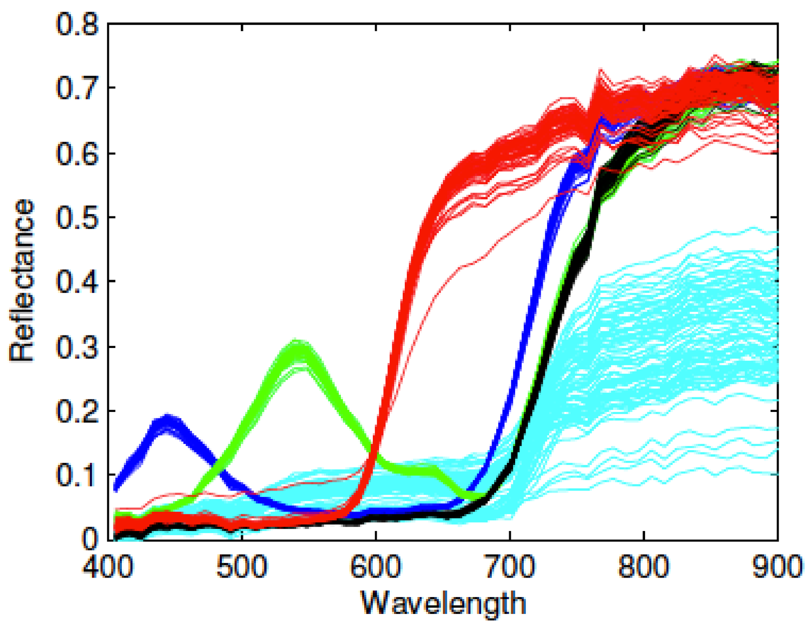



Figure 1: Spectral variability (P. Gader, A. Zare, R. Close, J. Aitken, G. Tuell, MUUFL Gulfport Hyperspectral and LiDAR Airborne Data Set, University of Florida, Gainesville, FL, Tech. Rep. REP-2013-570, Oct. 2013.)

## 2. Model

### Assumptions

- ▷ the $T$ images of the sequence share $K$ endmembers ($K$ known)
- ▷ the pixels of each image are similarly affected by spectral variability (first approximation).

### Perturbed linear mixing model (PLMM)

- pixel spectrum = linear combination of corrupted endmembers
- corrupted endmembers = endmembers affected by an additive time-varying perturbation vector

$$\mathbf{y}_{nt} = \sum_{k=1}^{K} a_{knt}\Big(\mathbf{m}_k + \mathbf{dm}_{kt}\Big) + \mathbf{b}_{nt} \qquad (1)$$

Matrix formulation

$$\mathbf{Y}_t = (\mathbf{M} + \mathbf{dM}_t)\mathbf{A}_t + \mathbf{B}_t \qquad (2)$$

| | |
|---|---|
| $N$ | number of pixels |
| $L$ | number of spectral bands |
| $K$ | number of endmembers |
| $\mathbf{Y}_t = [\mathbf{y}_{1t}, \dots, \mathbf{y}_{Nt}] \in \mathbb{R}^{L\times N}$ | $t$th hyperspectral image |
| $\mathbf{M} = [\mathbf{m}_1, \dots, \mathbf{m}_K] \in \mathbb{R}^{L\times K}$ | endmember matrix |
| $\mathbf{A}_t = [\mathbf{a}_{1t}, \dots, \mathbf{a}_{Nt}] \in \mathbb{R}^{K\times N}$ | $t$th abundance matrix |
| $\mathbf{dM}_t = [\mathbf{dm}_{1t}, \dots, \mathbf{dm}_{Kt}] \in \mathbb{R}^{L\times K}$ | $t$th variability matrix |

### Constraints

- abundance and endmembers (physical considerations)

$$\mathbf{M} \succeq \mathbf{0}_{L,K},\ \mathbf{A}_t \succeq \mathbf{0}_{K,N},\ \mathbf{A}_t^{\mathrm{T}}\mathbf{1}_K = \mathbf{1}_N,\ \forall t \in \{1,\dots,T\} \qquad (3)$$

