GlobalSIP 2015 – Monday 14.12. 2015

“Channel-Robust Compressed Sensing via Vector Pre-Quantization in Wireless Sensor Networks”

*Markus Leinonen, Marian Codreanu, Markku Juntti

Centre for Wireless Communications (CWC), University of Oulu, Finland

CWC Oulu CENTRE FOR WIRELESS COMMUNICATIONS University of Oulu

UNIVERSITY of OULU OULUN YLIOPISTO

Contents

- Background
- System Model
- Problem Formulation
- Design Approach
- Encoder-Decoder Optimization
- Algorithm Implementation
- Numerical Results
- Conclusions

Background: Wireless Sensor Network Monitoring

- Measuring & monitoring WSN applications
- Resource-limited sensors
  - ➢ Wireless access is energy-consuming
  - ➢ Limited computation power → ***simple encoding needed***
- Source compression for minimizing the information rate
- Real-world WSN signals ***sparse***, e.g., in surveillance, fault detection and spectrum sensing applications
- Signals typically continuous-valued
- ***Quantization*** inevitable for finite-rate communications

Background: Design Approaches

- Compressed sensing (CS)
  - ➢ Retrieve a high-dimensional sparse vector from few linear measurements
- Vector quantization (VQ)
  - ➢ Block source coding (cf. Shannon’s source coding theory)
- Channel-optimized VQ (COVQ)
  - ➢ Joint source-channel coding (JSCC)
  - ➢ Source-channel separation theorem: infinitely long block lengths
    - ❑ A practical method with constrained delay and complexity

System Model (1/2)

Fixed measurement matrix

- Noisy CS measurements [*M*] of *K*-sparse signal [*N*]: $\boxed{\boldsymbol{y} = \boldsymbol{\Phi x} + \boldsymbol{w}}$
- Pre-quantization of measurement space into *V* cells $\{\mathcal{S}_1, \ldots, \mathcal{S}_V\}$ with rate $\bar{R} = \log_2 V$ and indices $\mathcal{V} \triangleq \{1, \ldots, V\}$

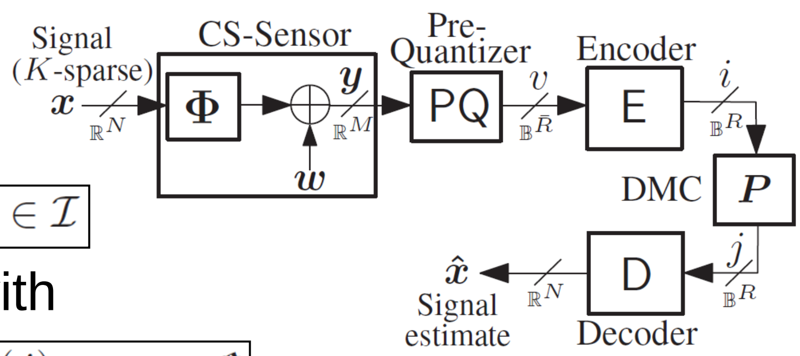
$$\boxed{\mathsf{PQ}(\boldsymbol{y}) = v, \text{ if } \boldsymbol{y} \in \mathcal{S}_v,\ v \in \mathcal{V}}$$

- Encoder is an index mapping $\pi : \mathcal{V} \mapsto \mathcal{I}$, $\pi = \{\pi(1), \ldots, \pi(V)\}$ with rate $R = \log_2 I$, $R \leq \bar{R}$, and indices $\mathcal{I} \triangleq \{1, \ldots, I\}$

  $\pi(v) \in \mathcal{I}$: encoding index of *v*:th cell

  $\pi^{-1}(i) = \{v | \pi(v) = i\}$: cell indices assigned to *i*:th encoding index

