OPTIMAL OPTIMAL PILOT LENGTH FOR UPLINK MASSIVE MIMO SYSTEMS WITH PILOT REUSE

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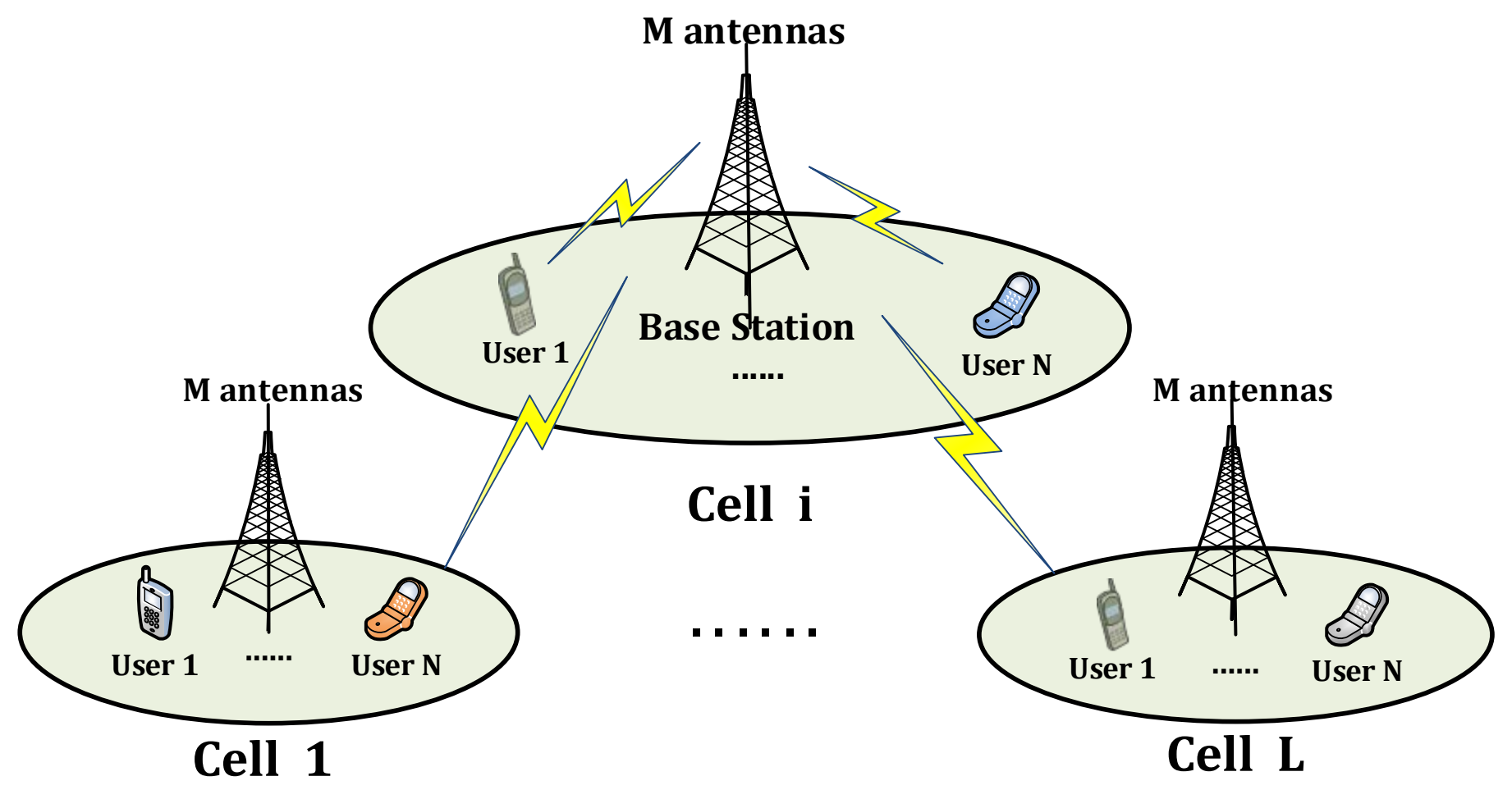
Introduction