- variability (modeling): small average temporal variability + upper bound for the instantaneous variability energy

$$\left\| \frac{1}{T}\sum_{t=1}^{T} \mathbf{dM}_t \right\|_{\mathrm{F}} \leq \kappa, \quad \|\mathbf{dM}_t\|_{\mathrm{F}} \leq \sigma, \quad \forall t \in \{1,\dots,T\} \qquad (4)$$

## 3. Problem formulation

- Two-stage stochastic problem, associated with the empirical risk minimization

$$\min_{\mathbf{M}\in\mathcal{M}} \frac{1}{T}\sum_{t=1}^{T} h(\mathbf{Y}_t, \mathbf{M}) + \beta\Psi(\mathbf{M}) \qquad (5)$$

$$h(\mathbf{Y}_t, \mathbf{M}) = \min_{(\mathbf{A},\mathbf{dM})\in\mathcal{A}_K\times\mathcal{D}_t} f(\mathbf{Y}_t, \mathbf{M}, \mathbf{A}, \mathbf{dM}) \qquad (6)$$

- ▷ $f$ : regularized instantaneous discrepancy measure
- ▷ $h$ : cost of the $t$th optimal decision to update the endmember matrix $\mathbf{M}$ given the data available at time $t$
- ▷ $\Psi$ : endmember regularization.
- ▷ $\mathcal{M} = \{\mathbf{M} : \mathbf{M} \succeq \mathbf{0}_{L,K}\}$
- ▷ $\mathcal{A}_K = \{\mathbf{A} : \mathbf{A} \succeq \mathbf{0}_{K,N},\ \mathbf{A}^{\mathrm{T}}\mathbf{1}_K = \mathbf{1}_N\}$
- ▷ $\mathcal{D}_t = \{\mathbf{dM} : \|\mathbf{dM}\|_{\mathrm{F}} \leq \sigma\} \cap \{\mathbf{dM} : \|\mathbf{dM} + \mathbf{E}_{t-1}\|_{\mathrm{F}} \leq t\kappa\}$
- ▷ $\mathbf{E}_t = \sum_{i=1}^{t} \mathbf{dM}_i$.
- White Gaussian noise assumption

$$f(\mathbf{Y}_t, \mathbf{M}, \mathbf{A}, \mathbf{dM}) = \frac{1}{2}\|\mathbf{Y}_t - (\mathbf{M} + \mathbf{dM})\mathbf{A}\|_{\mathrm{F}}^2 + \alpha\Phi_t(\mathbf{A}) + \gamma\Upsilon_t(\mathbf{dM}) \qquad (7)$$

- ▷ $\Phi_t$, $\Upsilon_t$ : appropriate regularizations
- ▷ trade-off between the data fitting term and the penalties $\Phi_t(\mathbf{A})$, $\Psi(\mathbf{M})$ and $\Upsilon_t(\mathbf{dM})$ controlled by $(\alpha, \beta, \gamma)$.

### Abundance and variability regularization

Moderate/smooth changes assumed from one image to another

$$\Phi_t(\mathbf{A}) = \frac{1}{2}\|\mathbf{A} - \mathbf{A}_{t-1}\|_{\mathrm{F}}^2 \qquad (8)$$

$$\Upsilon_t(\mathbf{dM}) = \frac{1}{2}\|\mathbf{dM} - \mathbf{dM}_{t-1}\|_{\mathrm{F}}^2 \qquad (9)$$

### Endmember regularization

Constrains the volume of the simplex whose vertices are the endmember signatures

$$\Psi(\mathbf{M}) = \frac{1}{2}\sum_{i=1}^{K}\left(\sum_{\substack{j=1\\ j\neq i}}^{K} \|\mathbf{m}_i - \mathbf{m}_j\|_2^2\right). \qquad (10)$$

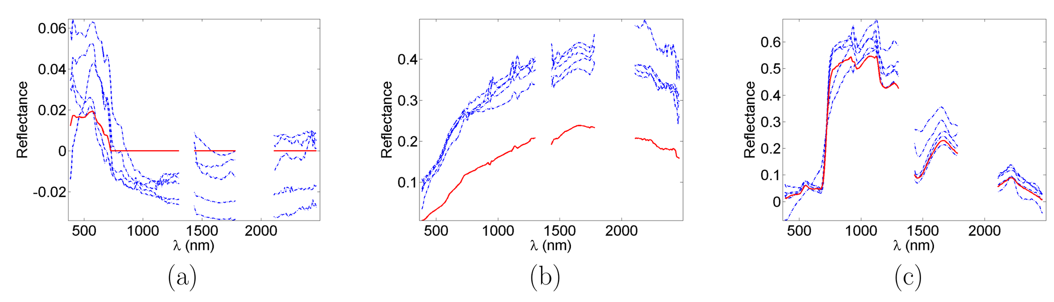



Figure 2: Example of endmembers (in red) and variability (in blue) obtained when removing the constraint on the averaged variability.

