Superimposed Pilots : An Alternative Pilot Structure to Mitigate Pilot Contamination in Massive MIMO

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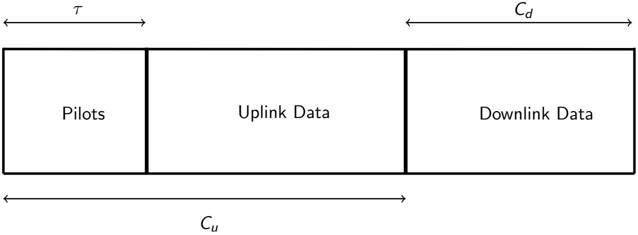
Outline

- Channel Estimation and Pilot Contamination in Massive MIMO
- Channel Estimation Using Superimposed Pilots
- Iterative Channel Estimation
- Simulation Results
- Some Comments on Performance

Channel Estimation in Massive MIMO

- Uplink time slot is partitioned for transmitting uplink pilots and data



- Orthogonal pilots are used for estimating the channel
- Pilots are shared between neighboring cells

Channel Estimation in Massive MIMO

- Received pilot sequence at the j^{th} BS (implicitly assumed in equations) with M antennas and K users per cell

$$\mathbf{Y} = \sum_{\ell=0}^{L-1} \mathbf{H}_\ell \mathbf{\Phi}^T + \mathbf{W}$$

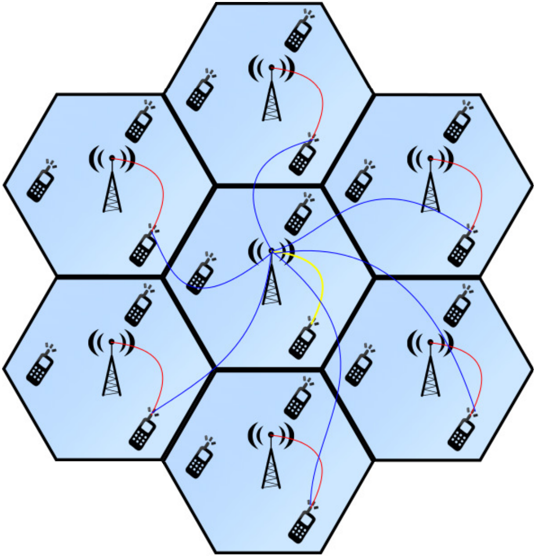
$\mathbf{H}_\ell$: Channel matrix between the j^{th} BS and users in the ℓ^{th} cell
$\mathbf{\Phi}$: $\tau \times K$ Matrix of orthogonal pilot sequences ($\mathbf{\Phi}^T \mathbf{\Phi}^* = \tau \mathbf{I}$)
$\mathbf{W}$: Additive white Gaussian noise at j^{th} BS

- LS channel estimate

$$\widehat{\mathbf{h}}_{j,m}^{\text{CP}} \triangleq \frac{1}{\tau}\mathbf{Y}\boldsymbol{\phi}_m^* = \mathbf{h}_{j,m} + \sum_{\substack{\ell=0 \\ \ell \neq j}}^{L-1} \mathbf{h}_{\ell,m} + \frac{1}{\tau}\mathbf{W}\boldsymbol{\phi}_m^*$$

$\mathbf{h}_{\ell,k}$: k^{th} column of $\mathbf{H}_\ell$, $\mathbf{h}_{\ell,k} \sim \mathcal{CN}(0, \beta_{\ell,k}\mathbf{I})$
$\boldsymbol{\phi}_m$: m^{th} column of $\mathbf{\Phi}$

Pilot Contamination



Pilot Contamination

- SINR in the uplink and downlink

$$\underset{M\to\infty}{\mathrm{SINR}_{j,m}^{\mathrm{CP-ul}}} = \underset{M\to\infty}{\mathrm{SINR}_{j,m}^{\mathrm{CP-dl}}} = \frac{\beta_{j,m}^2}{\sum_{\ell\neq j}\beta_{\ell,m}^2}$$

$\beta_{\ell,k}$: Large-scale path-loss coefficient between j^{th} BS and the k^{th} user in the ℓ^{th} cell