An important practical issue in massive MIMO is channel estimation, which is typically performed using uplink pilot training. Users employing the same pilot sequence give rise to the pilot contamination issue, which persists even with unlimited BS antennas[1-3]. To diminish this effect, more orthogonal pilot sequences can be used to expand the distance between interference users. This increases the pilot length, following for improved channel estimation, however, it also reduce the time for data transmission. Here, we are interested in determining the optimal pilot length that maximizes the sum rate. A similar problem has been addressed in some previous works, but pilot reuse is not considered or not present optimization results with finite number of antennas[4,5].

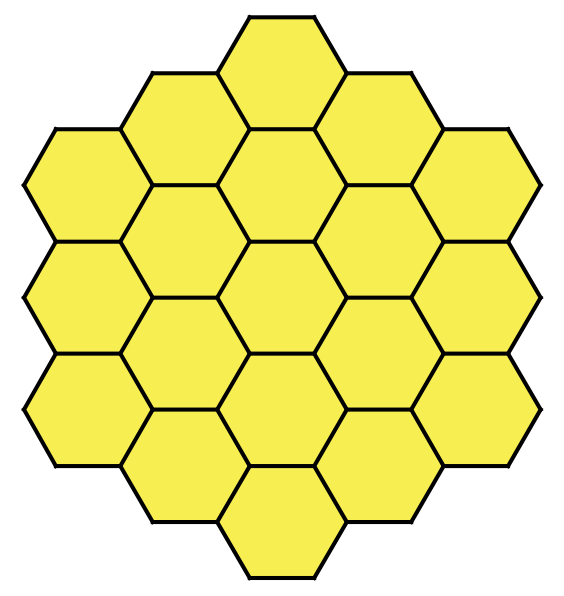
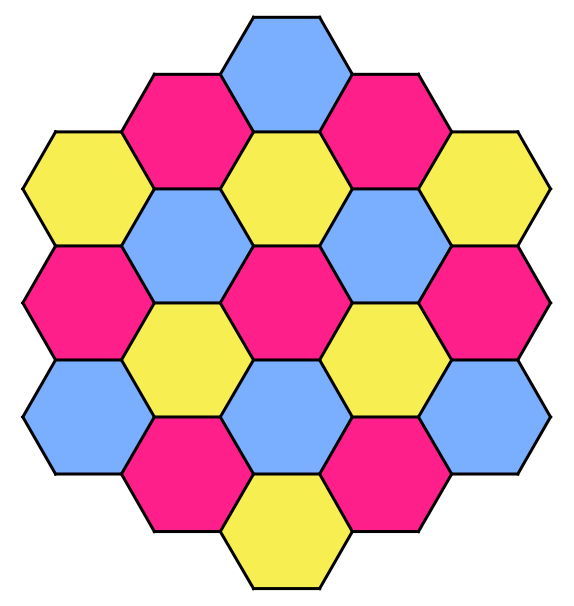
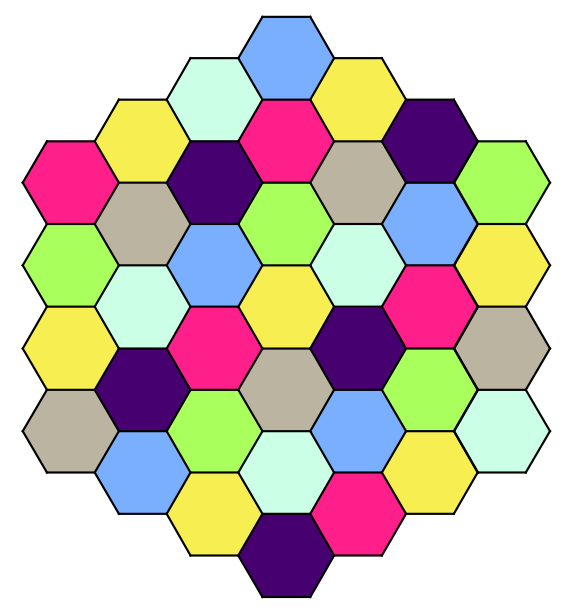
In this paper, accounting for pilot reuse effects, we derive tractable expressions for the achievable uplink rate with MRC and ZF receivers. Through these analytical results, the optimal pilot length which maximizes the sum rate per cell is put forward. It is found that a larger pilot reuse factor can improve the rate performance. In the asymptotic massive MIMO regime, the optimal pilot length is shown to be the minimum feasible value which ensures orthogonality within the set of pilots. However, for moderate numbers of BS antennas, the optimal pilot length is above the minimum feasible value, and diminishes towards the minimum feasible value as the pilot length grows. Our analytical results are validated via simulations.