## 4. An online algorithm

### Structure of the online algorithm

- whenever an image $\mathbf{Y}_t$ is received, local abundance and variability estimation by a proximal alternating linearized minimization (PALM) algorithm
  - ▷ PALM guaranteed to converge to a critical point of the non-convex problem (6)
- endmembers updated by proximal gradient descent steps
- possibility to add a forgetting factor $\xi \in ]0,1]$
- provided problem (6) exclusively admits locally unique critical points, Algo. 1 converges to a critical point of Problem (5) as $T \to +\infty$.

**Algorithm 1:** Online unmixing algorithm.

**Data**: $\mathbf{M}_0$, $\mathbf{A}_0$, $\mathbf{dM}_0$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\xi \in ]0,1]$
**begin**
- $\mathbf{C}_0 \leftarrow \mathbf{0}_{K,K}$;
- $\mathbf{D}_0 \leftarrow \mathbf{0}_{L,K}$;
- $\mathbf{E}_0 \leftarrow \mathbf{0}_{L,K}$;
- **for** $t = 1$ **to** $T$ **do**
  - **a** Random selection of an image $\mathbf{Y}_t$ ;
  - // Abundance and variability estimation by PALM
  - **b** $(\mathbf{A}_t, \mathbf{dM}_t) \in \arg\min_{(\mathbf{A},\mathbf{dM})\in\mathcal{A}_K\times\mathcal{D}_t} f(\mathbf{Y}_t, \mathbf{M}_t, \mathbf{A}, \mathbf{dM})$;
  - $\mathbf{C}_t \leftarrow \xi\mathbf{C}_{t-1} + \mathbf{A}_t\mathbf{A}_t^{\mathrm{T}}$ ;
  - $\mathbf{D}_t \leftarrow \xi\mathbf{D}_{t-1} + (\mathbf{dM}_t\mathbf{A}_t - \mathbf{Y}_t)\mathbf{A}_t^{\mathrm{T}}$ ;
  - $\mathbf{E}_t \leftarrow \xi\mathbf{E}_{t-1} + \mathbf{dM}_t$ ;
  - // Endmember update
  - **c** $\mathbf{M}_t \leftarrow \arg\min_{\mathbf{M}\in\mathcal{M}} \frac{1}{t}\left[\frac{1}{2}\mathrm{Tr}(\mathbf{M}^{\mathrm{T}}\mathbf{M}\mathbf{C}_t) + \mathrm{Tr}(\mathbf{M}^{\mathrm{T}}\mathbf{D}_t)\right] + \beta\Psi(\mathbf{M})$

**Result**: $\mathbf{M}_T$, $(\mathbf{A}_t)_{t=1,\dots,T}$, $(\mathbf{dM}_t)_{t=1,\dots,T}$

## 5. Experiment with synthetic data

- Method evaluated on 15 linear mixtures of size 31 × 30, composed of 413 bands
- No pure pixel, mixtures corrupted by an additive white Gaussian noise to ensure SNR = 30 dB
- Abundance and endmembers initialized with VCA/FCLS
- Simulation scenario: Algo. 1 run for 50 cycles through the whole dataset, PALM and proximal gradient descent stopped after 50 iterations, $\xi = 0.99$, $\alpha = 3.9 \times 10^{-2}$, $\beta = 5.4 \times 10^{-4}$, $\gamma = 3.2 \times 10^{-4}$, $\sigma^2 = 12.4$, $\kappa^2 = 1.9$.