- Corresponding rates

$$\mathrm{R}_{j,m}^{\mathrm{CP-ul}} = \frac{(C_u-\tau)}{C}\log_2\left(1+\mathrm{SINR}_{j,m}^{\mathrm{CP-ul}}\right)$$
$$\mathrm{R}_{j,m}^{\mathrm{CP-dl}} = \frac{C_d}{C}\log_2\left(1+\mathrm{SINR}_{j,m}^{\mathrm{CP-dl}}\right)$$

C : Coherence time of the channel. $C = C_u + C_d$

Superimposed Pilots

- Pilots of length C_u are transmitted simultaneously along with data
- If $KL \leq C_u$, each user is assigned a unique pilot
- Received signal at j^{th} BS (implicitly assumed in equations) when employing superimposed pilots

$$\mathbf{Y} = \sum_{\ell=0}^{L-1} \sum_{k=0}^{K-1} \mathbf{h}_{\ell,k} \left(\rho_{\ell,k}\mathbf{x}_{\ell,k} + \lambda_{\ell,k}\mathbf{p}_{\ell,k}\right)^T + \mathbf{W}$$

$\rho^2_{\ell,k}$ and $\lambda^2_{\ell,k}$: fractions of the transmit power reserved for pilot and data, respectively

$\mathbf{p}_{\ell,k}$: $C_u \times 1$ orthogonal pilot transmitted by the k^{th} user of the ℓ^{th} cell

- The total transmitted power is $\mu_{\ell,k} = \lambda^2_{\ell,k} + \rho^2_{\ell,k}$

Superimposed Pilots

- The least squares estimate of the channel

$$\begin{aligned}\widehat{\mathbf{h}}_{j,\ell,k} &= \mathbf{Y}\left(\lambda_{\ell,k}^2\,\mathbf{p}_{\ell,k}^H\mathbf{p}_{\ell,k}\right)^{-1}\lambda_{\ell,k}\mathbf{p}_{\ell,k}^* \\ &= \mathbf{h}_{\ell,k} + \frac{1}{C_u\lambda_{\ell,k}}\sum_{n=0}^{L-1}\sum_{p=0}^{K-1}\rho_{n,p}\mathbf{h}_{n,p}\mathbf{x}_{n,p}^T\mathbf{p}_{\ell,k}^* + \frac{1}{C_u\lambda_{n,p}}\mathbf{W}\mathbf{p}_{\ell,k}^*\end{aligned}$$

- The data, estimated using a matched filter

$$\begin{aligned}\widetilde{\mathbf{x}}_{j,m}^T &= \frac{1}{M\rho_{j,m}\beta_{j,m}}\widehat{\mathbf{h}}_{j,m}^H\left(\mathbf{Y} - \lambda_{j,m}\widehat{\mathbf{h}}_{j,m}\mathbf{p}_{j,m}^T\right) \\ \widehat{\mathbf{x}}_{j,m} &= \eta\left(\widetilde{\mathbf{x}}_{j,m}\right)\end{aligned}$$

$\eta(\cdot)$: element-by-element hard-decision function that replaces each element of the input vector with the constellation point that is closest in Euclidean distance to that element

Performance

- The MSE of the superimposed pilot based channel estimate

$$\mathrm{MSE}^{\mathrm{SP}}_{j,m} = \frac{1}{C_u \lambda^2_{j,m}} \left(\sum_{k=0}^{K-1} \rho^2_{j,k}\beta_{j,k} + \sum_{\substack{\ell=0 \\ \ell \neq j}}^{L-1} \sum_{k=0}^{K-1} \rho^2_{\ell,k}\beta_{\ell,k} \right) + \frac{\sigma^2}{C_u \lambda^2_{j,m}}$$

- The SINR after MF

$$\mathrm{SINR}^{\mathrm{SP-ul}}_{\substack{j,m \\ M\to\infty}} = \frac{\beta^2_{j,m}}{\frac{1}{C_u \lambda^2_{j,m}\rho^2_{j,m}} \sum_{\ell=0}^{L-1} \sum_{k=0}^{K-1} \rho^2_{\ell,k}\mu_{\ell,k}\beta^2_{\ell,k}}$$

- For comparison, recall the SINR of a system using conventional pilots

$$\mathrm{SINR}^{\mathrm{CP-ul}}_{\substack{j,m \\ M\to\infty}} = \frac{\beta^2_{j,m}}{\sum_{\ell \neq j} \beta^2_{\ell,m}}$$

