RADAR WAVEFORM SIDELOBE LEVEL OPTIMALITY AND SAMPLING

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Introduction

- Radar waveforms are often optimized to achieve minimal sidelobes
- Multistatic and MIMO radars require waveforms with low cross-correlation
- Typically oversampling is used in Rx
- Sampling rate can affect the ambiguity properties of the sampled waveforms in receivers

Contributions

- Theorem for narrowband waveforms with amplitude and phase coding
  - peak cross-correlation (PCC) level same regardless of oversampling rate
  - Peak sidelobe level (PSL) can increase

Problem Formulation

- Sample cross-ambiguity function

$$\chi_{ij}(\tau, F_D, T_s) = \left|T_s \sum_k s_i(kT_s)\sqrt{\gamma} s_j^*(\gamma k T_s + \tau) e^{j2\pi F_D k T_s}\right|^2$$

  - τ is the time delay
  - T_s is the sampling interval
  - F_D is the Doppler frequency
  - γ is the compression factor
- What is the impact of sampling rate change on the sample ambiguity function?
- Assumptions
  - narrowband waveforms
  - the symbol duration is an integer multiple of sampling interval T_s

$$\chi_{ij}(mT_s + \epsilon, f, T_s) = \chi_{ij}(mT_s, f, T_s)$$

⇒ Sample ambiguity constant for the duration of T_s

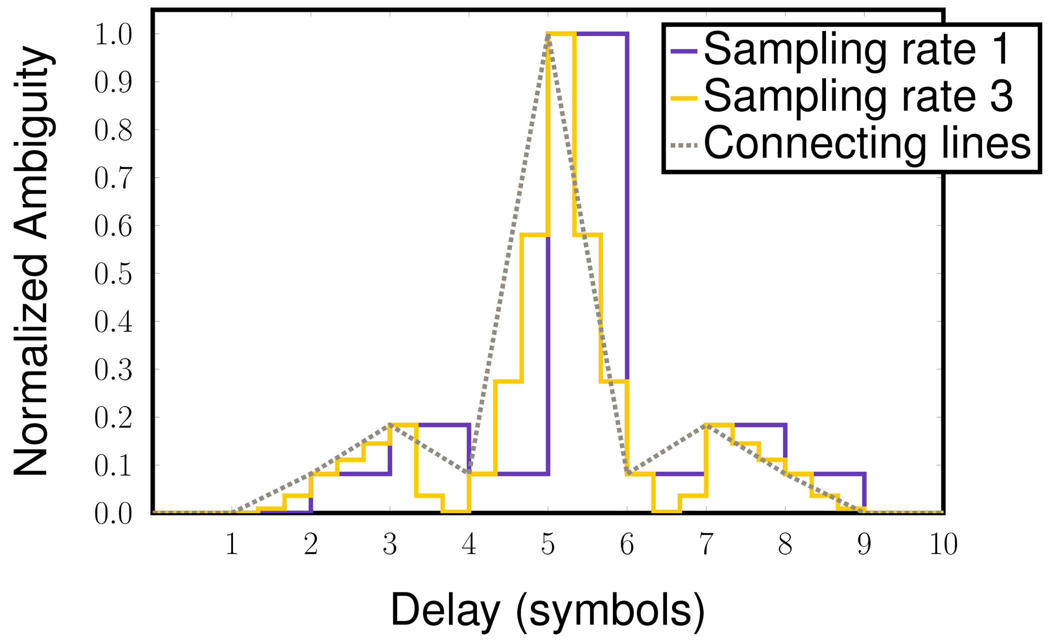
Theorem

- two narrowband sequences of rectangular pulses $s_i(t)$ and $s_j(t)$
- sampling rate $1/T_s$, an integer multiple of the symbol rate, M is a positive integer

- For $m = 0, 1, \ldots M$

$$\chi_{ij}\left(\left(n + \frac{m}{M}\right)T_s, F_D, \frac{1}{M}T_s\right) \le \left(1 - \frac{m}{M}\right)\chi_{ij}(nT_s, F_D, T_s) + \frac{m}{M}\chi_{ij}\left((n+1)T_s, F_D, T_s\right).$$