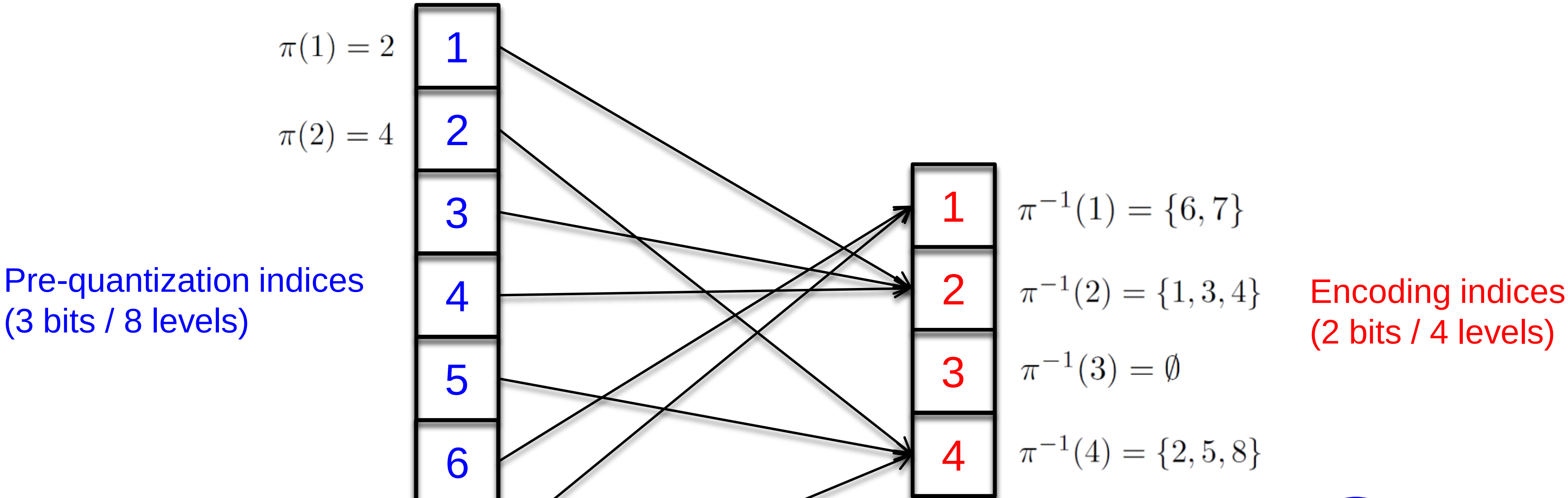
$$\boxed{\mathsf{E}(v) = i, \text{ if } \pi(v) = i,\ i \in \mathcal{I}}$$

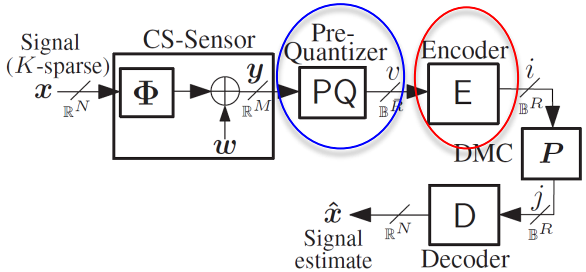


- Discrete memoryless channel is a mapping $I \rightarrow I$ with $\boxed{p_{ij} = \Pr(j|i),\ \ i, j \in \mathcal{I}}$
- Decoder is a mapping $\mathsf{D} : \mathcal{I} \rightarrow \mathfrak{C}$ with codevectors $\mathfrak{C} \triangleq \{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_I\}$, i.e., $\boxed{\mathsf{D}(j) = \boldsymbol{c}_j \in \mathfrak{C}}$

System Model (2/2)

- Illustration of assignment of pre-quantization cells into encoding indices with $\bar{R} = 3$ and $R = 2$





Problem Formulation

- Minimization of the end-to-end MSE distortion $D \triangleq \mathbb{E}\left[\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2^2\right]$
  - ➢ 1) Quantization error 2) Channel error 3) Reconstruction error
- Additional constraint for PQ, i.e., VQ with codebook $\mathfrak{G} \triangleq \{\boldsymbol{g}_1, \dots, \boldsymbol{g}_V\}$ and codepoints $\boldsymbol{g}_v \in \mathbb{R}^M$:
  - ➢ <u>Nearest-neighbor coding</u> to enforce controlled encoding complexity
- Marginalize over the PQ, E and D indices:

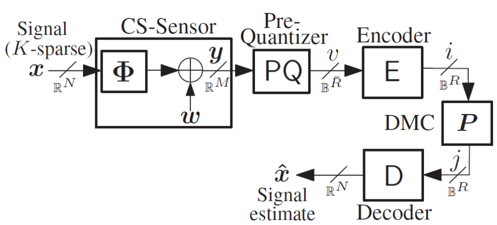
$$\begin{aligned} D &= \sum_{v,i,j} \Pr(v,i,j)\mathbb{E}\left[\|\boldsymbol{x} - \boldsymbol{c}_j\|_2^2 \middle| v,i,j\right] \\ &= \sum_{i,j} \sum_{v \in \pi^{-1}(i)} p_{ij} \Pr(v)\mathbb{E}\left[\|\boldsymbol{x} - \boldsymbol{c}_j\|_2^2 \middle| v,i,j\right] \\ &= \sum_{v,i,j} \delta_\pi(v,i) p_{ij} \Pr(v)\mathbb{E}\left[\|\boldsymbol{x} - \boldsymbol{c}_j\|_2^2 \middle| v,i,j\right] \end{aligned}$$

Markov property
$v \rightarrow i \rightarrow j$

- Joint optimization problem:

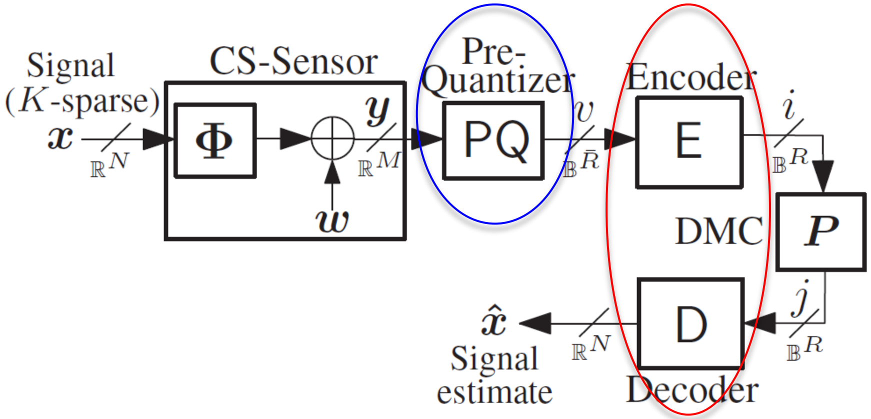
$$\begin{aligned} \text{min.} \quad & \sum_{v,i,j} \delta_\pi(v,i) p_{ij} \Pr(v)\mathbb{E}\left[\|\boldsymbol{x} - \boldsymbol{c}_j\|_2^2 \middle| v,i,j\right] \\ \text{s.t.} \quad & \pi(v) \in \mathcal{I}, \;\; v \in \mathcal{V} \\ & \mathcal{S}_v = \left\{\boldsymbol{y} : \|\boldsymbol{y} - \boldsymbol{g}_v\|_2^2 \leq \|\boldsymbol{y} - \boldsymbol{g}_{v'}\|_2^2, \; \forall v' \neq v\right\}, \; v \in \mathcal{V} \end{aligned}$$

with optimization variables $\{\boldsymbol{g}_v\}_{v=1}^V$, π , and $\{\boldsymbol{c}_j\}_{j=1}^I$



Optimization Approach

- The integer constraint makes the problem intractable
- Split the optimization into two **separate** steps:
  1. Optimization of PQ via $\{\boldsymbol{g}_v\}_{v=1}^{V}$
  2. Optimization of E-D pair via π and $\{\boldsymbol{c}_j\}_{j=1}^{I}$
- Alternating optimization in the spirit of the iterative Lloyd algorithm used in both steps
  - ➢ Optimize partition for fixed codevectors
  - ➢ Optimize codevectors for fixed partition



Pre-Quantization Optimization

- A reasonable choice is to minimize the MSE distortion induced by the measurement space discretization:

$$\tilde{D} \triangleq \sum_{v=1}^{V} \Pr(v)\mathbb{E}\left[\|\boldsymbol{y} - \boldsymbol{g}_v\|_2^2 | v\right]$$