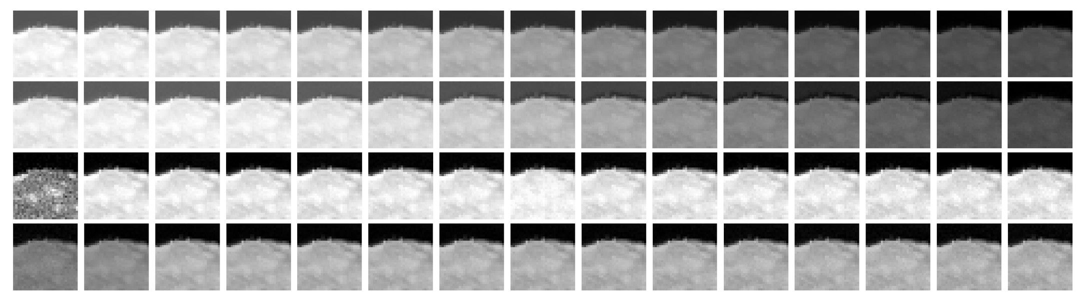


Figure 3: Abundance maps of $\mathbf{m}_{1t}$ [0: black, 1: white] (rows: true maps, proposed method, VCA/FCLS, SISAL/FCLS).

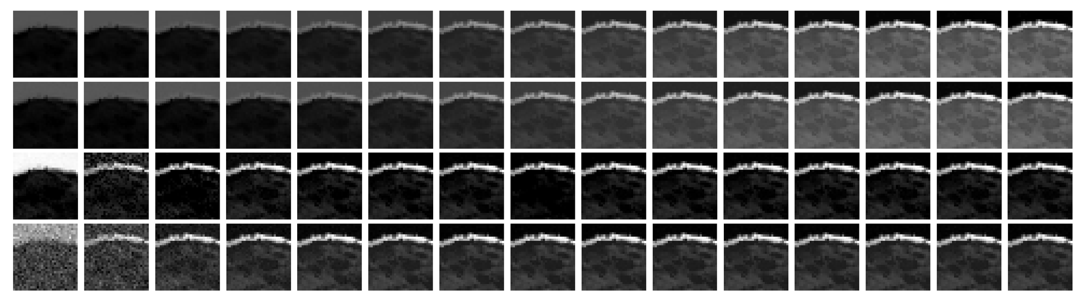


Figure 4: Abundance maps of $\mathbf{m}_{2t}$ [0: black, 1: white] (rows: true maps, proposed method, VCA/FCLS, SISAL/FCLS).

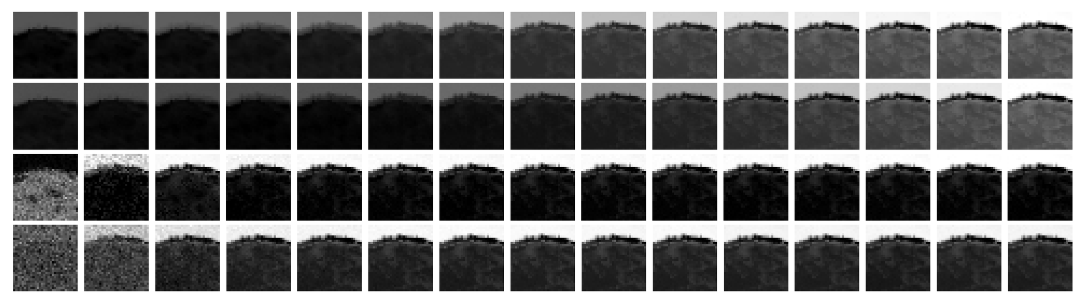


Figure 5: Abundance maps of $\mathbf{m}_{3t}$ [0: black, 1: white] (rows: true maps, proposed method, VCA/FCLS, SISAL/FCLS).

