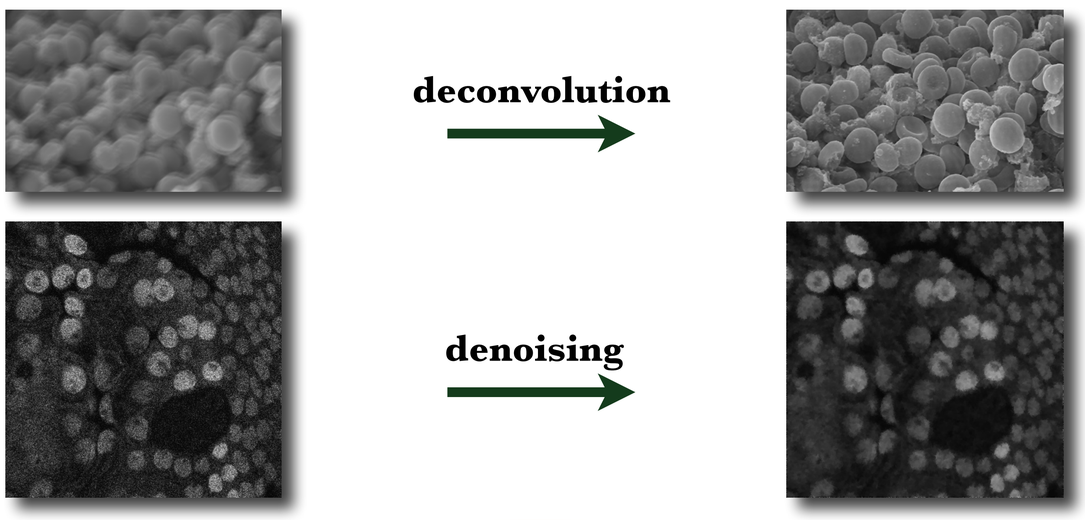
Bernstein Filter: a new solver for mean curvature regularized models

Yuanhao Gong
National University of Singapore

Introduction :

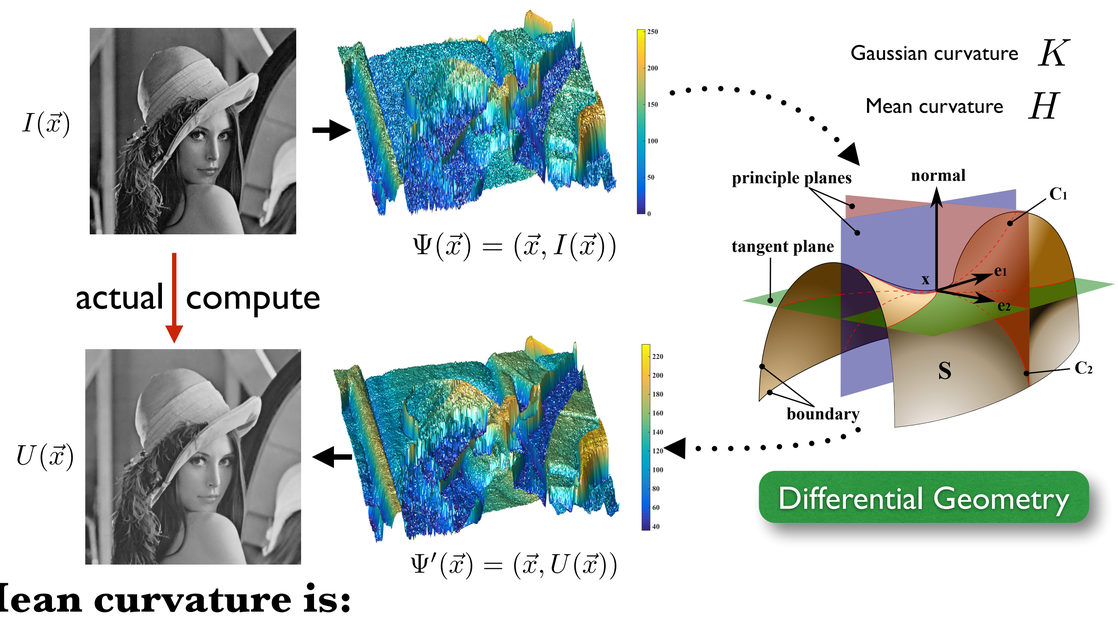
Signal processing problems are usually ill-posed, where a prior or regularization is needed.



Mean Curvature is a good prior

Mean Curvature Regularization:

Images are surfaces:



Mean curvature is:

$$H(U(\vec{x})) = \frac{(1+U_y^2)U_{xx} - 2U_xU_yU_{xy} + (1+U_x^2)U_{yy}}{2(1+U_x^2+U_y^2)^{\frac{3}{2}}}$$

Contribution:

We prove that:

mean curvature is a convex term

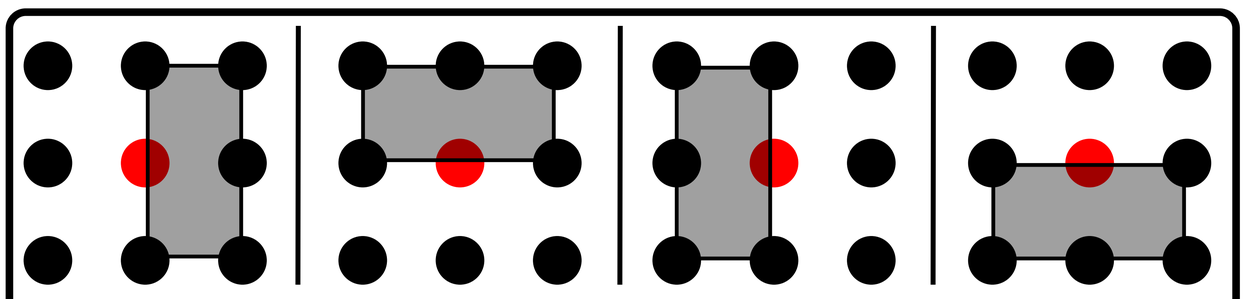
We show that:

Bernstein Filter is fast and effective

Convexity:

According to Bernstein Theorem, minimizing mean curvature is assuming that the signal is piece-wise linear. Based on this linearity, we can prove that mean curvature regularization term is convex.