Implications

$$\underset{M\to\infty}{\mathrm{SINR}_{j,m}^{\mathrm{SP-ul}}} = \frac{\beta_{j,m}^2}{\frac{1}{C_u \lambda_{j,m}^2 \rho_{j,m}^2} \sum_{\ell=0}^{L-1} \sum_{k=0}^{K-1} \rho_{\ell,k}^2 \mu_{\ell,k} \beta_{\ell,k}^2} \qquad \underset{M\to\infty}{\mathrm{SINR}_{j,m}^{\mathrm{CP-ul}}} = \frac{\beta_{j,m}^2}{\sum_{\ell\neq j} \beta_{\ell,m}^2}$$

- Power control is essential for a system with superimposed pilots
- There is a coherence time beyond which a system with superimposed pilots performs better than a system with conventional pilots, i.e., if $\beta_{j,m} = 1,\ \forall m,\ \beta_{\ell,m} = \beta,\ \forall \ell \neq j, m$, and $\rho_{j,m}^2 = \lambda_{j,m}^2, \forall\, j, m$, then

$$\underset{M\to\infty}{\mathrm{SINR}_{j,m}^{\mathrm{SP-ul}}} > \underset{M\to\infty}{\mathrm{SINR}_{j,m}^{\mathrm{CP-ul}}} \quad \text{if} \quad C_u > 2K\left(1 + \frac{1}{(L-1)\beta^2}\right)$$

- This dependence on C_u can be reduced by using an iterative data aided scheme.

Iterative Channel Estimation

- Replacing k, ℓ with a single index n such that $0 \leq n \leq N-1$, where $N \triangleq KL$, received signal at the j^{th} BS when employing superimposed pilots

$$\mathbf{Y} = \sum_{n=0}^{N-1} \mathbf{h}_n \left(\rho_n \mathbf{x}_n + \lambda_n \mathbf{p}_n\right)^T + \mathbf{W}$$

- Users are arranged in decreasing order of their path-loss coefficients, i.e., $\beta_0 > \beta_1 > \ldots > \beta_{N-1}$
- $\mathcal{U}_m^{(i)}$ is the set of users whose channel and data vectors are used in feedback to estimate channel of m^{th} user in i^{th} iteration

Iterative Channel Estimation

- Set $\widehat{\mathbf{h}}_m^{(0)} = \mathbf{0}, \ \forall \ m$ and $\widehat{\mathbf{x}}_m^{(0)} = \mathbf{0}, \ \forall \ m = 0, \ldots, N-1$
- Channel estimation step

$$\widehat{\mathbf{h}}_m^{(i)} = \frac{1}{C_u \lambda_m} \left[\mathbf{Y} - \sum_{\substack{n=0 \\ n \in \mathcal{U}_m^{(i)}}}^{m-1} \rho_n \widehat{\mathbf{h}}_n^{(i)} \left(\widehat{\mathbf{x}}_n^{(i)} \right)^T - \sum_{\substack{n=m \\ n \in \mathcal{U}_m^{(i)}}}^{N-1} \rho_n \widehat{\mathbf{h}}_n^{(i-1)} \left(\widehat{\mathbf{x}}_n^{(i-1)} \right)^T \right]$$

- Data estimation step

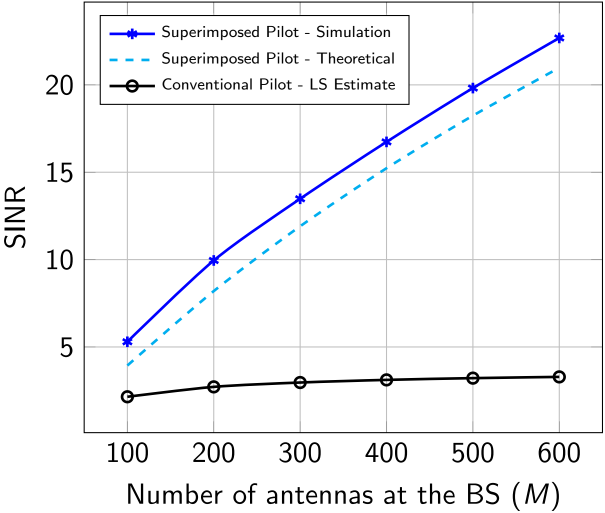
$$\left(\widetilde{\mathbf{x}}_m^{(i)} \right)^T = \frac{1}{M \rho_m \beta_m} \left(\widehat{\mathbf{h}}_m^{(i)} \right)^H \left(\mathbf{Y} - \lambda_m \widehat{\mathbf{h}}_m^{(i)} \mathbf{p}_m^T \right)$$