System Model

Consider uplink transmission in a cellular network consisting of L cells, each with one M-antenna BS and N single antenna users.



Channel estimation is performed using uplink pilots. Users in one cell transmit orthogonal pilot sequences, and pilots can be reused among different cells with a factor ρ.

Method

- ❑ Define a mapping function to clarify the relationship of cell index and pilot reuse groups

$$f_1(i) = q$$

The nth user in the ith cell use the $\tau \times 1$ pilot vector $\sqrt{\tau}\boldsymbol{\psi}_{ni}$, which satisfies

$$\boldsymbol{\psi}_{ni}^H \boldsymbol{\psi}_{cl} = \begin{cases} 1, & f_1(i) = f_1(l), n = c \\ 0, & otherwise \end{cases}$$

The MMSE estimate of channel vector $\hat{\boldsymbol{g}}_{ini}$ is[6]

$$\hat{\boldsymbol{g}}_{ini} = \left(\sum_{\substack{l=1 \\ f_1(l)=f_1(i)}}^{L} g_{inl} + \frac{1}{\sqrt{\tau p_u}} \mathbf{w}_i \right) \frac{\tau p_u \beta_{icl}}{1 + \tau p_u \sum_{\substack{l'=1 \\ f_1(l')=f_1(l)}}^{L} \beta_{icl'}}$$

- ❑ Find the optimal pilot length, i.e., to solve the following optimization problem

$$\tau^* = \arg \max_{N\rho \le \tau \le T} \frac{T-\tau}{T} \sum_{n=1}^{N} R_{ni}$$

The key procedure is to find a tractable expression for the achievable rate R_{ni}.

- ❑ For MRC receivers, we present an approximation[7]

$$R_{ni}^{MRC} \approx \tilde{R}_{ni}^{MRC} = \log_2 \left(1 + \frac{(M+1) p_u^2 \beta_{ini}^2 \tau}{\Delta_1 \tau + \left(p_u \sum_{l=1}^{L} \sum_{c=1}^{N} \beta_{icl} + 1 \right)} \right)$$

For ZF receivers, we present a lower bound[8]

$$R_{ni}^{ZF} \ge \tilde{R}_{ni}^{ZF} = \log_2 \left(1 + \frac{(M-N) p_u^2 \beta_{ini}^2 \tau}{\Delta_2 \tau + \left(p_u \sum_{l=1}^{L} \sum_{c=1}^{N} \beta_{icl} + 1 \right)} \right)$$

- ❑ The optimal pilot length should not be dependent on a specific user location. To remove the dependence:

Central users use power control to compensate

$p_u \beta_{ici} = \lambda_{ii}$ $\beta_{icl} = \lambda_{il}$

Other users use the average value to replace

Applying these assumptions into achievable rate (take MRC receivers for example), we can get

$$\tilde{R}_{ni}^{MRC,\lambda} = \log_2 \left(1 + \frac{(M+1) \lambda_{ii}^2 \tau}{\Delta_3 \tau + \left(N\lambda_{ii} + N p_u \sum_{l=1, l \ne i}^{L} \lambda_{il} + 1 \right)} \right)$$

- ❑ The original optimization problem can be simplified as

$$\tau^* = \arg \max_{N\rho \le \tau \le T} N \frac{T-\tau}{T} \tilde{R}_{ni}^{MRC,\lambda}$$