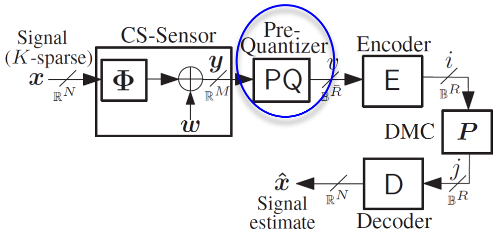
- Due to the MSE distortion criterion, the alternating optimization for PQ results in the traditional VQ with
  - Optimal partition (nearest-neighbor condition):

$$\mathcal{S}_v^* = \left\{\boldsymbol{y} : \|\boldsymbol{y} - \boldsymbol{g}_v\|_2^2 \leq \|\boldsymbol{y} - \boldsymbol{g}_{v'}\|_2^2, \forall v' \neq v\right\}, \ v \in \mathcal{V}$$

  - Optimal codepoints (centroid condition):

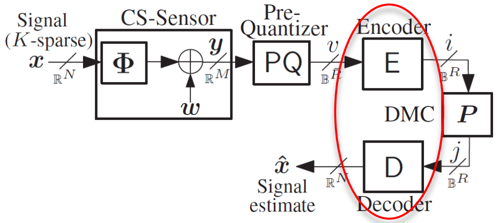
$$\boldsymbol{g}_v^* := \mathbb{E}[\boldsymbol{y}|\boldsymbol{y} \in \mathcal{S}_v] = \frac{1}{\Pr(v)} \int_{\mathcal{S}_v} \boldsymbol{y} f(\boldsymbol{y}) \mathrm{d}\boldsymbol{y}, \ \ v \in \mathcal{V}$$

VQ training via the iterative Lloyd algorithm by successively applying the necessary optimality conditions



Joint Encoder-Decoder Optimization (1/3)

- Optimize E-D for a fixed PQ, i.e., fixed PQ cells $\{\mathcal{S}_v\}_{v\in\mathcal{V}}$
- Minimum mean-square error (MMSE) estimator of the source given noisy CS measurements: $\hat{\boldsymbol{x}}(\boldsymbol{y}) \triangleq \mathbb{E}[\boldsymbol{x}|\boldsymbol{y}] \in \mathbb{R}^N$
  - Closed-form solution available
  - Exponential complexity proportional to the number of supports $\binom{N}{K}$
- Alternating optimization of E and D
  - Practical training via the principles of the iterative Lloyd algorithm



Joint Encoder-Decoder Optimization (2/3)

- The optimization of E separates into *V* subproblems, namely

$$D = \sum_{v,i,j} \delta_\pi(v,i) p_{ij} \int_{\boldsymbol{y}} \Pr(v|\boldsymbol{y}) \mathbb{E}\big[d_j \big| v,i,j,\boldsymbol{y}\big] f(\boldsymbol{y}) \mathrm{d}\boldsymbol{y}$$

$$= \sum_{v,i,j} \delta_\pi(v,i) p_{ij} \int_{\mathcal{S}_v} \mathbb{E}\big[d_j \big| v,i,j,\boldsymbol{y}\big] f(\boldsymbol{y}) \mathrm{d}\boldsymbol{y}$$

$$= \sum_{v,i} \delta_\pi(v,i) \boxed{D(v,i)}$$

→ **Distortion when *v*:th cell is assigned to *i*:th encoding index**

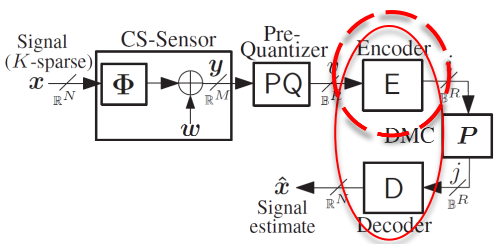
- Optimal encoder index:

$$\pi_v^* := \operatorname*{argmin}_{i\in\mathcal{I}} \left\{ \sum_j p_{ij} \int_{\mathcal{S}_v} \mathbb{E}\big[\|\boldsymbol{x} - \boldsymbol{c}_j\|_2^2 | v,i,j,\boldsymbol{y}\big] f(\boldsymbol{y}) \mathrm{d}\boldsymbol{y} \right\}$$