$$\widehat{\mathbf{x}}_m^{(i)} = \eta \left(\widetilde{\mathbf{x}}_m^{(i)} \right)$$

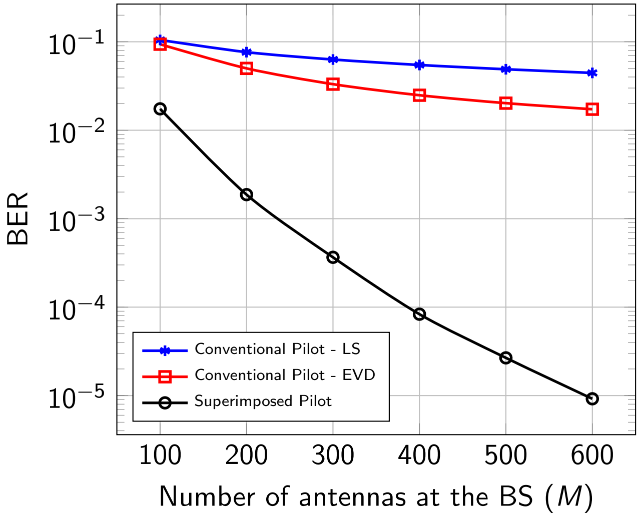
Simulation Setup

- $M = 300$ antennas
- $C_u = 100$ symbols
- 6 interfering cells ($L = 6$)
- $K = 5$ users per cell
- Cell Radius - 1km
- Users are equally spaced on a circle of radius 800m
- Path-loss exponent - 3

SINR Performance



BER Performance



Performance Benefits

- Superior uplink SINR performance in scenarios with high pilot contamination.
- As $M \to \infty$, optimal values of $\rho_{j,m} \to 0$ and $\lambda_{j,m} \to 1$, resulting in an interference-free downlink among the KL users
- Can augment systems with conventional pilots to increase the spectral efficiency

Caveats of Using Superimposed Pilots

- Number of users in all cells combined have to be less than uplink duration, i.e., $KL \leq C_u$
- Is superior to conventional pilots in uplink performance, in scenarios with high pilot contamination
- Uplink performance depends on $\frac{KL}{C}$
- $M \gg LK$ instead of $M \gg K$
- Power control is mandatory
- Coordination between cells is required for assigning pilots and for learning the path-loss coefficients $\beta_{j,\ell,k}$ of all users

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Thank You

Backup Slides

Choice of $\mathcal{U}_m^{(i)}$ - Assumptions

- $\mathbf{e}_m^{(i)} \triangleq \mathbf{x}_m - \widetilde{\mathbf{x}}_m^{(i)}$ is the error in the matched filtered data.
- $\Delta\mathbf{x}_m^{(i)} \triangleq \mathbf{x}_m - \widehat{\mathbf{x}}_m^{(i)}$ is the error vector after slicing.
- $\Delta\mathbf{h}_m^{(i)} \triangleq \mathbf{h}_m - \widehat{\mathbf{h}}_m^{(i)}$ is the error in the channel estimate.
- Assuming that the elements of $\mathbf{e}_m^{(i)}$ are i.i.d. circular complex-Gaussian random variables with zero mean and variance $\mathcal{I}_m^{(i)}$.
- In deriving $\mathcal{I}_m^{(i)}$, the following simplifications have been made to obtain a closed form expression:
  1. $\mathbf{e}_m^{(i)}$ is independent of $\mathbf{x}_k$ and $\mathbf{W}$, $\forall\ k, i$.
  2. $\Delta\mathbf{x}_m^{(i)}$ is independent of $\mathbf{x}_k$, $\mathbf{W}$, and $\mathbf{h}_k$, $\forall\ k, i$.
  3. $\Delta\mathbf{x}_m^{(i)}$ is independent of $\Delta\mathbf{x}_k^{(p)}$, $\forall p \neq i, m \neq k$ and the elements of $\Delta\mathbf{x}_m^{(i)}$ are i.i.d.
  4. $\Delta\mathbf{h}_m^{(i)}$ is independent of $\mathbf{x}_k$, $\mathbf{W}$, and $\Delta\mathbf{x}_k^{(p)}$, $\forall\ k, p$.