Bernstein Filter:



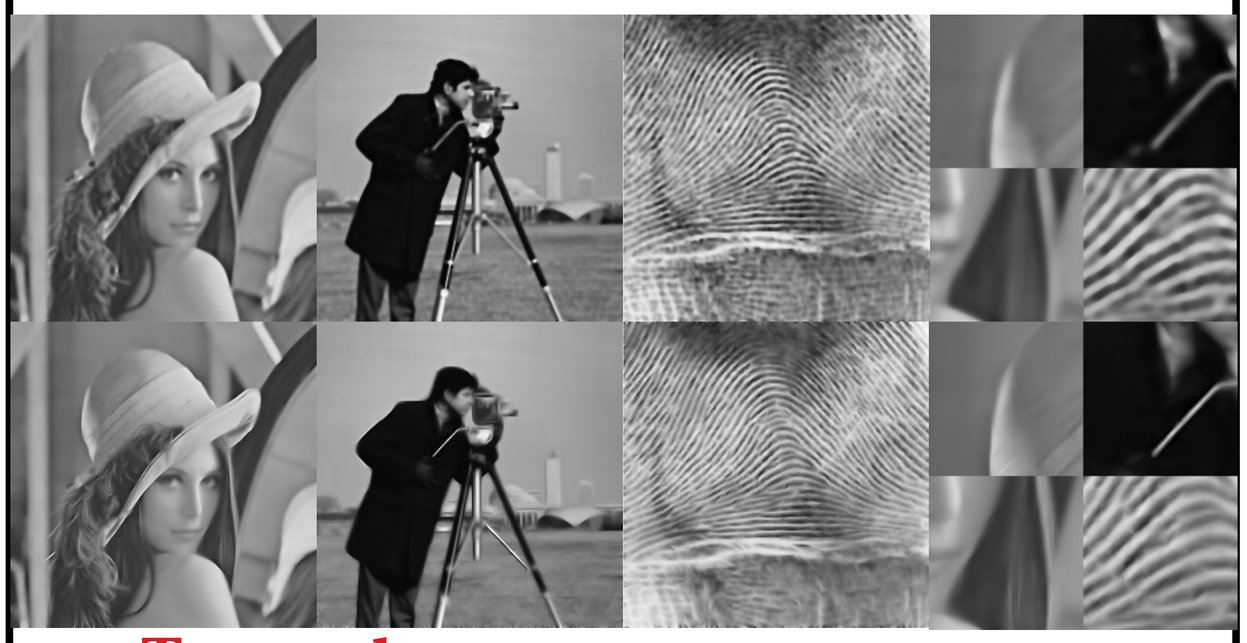
Impulsing the linearity on the four half-windows (above) by Least Square Regression, we get the Bernstein Filter:

Algorithm 1 Bernstein Filter

Require: IterationNum, $I(x_i, y_j)$
  $U^0(x_i, y_j) = I(x_i, y_j)$, $t = 0$
  while $t <$ IterationNum **do**
    for i=2:M-1,j=2:N-1 **do**
      $d_1 = \frac{1}{2}\left[U^t(x_{i-1}, y_j) + U^t(x_{i+1}, y_j)\right] - U^t(x_i, y_j)$
      $d_2 = \frac{1}{2}\left[U^t(x_i, y_{j-1}) + U^t(x_i, y_{j+1})\right] - U^t(x_i, y_j)$
      find d_m such that $|d_m| = \min_{k=1,2}\{|d_k|\}$
      $U^{t+1}(x_i, y_j) = U^t(x_i, y_j) + d_m$
    end for
    $t = t + 1$
  end while
Ensure: $U(x_i, y_j)$

Experiments:

Results from Multi Grid Solver(first row) and Bernstein Filter(second row) are similar because both solve the same variational model. However, our filter is much faster.



Two or three orders of magnitude FASTER!

solver (language)	Multigrid (Matlab)	Our filter (Matlab)	Our filter (C++)
Lena	183	1.1	0.025
Cameraman	648	1.1	0.025
Fingerprint	587	1.1	0.025

Table 1: time in seconds on 512×512 images. Our filter runs 30 iterations.

Contact and free! software:

gongyuanhao@gmail.com

https://github.com/YuanhaoGong/CurvatureFilter

Curvature Filter

Yuanhao Gong, Spectrally Regularized Surfaces, PhD Thesis, ETH Zurich, NO. 22616
Yuanhao Gong et al. *A natural scene gradient distribution prior and its application in light microscopy image processing, IEEE J-STSP, 2016.*
Yuanhao Gong et al., *Local Weighted Gaussian Curvature for Image Processing,* **ICIP2013, oral.**
Yuanhao Gong et al., *Coupled signed distance functions for implicit surface reconstruction*, **ISBI2012, Best Paper Award.**