$$:= \operatorname*{argmin}_{i\in\mathcal{I}} \left\{ \sum_j p_{ij} \int_{\mathcal{S}_v} \Big(\|\boldsymbol{c}_j\|_2^2 - 2\boldsymbol{c}_j^{\mathrm{T}} \mathbb{E}[\boldsymbol{x}|\boldsymbol{y}] \Big) f(\boldsymbol{y}) \mathrm{d}\boldsymbol{y} \right\}$$

$$:= \operatorname*{argmin}_{i\in\mathcal{I}} \left\{ \sum_j p_{ij} \int_{\mathcal{S}_v} \Big(\|\boldsymbol{c}_j\|_2^2 - 2\boldsymbol{c}_j^{\mathrm{T}} \hat{\boldsymbol{x}}(\boldsymbol{y}) \Big) f(\boldsymbol{y}) \mathrm{d}\boldsymbol{y} \right\}$$

  - ➢ Weighted nearest-neighbor condition

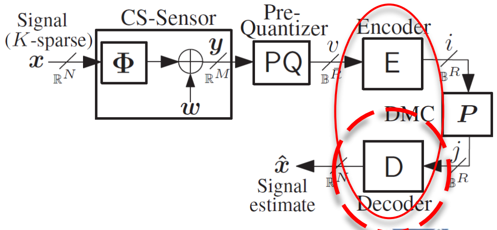


Joint Encoder-Decoder Optimization (3/3)

- The MSE marginalized over channel output indices as $\sum_j \Pr(j)\mathbb{E}\left[\|\boldsymbol{x}-\boldsymbol{c}_j\|_2^2 \middle| j\right]$ results in $\boldsymbol{c}_j^* := \operatorname{argmin}_{\boldsymbol{c}_j} \mathbb{E}\left[\|\boldsymbol{x}-\boldsymbol{c}_j\|_2^2 \middle| j\right]$
- The optimal codevector:

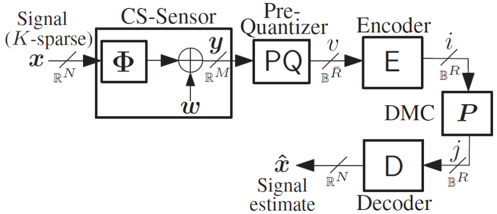
$$\begin{aligned}
\boldsymbol{c}_j^* &= \mathbb{E}\left[\boldsymbol{x} \middle| j\right] \\
&= \sum_v \sum_i \int_{\boldsymbol{y}} \Pr(v,i|j)\mathbb{E}\left[\boldsymbol{x} \middle| v,i,j,\boldsymbol{y}\right] f(\boldsymbol{y}|v,i,j)\mathrm{d}\boldsymbol{y} \\
&= \sum_{v,i} \int_{\boldsymbol{y}} \Pr(i|j)\Pr(v|i)\hat{\boldsymbol{x}}(\boldsymbol{y}) f(\boldsymbol{y}|v,i)\mathrm{d}\boldsymbol{y} \\
&= \sum_{v,i} \int_{\boldsymbol{y}} p_{ij} \frac{\Pr(i)\Pr(v|i)\Pr(v,i|\boldsymbol{y})}{\Pr(j)\Pr(v,i)} \hat{\boldsymbol{x}}(\boldsymbol{y}) f(\boldsymbol{y})\mathrm{d}\boldsymbol{y} \\
&= \frac{1}{\Pr(j)} \sum_{i=1}^{I} p_{ij} \sum_{v\in\pi^{-1}(i)} \int_{\mathcal{S}_v} \hat{\boldsymbol{x}}(\boldsymbol{y}) f(\boldsymbol{y})\mathrm{d}\boldsymbol{y}
\end{aligned}$$