$\mathcal{S}_1(\tau) = N \frac{T-\tau}{T} \tilde{R}_{ni}^{MRC,\lambda}$ is a concave function, and the above problem can be solve by the following lemma

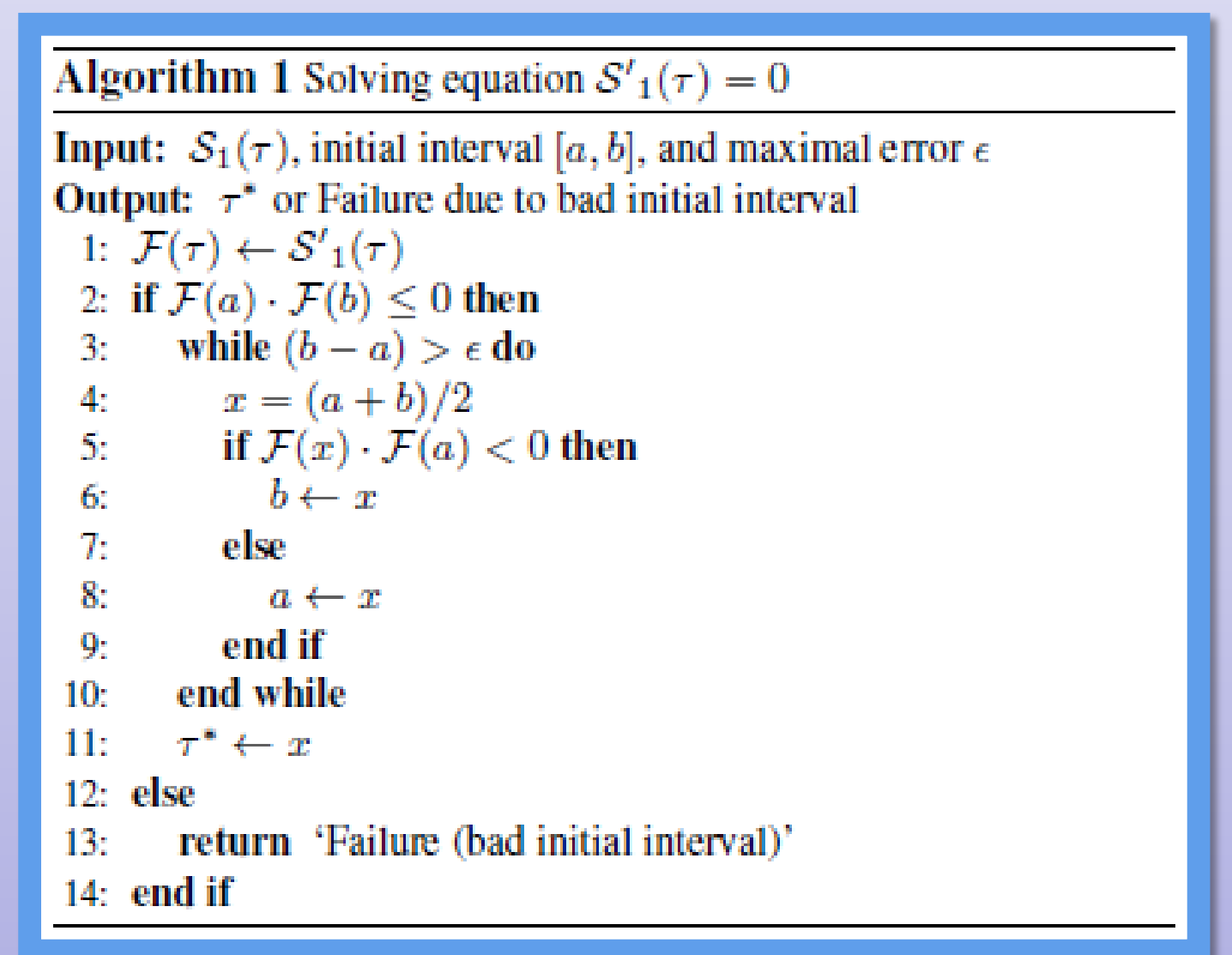
If a_1, a_2, b_1, b_2, b_3 are all positive,

$$s_1(x) = (a_1 - a_2 x) log_2 \left(1 + \frac{b_1 x}{b_2 x + b_3} \right)$$

is concave on $x \in (0, \infty)$, and has a maximum value at x^*, which is the solution of $s_1'(x)$ =0.

