MULTIPLE SCATTERING EFFECTS ON THE LOCALIZATION OF TWO POINT SCATTERERS

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Introduction

- Multiple scattering is commonly ignored in sensing signal processing research because of small physical energy of the higher-order scattering components
- Multiple scattering in general can significantly increase the estimation precision of point scatterers [Shi and Nehorai, 2007]
- Under some scenarios, multiple scattering does not always lead to an improvement [Sentenac et al., 2007; Chen and Zhong, 2010]
- Identifying conditions under which multiple scattering is beneficial or detrimental to estimation is an open problem

Problem Statement

- Two point scatterers at unknown positions x_1 and x_2
- Known scattering coefficients τ_1 and τ_2 of the two scatterers
- N antennas at known positions $\alpha_1, \alpha_2, \ldots, \alpha_N$
- Known background Green function G

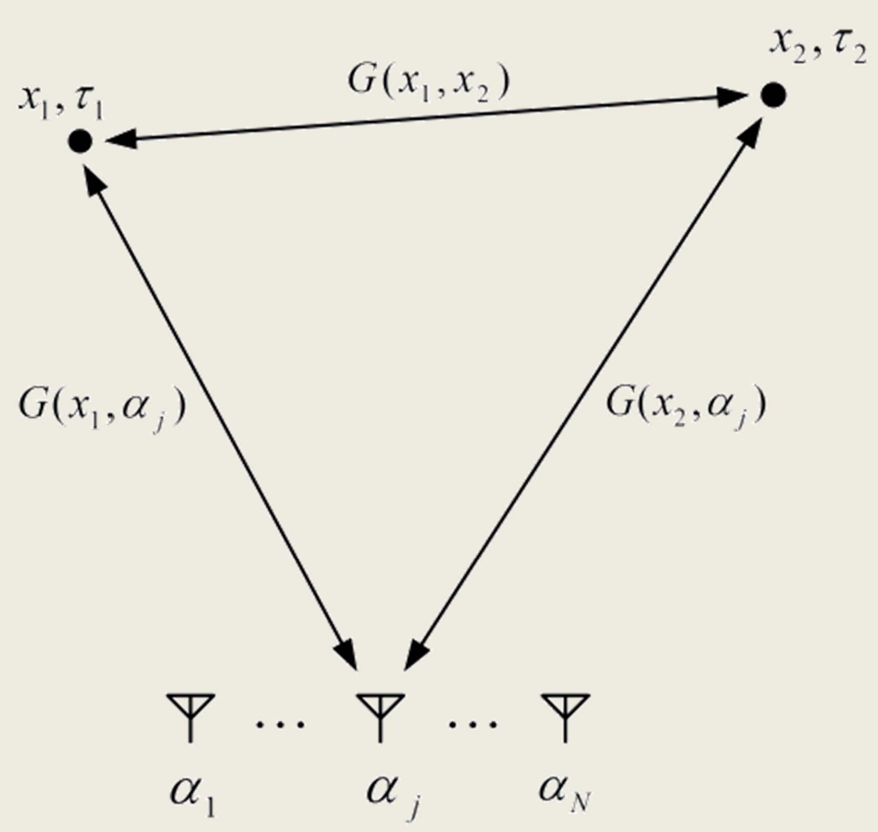

Figure 1. Illustration of the multistatic setup.

Multistatic model **with multiple scattering (Foldy-Lax Model)** can be formulated in the closed form as [Shi and Nehorai, 2005]

$$K_{\mathrm{FL}} = A(T^{-1} - S)^{-1} A^T$$

where $A = [\boldsymbol{g}(\boldsymbol{x}_1), \boldsymbol{g}(\boldsymbol{x}_2)]$, $T = \mathrm{diag}\{\tau_1, \tau_2\}$, $S = \begin{bmatrix} 0 & G(\boldsymbol{x}_1, \boldsymbol{x}_2) \\ G(\boldsymbol{x}_1, \boldsymbol{x}_2) & 0 \end{bmatrix}$

$$\boldsymbol{g}(\boldsymbol{x}_1) = [G(\boldsymbol{x}_1, \boldsymbol{\alpha}_1), G(\boldsymbol{x}_1, \boldsymbol{\alpha}_2), \ldots, G(\boldsymbol{x}_1, \boldsymbol{\alpha}_N)]^T$$
$$\boldsymbol{g}(\boldsymbol{x}_2) = [G(\boldsymbol{x}_2, \boldsymbol{\alpha}_1), G(\boldsymbol{x}_2, \boldsymbol{\alpha}_2), \ldots, G(\boldsymbol{x}_2, \boldsymbol{\alpha}_N)]^T .$$