➢ A weighted centroid condition

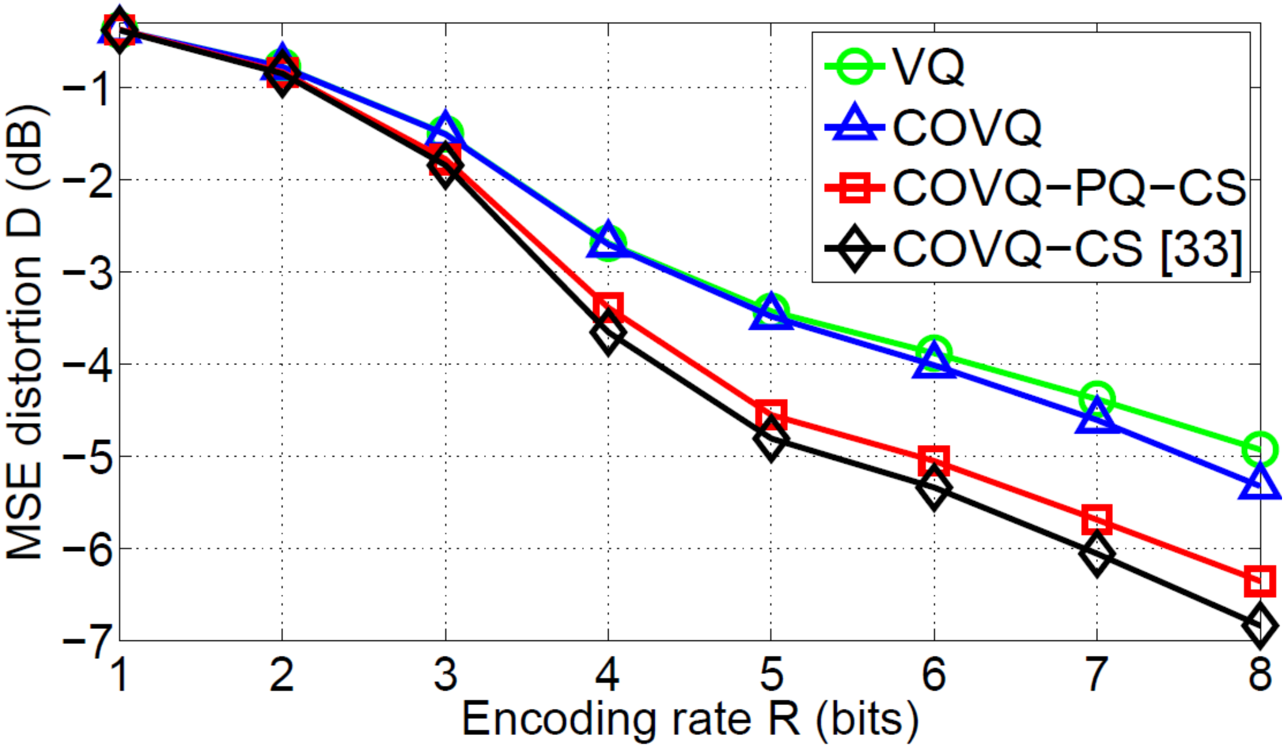


Algorithm Implementation

- Encoding of each measurement vector requires
  1. Table look-ups (V) at PQ: $v^* := \mathrm{argmin}_{v \in \mathcal{V}} \{\|\boldsymbol{y} - \boldsymbol{g}_v^*\|_2^2\}$ using codebook $\mathfrak{G}^* = \{\boldsymbol{g}_1^*, \ldots, \boldsymbol{g}_V^*\}$
  2. A simple reference at E: $i^* = \pi^*(v)$ using $\pi^* = \{\pi^*(1), \ldots, \pi^*(V)\}$
- Tolerable encoding complexity via adjusting the PQ rate
  - Higher PQ rate refines the approximation of measurement space → smaller end-to-end distortion
  - Lower PQ rate decreases the encoding complexity
- Decoding of each received index *j* requires
  - A simple reference $\mathrm{D}(j) = \boldsymbol{c}_j \in \mathfrak{C}$ using $\mathfrak{C}^* = \{\boldsymbol{c}_1^*, \ldots, \boldsymbol{c}_I^*\}$



Numerical Results (1/2)



Average MSE distortion for varying values of encoding rate R with $\sigma_w^2 = 0.008$ and $p_\epsilon = 0.99$ for $N = 7$, $M = 5$, and $K = 2$.