- Explanation
  - Take points at which the sample ambiguity of the sequence changes
  - Connect the points with lines
  - Take points at which the sample ambiguity of the *oversampled* sequence changes
  - Those points will be below or on the lines



Consequences

- Oversampling cannot increase the height of the peaks of the ambiguity function.
- Peak cross-correlation does not increase $\mathrm{PCC}_k(T_s/M) \le \mathrm{PCC}_k(T_s)$.
- Sidelobes can form in the auto-ambiguity, $\mathrm{PSL}_k(T_s/M) > \mathrm{PSL}_k(T_s)$ possible

Numerical Examples

- Single QPSK waveform with 12 symbols
- Optimize peak sidelobe level
- Sampling rates $M = 1, 2, 3$
- Optimization by exhaustive search
- Waveform 1 optimal for $M = 1$ etc.
- Peak sidelobe level of the optimized waveforms

Sequence	Oversampling Rate		
	1	2	3
Waveform 1	0.1497	0.4184	0.1870
Waveform 2	0.1688	0.1389	0.1389
Waveform 3	0.1880	0.2101	0.1254

- The optimal waveform clearly depends on the sampling rate
- Ambiguity function cuts along the delay axis:

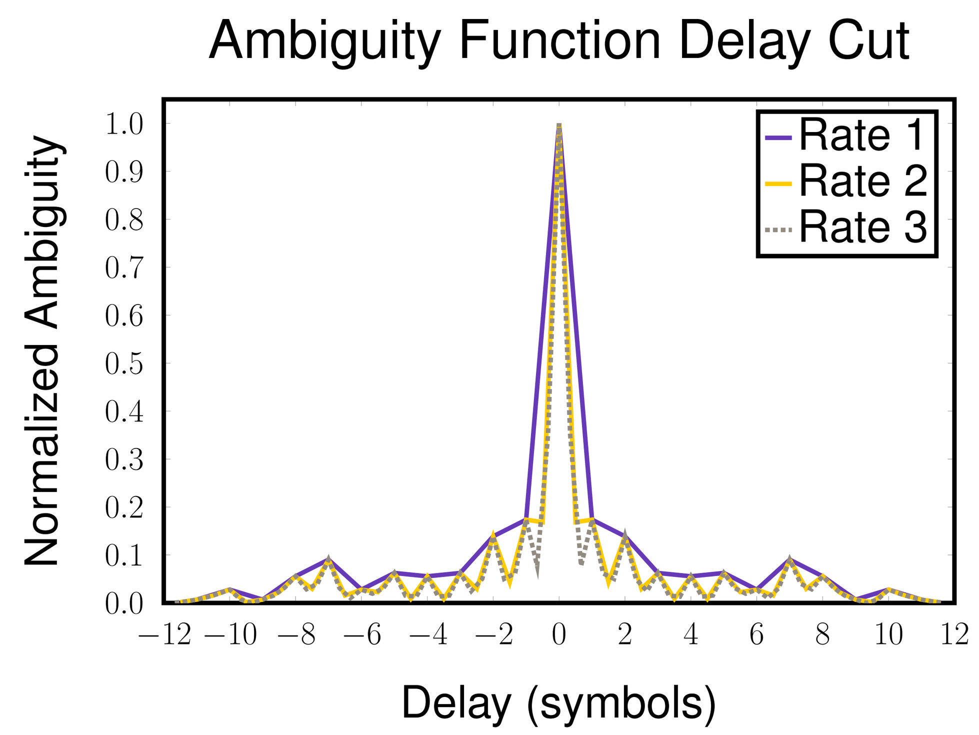


Figure 1: Ambiguity function cut of Waveform 1 at zero Doppler

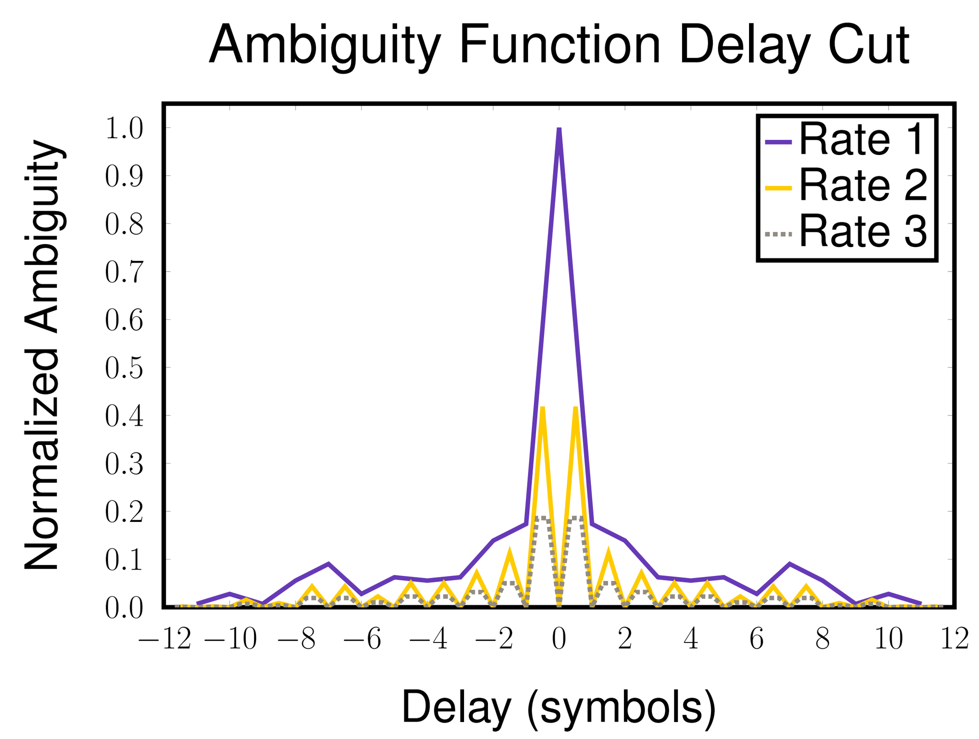


Figure 2: Ambiguity function cut of Waveform 1 at Doppler frequency equal to the symbol frequency. High sidelobes have developed for the oversampled waveforms.

- sidelobes have formed for nonzero Doppler
- PSL and PCC as functions of the sampling rate:

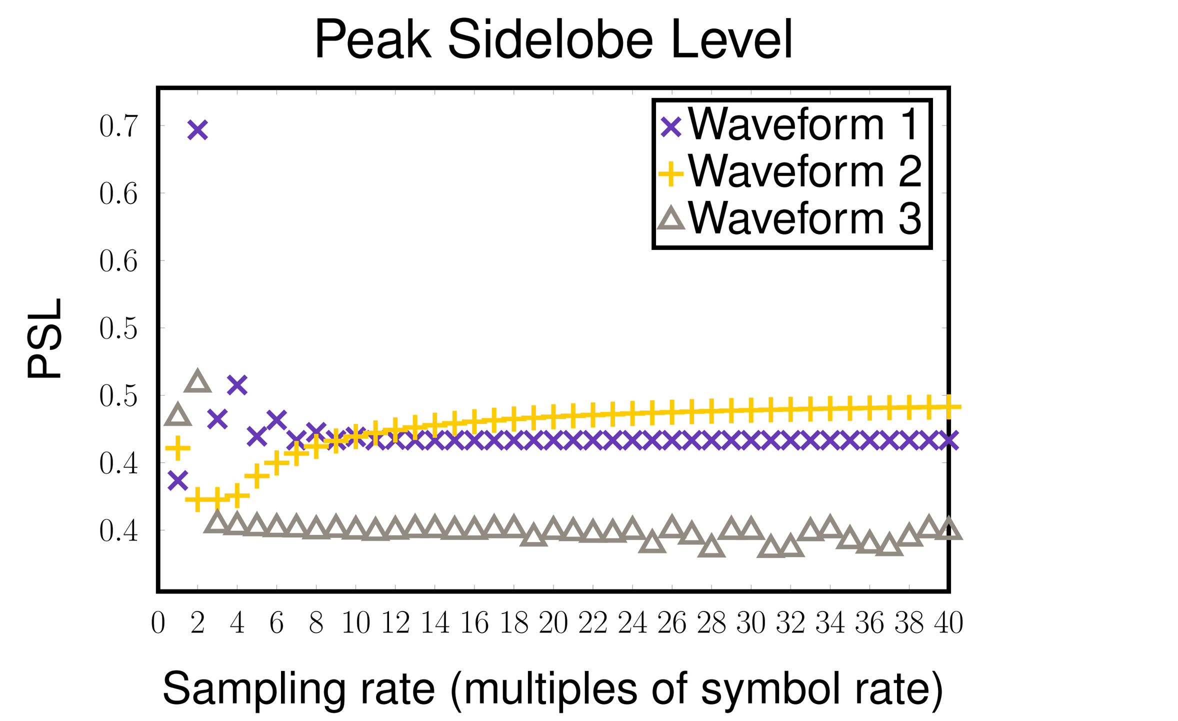


Figure 3: Peak sidelobe level of the waveforms for different sampling rates. The sampling rate is given as a multiple of the symbol rate. The PSL can increase or decrease as the sampling rate is increased.

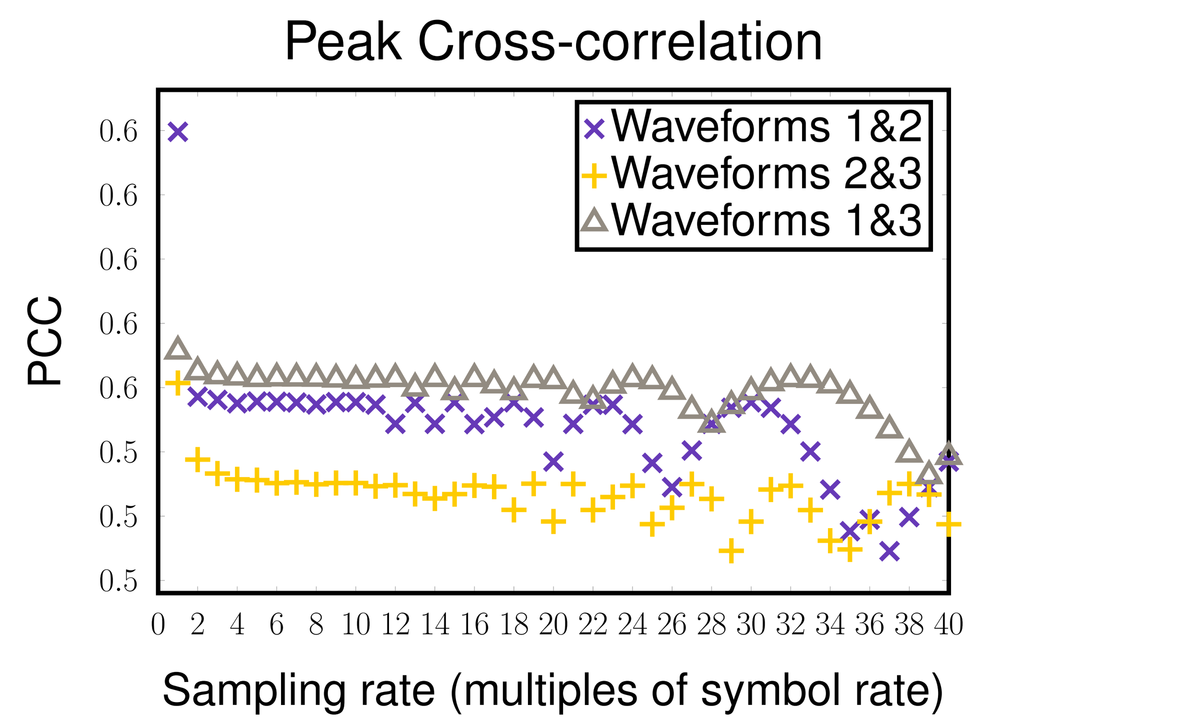


Figure 4: Peak cross-correlation of the waveforms for different sampling rates. The sampling rate is given as a multiple of the symbol rate. PCC does not exceed the level at the critical sampling.

- Both PSL and PCC vary, PSL can increase

Summary

- Digital signal processing and oversampling often used in radar receivers
- Sampling rate affects the ambiguity properties
  - Peak cross-correlation does not increase
  - Increase of peak sidelobe level possible!
- Sampling rate should be considered already during the waveform optimization