Choice of $\mathcal{U}_m^{(i)}$

- The user set $\mathcal{U}_m^{(i)}$ is selected as

$$\mathcal{U}_m^{(i)} = \arg \min_{\mathcal{U} \in \mathcal{P}(\mathcal{S})} \left\{ \mathcal{I}_m^{(i)} (\mathcal{U}) \right\}$$

$\mathcal{I}_m^{(i)}(\mathcal{U})$ is the interference power

$$\mathcal{I}_m^{(i)} (\mathcal{U}) \approx \frac{1}{\beta_m^2} \left(\frac{1}{M\rho_m^2} \sum_{\substack{n=0 \\ n \neq m}}^{N-1} \beta_n \beta_m + \frac{\sigma^2 \beta_m}{M\rho_m^2} + \frac{1}{M^2 \rho_m^2} \psi_m^{(i)}(\mathcal{U}) \right)$$

Choice of $\mathcal{U}_m^{(i)}$

- $\psi_m^{(0)} = 0$. For $i \geq 1$

$$\psi_m^{(i)} = \frac{M^2}{C_u \lambda_m^2} \left[\sum_{\substack{n=0 \\ n \notin \mathcal{U}_m^{(i)}}}^{N-1} \rho_n^2 \beta_n^2 + \sum_{\substack{n=0 \\ n \in \mathcal{U}_m^{(i)}}}^{m-1} \rho_n^2 \beta_n^2 \alpha_n^{(i)} + \sum_{\substack{n=m \\ n \in \mathcal{U}_m^{(i)}}}^{N-1} \rho_n^2 \beta_n^2 \alpha_n^{(i-1)} + o\left(\frac{N}{M}\right) \right]$$

$\alpha_k^{(i)} \triangleq \mathbb{E}\left\{ \left| \left[\Delta \mathbf{x}_k^{(i)} \right]_n \right|^2 \right\}$ is the variance of $\Delta \mathbf{x}_m^{(i)} \triangleq \mathbf{x}_m - \widehat{\mathbf{x}}_m^{(i)}$

- For a P-QAM constellation

$$\alpha_k^{(i)} = \begin{cases} \frac{24}{\sqrt{P}(\sqrt{P}+1)} Q\left(\sqrt{\frac{\frac{3}{(P-1)}}{\mathcal{I}_k^{(i)}}} \right), & i \geq 1 \\ 1, & i = 0 \end{cases}$$

Choice of $\mathcal{U}_m^{(i)}$

- The optimal set is given as

$$\mathcal{U}_m^{(i)} = \Big\{ k \in \mathbb{N} \,\Big|\, \alpha_k^{(i)} < \gamma_k^{(i)} \text{ when } k < m \\ \text{and } \alpha_k^{(i-1)} < \gamma_k^{(i-1)} \text{ when } k \geq m \Big\}$$

$$\gamma_k^{(i)} \triangleq \frac{\left\{ \beta_k^2 + \frac{1}{M} \sum_{n=0}^{N-1} \beta_k \beta_n - \frac{\psi_k^{(i)} \big|_{k \in \mathcal{U}_m^{(i)}}}{M^2} \right\}}{\left\{ \beta_k^2 + \frac{1}{M} \sum_{n=0}^{N-1} \beta_k \beta_n + \frac{\psi_k^{(i)} \big|_{k \in \mathcal{U}_m^{(i)}}}{M^2} \right\}}$$

- Alternatively, a fixed conservative threshold can be chosen as

$$\mathcal{U}_{\text{fixed}} = \Big\{ m \in \mathbb{N} \,\Big|\, \mathcal{I}_m^{(2)}(\{m\}) < \mathcal{I}_m^{(2)}(\varnothing) = \mathcal{I}_m^{(1)}(\varnothing) \Big\}$$