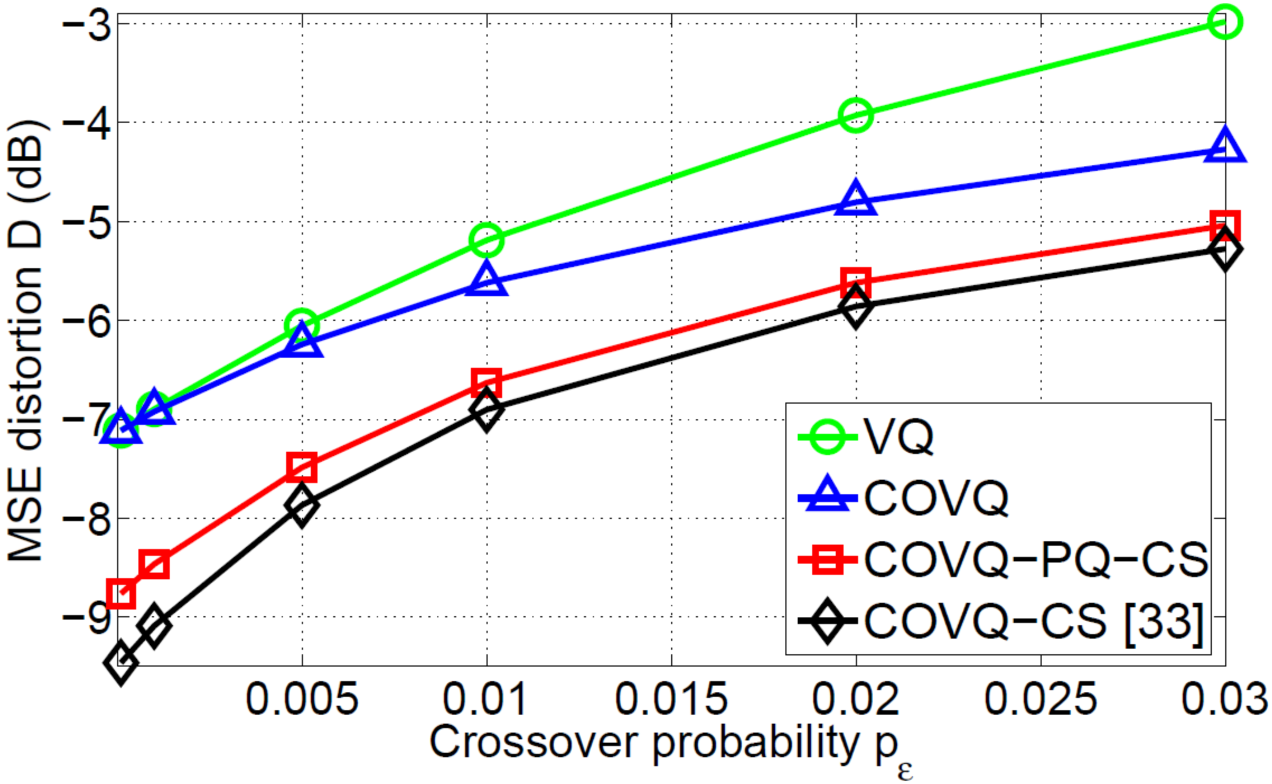
VQ: CS-blind & Channel-blind

COVQ: CS-blind & Channel-aware

COVQ-PQ-CS (Proposed): CS-aware & Channel-aware with controlled encoding complexity

COVQ-CS: CS-aware & channel-aware with "full" encoding complexity

Numerical Results (2/2)



Average MSE distortion for varying values of crossover probability p_ϵ with $R = 8$ and $\sigma_w^2 = 0$ for $N = 8$, $M = 6$, and $K = 2$.

VQ: CS-blind & Channel-blind

COVQ: CS-blind & Channel-aware

COVQ-PQ-CS (Proposed): CS-aware & Channel-aware with controlled encoding complexity

COVQ-CS: CS-aware & channel-aware with "full" encoding complexity

Conclusions

- We proposed a novel finite-rate communication method for efficient and robust acquisition of sparse sources over noisy channels with controlled encoding complexity
- The results illustrated that the sparse signal structure and the existence of channel noise, respectively, necessitates
  - ***CS-awareness***
  - ***Channel-awareness***

Thank You For Your Interest!

Questions?