Results

- ❑ The optimal pilot length (take MRC receiver for example) can be demonstrated as:
  - ✓ If $\mathcal{S}_1'(N\rho) > 0$, the optimal τ^* is obtained by Algorithm 1 and rounded to the nearest larger/smaller integer;
  - ✓ If $\mathcal{S}_1'(N\rho) \le 0$, the optimal $\tau^* = N\rho$



Algorithm 1 Solving equation $\mathcal{S}'_1(\tau) = 0$

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Input: $\mathcal{S}_1(\tau)$, initial interval $[a, b]$, and maximal error $\epsilon$
Output: $\tau^*$ or Failure due to bad initial interval
$\mathcal{F}(\tau) \leftarrow \mathcal{S}'_1(\tau)$
if $\mathcal{F}(a) \cdot \mathcal{F}(b) \le 0$ then
    while $(b - a) > \epsilon$ do
        $x = (a + b)/2$
        if $\mathcal{F}(x) \cdot \mathcal{F}(a) < 0$ then
            $b \leftarrow x$
        else
            $a \leftarrow x$
        end if
    end while
    $\tau^* \leftarrow x$
else
    return 'Failure (bad initial interval)'
end if
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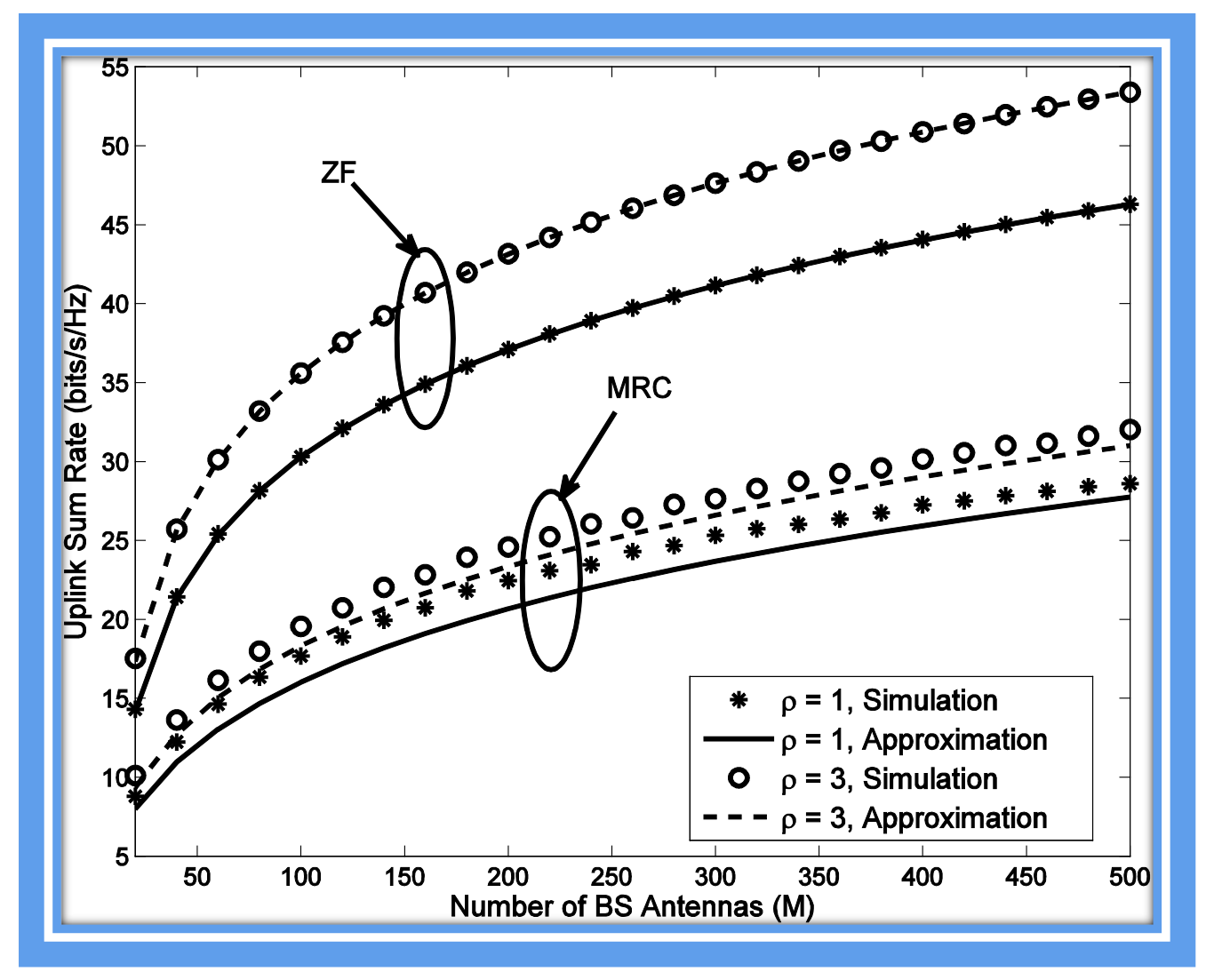
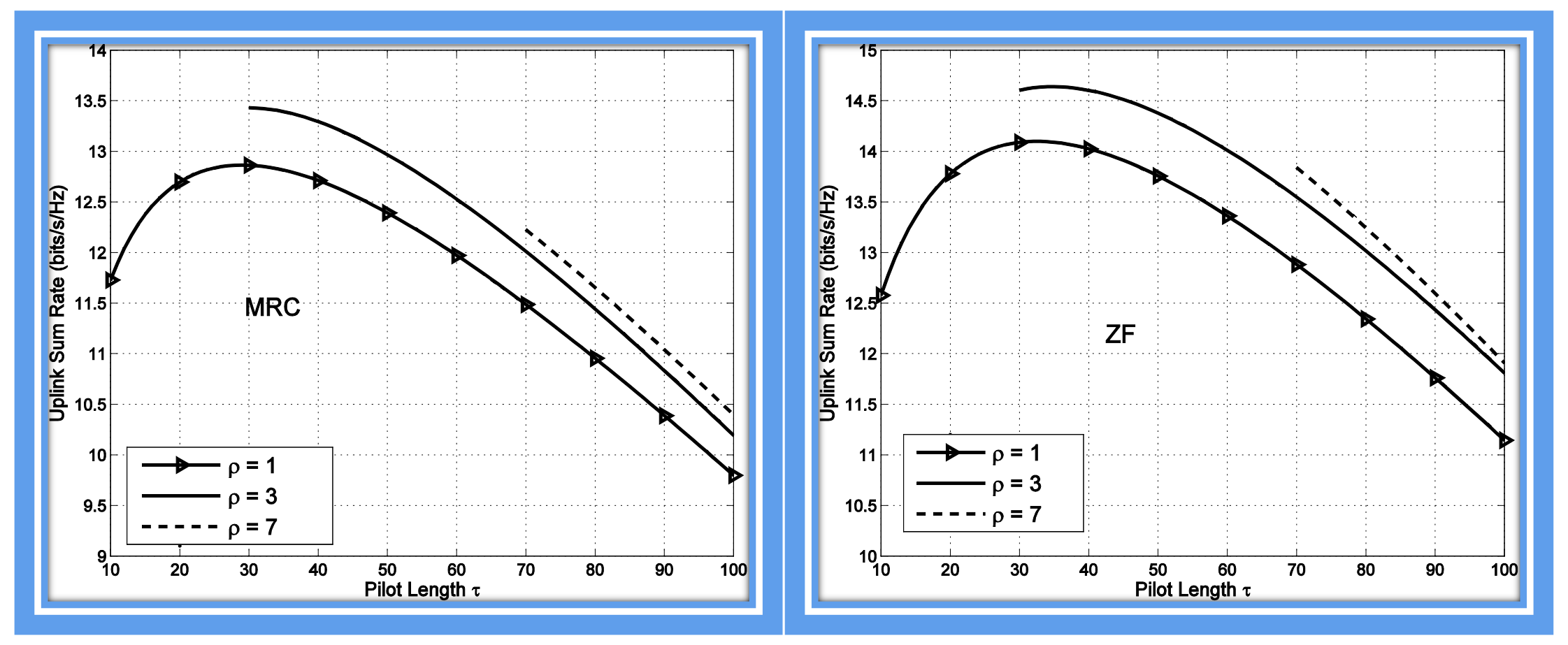
  - ✓ With infinite number of antennas, the optimal $\tau^* = N\rho$