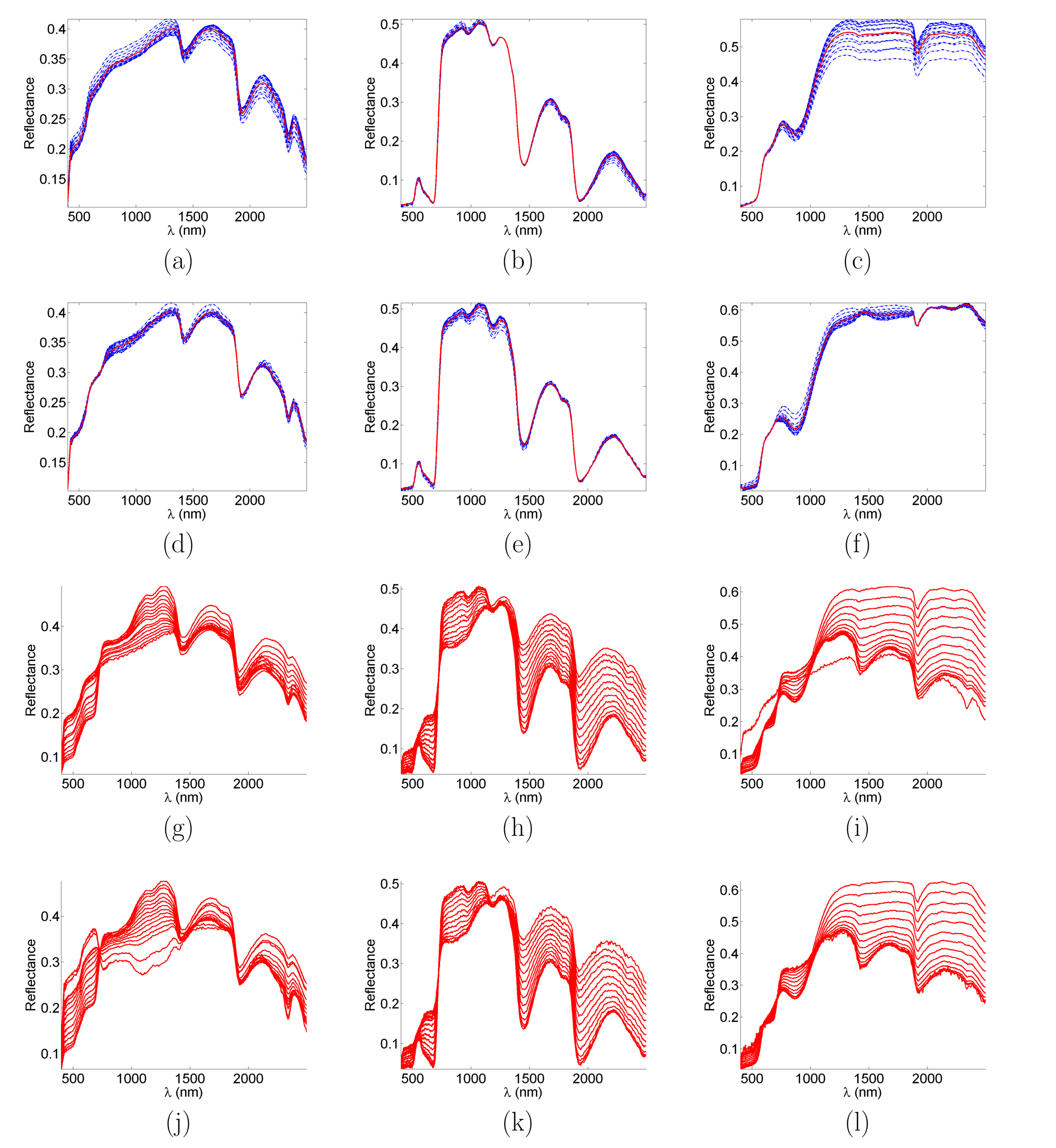



Figure 6: Corresponding endmembers [rows: true endmembers (in red) and variability (in blue), proposed method, VCA, SISAL].

Table 1: Simulation results obtained with synthetic data (GMSE($\mathbf{A}$)×$10^{-2}$, GMSE($\mathbf{dM}$)×$10^{-4}$, RE ×$10^{-5}$).

| | VCA/FCLS | SISAL/FCLS | Prop. method |
|---|---|---|---|
| aSAM($\mathbf{M}$) (°) | 8.9792 | 8.6685 | **1.9898** |
| GMSE($\mathbf{A}$) | 6.67 | 3.90 | **0.47** |
| GMSE($\mathbf{dM}$) | / | / | **3.07** |
| RE | 9.59 | **9.49** | 9.63 |
| time (s) | **2** | 2.2 | 561 |

## 6. Conclusion and future work

- ▶ Proposition of an online hyperspectral unmixing algorithm accounting for endmember temporal variability
- ▷ Consider abrupt endmember changes (common in real data)
- ▷ Incorporate spatial variability
- ▷ Find automatic rules to adjust the regularization parameters

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