Multistatic model **without multiple scattering (Born approximation Model)** is

$$K_{\mathrm{B}} = ATA^T = \tau_1 \boldsymbol{g}(\boldsymbol{x}_1)\boldsymbol{g}^T(\boldsymbol{x}_1) + \tau_2 \boldsymbol{g}(\boldsymbol{x}_2)\boldsymbol{g}^T(\boldsymbol{x}_2) .$$

Problem: **Under what conditions multiple scattering is beneficial or detrimental to the estimation of x_1 and x_2?**

Fisher Information Comparison

Assuming the noises are additive, independent, and identically distributed following a multivariate, complex, circularly symmetric Gaussian with variance σ^2, the Fisher information matrices with and without multiple scattering can be found as [Shi and Nehorai, 2007]

$$\mathcal{I}_{\mathrm{FL}}(\boldsymbol{x}) = \frac{2}{\sigma^2}\Re\{D_{\mathrm{FL}}^H D_{\mathrm{FL}}\},\quad D_{\mathrm{FL}} = A(T^{-1}-S)^{-1} \otimes \mathbf{1}_n^T \odot B - [A(T^{-1}-S)^{-1} \otimes A(T^{-1}-S)^{-1}]C + B \odot [A(T^{-1}-S)^{-1} \otimes \mathbf{1}_n^T]$$

$$\mathcal{I}_{\mathrm{B}}(\boldsymbol{x}) = \frac{2}{\sigma^2}\Re\{D_{\mathrm{B}}^H D_{\mathrm{B}}\},\quad D_{\mathrm{B}} = AT \otimes \mathbf{1}_n^T \odot B + B \odot (AT \otimes \mathbf{1}_n^T)$$

where $B = [\boldsymbol{b}(\boldsymbol{x}_1), \boldsymbol{b}(\boldsymbol{x}_2)]$, $\boldsymbol{b}(\boldsymbol{x}_m) = \partial \boldsymbol{g}(\boldsymbol{x}_m)/\partial \boldsymbol{x}_m^T$

$C = [\boldsymbol{c}^T(\boldsymbol{x}_1), \boldsymbol{c}^T(\boldsymbol{x}_2)]^T$, $\boldsymbol{c}(\boldsymbol{x}_m) = \partial \boldsymbol{s}(\boldsymbol{x}_m)/\partial \boldsymbol{x}^T$.

For homogeneous background in the three-dimensional space

$$G(\boldsymbol{x}_m, \boldsymbol{\alpha}_j) = -e^{ikR_{m,j}}/4\pi R_{m,j} ,\quad R_{m,j} = |\boldsymbol{x}_m - \boldsymbol{\alpha}_j|$$
$$G(\boldsymbol{x}_1, \boldsymbol{x}_2) = -e^{ikR}/4\pi R ,\quad R = |\boldsymbol{x}_1 - \boldsymbol{x}_2| .$$

Answer: **The following gives a set of sufficient conditions under which multiple scattering is beneficial to the estimation of x_1 and x_2.**

- Far-field condition: $R_{m,j} \gg 1$
- Far-field and monostatic condition: $\overrightarrow{\boldsymbol{x}_m - \boldsymbol{\alpha}_j} \approx \overrightarrow{\boldsymbol{x}_m - \boldsymbol{\alpha}}$
- Well-separated condition: $R \gg 1$
- Weak interaction condition: $\det\left((T^{-1} - S)^{-1}\right) \approx \tau_1\tau_2$
- Well-resolved condition: $\boldsymbol{g}^H(\boldsymbol{x}_1)\boldsymbol{g}(\boldsymbol{x}_2) \approx 0$

In this case, the difference between the two Fisher information matrices

$$\mathcal{I}_{\mathrm{FL}}(\boldsymbol{x}) - \mathcal{I}_{\mathrm{B}}(\boldsymbol{x}) \approx \frac{4}{\sigma^2}k^2|\tau_1|^2|\tau_2|^2|G(\boldsymbol{x}_1, \boldsymbol{x}_2)|^2||\boldsymbol{g}(\boldsymbol{x}_1)||_{\mathrm{F}}^2||\boldsymbol{g}(\boldsymbol{x}_2)||_{\mathrm{F}}^2(\boldsymbol{y}_1 - \boldsymbol{y}_2)(\boldsymbol{y}_1 - \boldsymbol{y}_2)^T$$

is semi-positive definitive, where

$\boldsymbol{y}_1 = (\overrightarrow{\boldsymbol{x}_1 - \boldsymbol{\alpha}}^T, \overrightarrow{\boldsymbol{x}_2 - \boldsymbol{\alpha}}^T)^T$ and $\boldsymbol{y}_2 = (\overrightarrow{\boldsymbol{x}_1 - \boldsymbol{x}_2}^T, \overrightarrow{\boldsymbol{x}_2 - \boldsymbol{x}_1}^T)^T$.