- ❑ Use 10^4 realizations of β_{icl} to get the average value, λ_{il}. With fixed τ, compare simulation and analytical results. We can see a close agreement between the simulation and analytical results, especially for the ZF receiver. A larger ρ improves the uplink performance. Next, use analytical results to explore the optimal pilot length.



- ❑ Due to $\tau \ge N\rho$, curves with different ρ have a different starting point. It can be found that when $\rho = 1$, as expected, the sum rate first increases and then decreases. For larger ρ, the sum rate almost monotonically decreases. This can be explained by the benefit brought by a slightly better estimation is negligible as compared with the loss in transmission time.



Conclusion

In this paper, we study the achievable uplink rate of multicell massive MIMO systems with MRC and ZF receivers.

- ❑ Allowing pilots orthogonality across different cells, we obtain the optimal pilot length that maximizes the uplink sum rate, and the results applies for any number of BS antennas.
- ❑ Larger pilot reuse factor can improve the rate performance, and as it grows, the optimal pilot length turned closer to the minimum feasible value.
- ❑ As the pilot reuse factor grows, the benefit of increasing pilot length turns less significant.

Future Work

This work focuses on the case where number of users and the pilot reuse pattern is fixed. This is a start of our research. Next we will continue this work to more complicated cases.

- ❑ Find the optimal pilot length with unknown number of users and pilot reuse pattern.
- ❑ Find the optimal pilot reuse pattern with known or unknown pilot length.
- ❑ Optimize the pilot length, number of users and pilot reuse pattern at the same time.

These works are ongoing. Please looking forward to our journal version paper.

References

[1] T. L. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas,” IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
[2] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, “Pilot contamination and precoding in multicell TDD systems,” IEEE Trans. Wireless Commun., vol. 10, no. 8, pp. 2640–2651, Aug. 2011.
[3] F. Fernandes, A. Ashikhmin, and T. L. Marzetta, “Intercell interference in noncooperative TDD large scale antenna systems,” IEEE J. Sel. Areas Commun., vol. 31, no. 2, pp. 192–201, Feb. 2013.
[4] H. Q. Ngo, M. Matthaiou, and E. G. Larsson, “Massive MIMO with optimal power and training duration allocation,” IEEE Wireless Commun. Lett., vol. 2, no. 6, pp. 605–608, Sept. 2014.
[5] E. Björnson, E. G. Larsson, and M. Debbah, “Massive MIMO for maximal spectral efficiency: How many users and pilots should be allocated?” IEEE Trans. Wireless Commun., accepted to appear.
[6] S. M. Kay, Fundamentals of Statistical Signal Srocessing: Estimation Theory (Volume 1). Prentice Hall, 1993.
[7] Q. Zhang, S. Jin, K.-K. Wong, H. Zhu, and M. Matthaiou, “Power scaling of uplink massive MIMO systems with arbitrary-rank channel means,” IEEE J. Sel. Topics Signal Process., vol. 8, no. 5, pp. 966–981, Oct. 2014.
[8] D. Gore, R. W. Heath Jr., and A. Paulraj, “Transmit selection in spatial multiplexing systems,” IEEE Commun. Lett., vol. 6, no. 11, pp. 491–493, Nov. 2002.