Numerical Example

Consider a two-dimensional multistatic setup with a uniform linear array located between (-5,0) and (5,0) with a spacing of 0.5 between adjacent elements (unit in wavelength). The Green function is a far-field approximation of the zero-order Hankel function of the first kind. The two scatterers are located on the line y = 40 and are symmetric about the y-axis.

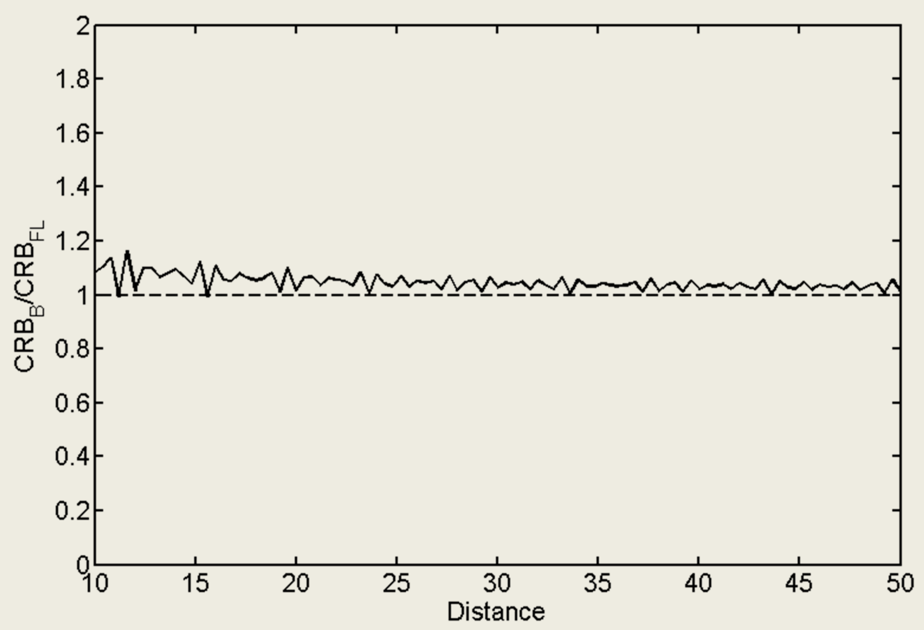

Figure 2. tr $\mathrm{CRB_B}(x)$ / tr $\mathrm{CRB_{FL}}(x)$ as a function of the distance between the two scatterers, $\tau_1 = \tau_2 = 1$.

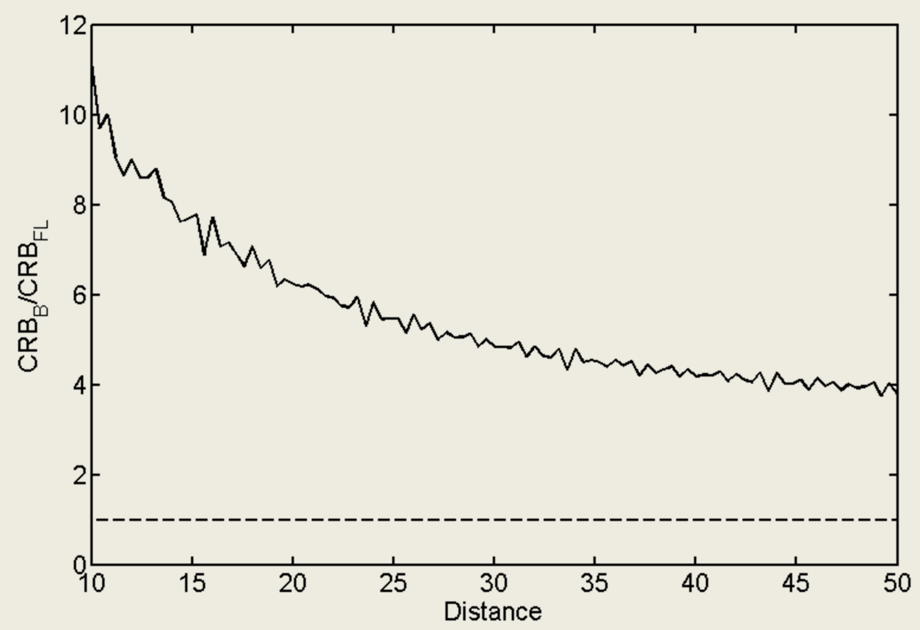

Figure 3. tr $\mathrm{CRB_B}(x)$ / tr $\mathrm{CRB_{FL}}(x)$ as a function of the distance between the two scatterers, , $\tau_1 = 1$, $\tau_2 = 10$.

Conclusion

- We compared analytically the Fisher information matrices for estimating locations of two point scatterers when multiple scattering exists and does not exist
- Multiple scattering improves the estimation of directions of arrival when the two scatterers are in far-field and well resolved
- When natural multiple scattering is weak, an artificial scatterer can be introduced to enhance the system performance

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