

## Introduction

### Motivation

- Parametric Bayesian spectral estimation outperforms the conventional Fourier Transform (FT) based methods at the expense of computational time.
- Increased frequency resolution in ultrasound signal analysis may reveal new diagnostic information.

### Research Question

- Can ultrasound signals be fully characterized (frequency, amplitude, phase, noise) based on Bayesian spectral estimation?

## Summary

### Background

- In ultrasound, the reflected echoes from point scatterers, are short sinusoidal signals with many closely spaced frequency components.
- The frequency resolution attained by the FT is limited by the small number of signal samples.
- Prior knowledge and parametric spectral estimation can be used to extract previously hidden signal characteristics.

### Objective

- Reconstruct real ultrasound signals based on an already existing rjMCMC algorithm.
- Reduce the calculation load the method introduces.
- Compare the initial and reconstructed signal and identify the benefits of the parametric estimation.

### Approach

- Apply a modified rjMCMC algorithm to the acquired ultrasound point scatter data.
- Extract a reasonable summary of the algorithm's output through clustering, outlier rejection and signal comparison.

### Results

- The correlation coefficient between the original and the (noise-free) reconstructed signal is measured to 0.987.
- The minimum difference between neighboring frequencies that are both identified by the parametric approach is 110 kHz, whereas the FT theoretical limit is 220 kHz.
- The burn-in period of a single algorithm realization is reduced by 20%

## Bayesian Inference

- Parametric estimation assumes a signal model and the received echoes are represented as a sum of sines and cosines in white Gaussian noise.
- A prior distribution is selected for each of the unknown model parameters:

Parameter	Prior distribution
Model order ( $k$ )	Truncated Poisson Distribution
Frequencies ( $\omega_k$ )	Uniform Distribution
Amplitudes ( $a_k$ )	Multivariate Normal Distribution
Noise variance ( $\sigma_k^2$ )	Jeffrey's uninformative prior
Hyperparameter of $k$ ( $\Lambda$ )	Gamma Distribution
Hyperparameter of $a_k$ ( $\delta^2$ )	Inverse Gamma Distribution

- The joint prior distribution is given by the product of all individual priors and the joint posterior distribution can be calculated as:

$$p(\Psi | y) = \frac{p(\Psi)p(y | \Psi)}{p(y)} \propto p(\Psi)p(y | \Psi), \quad (1)$$

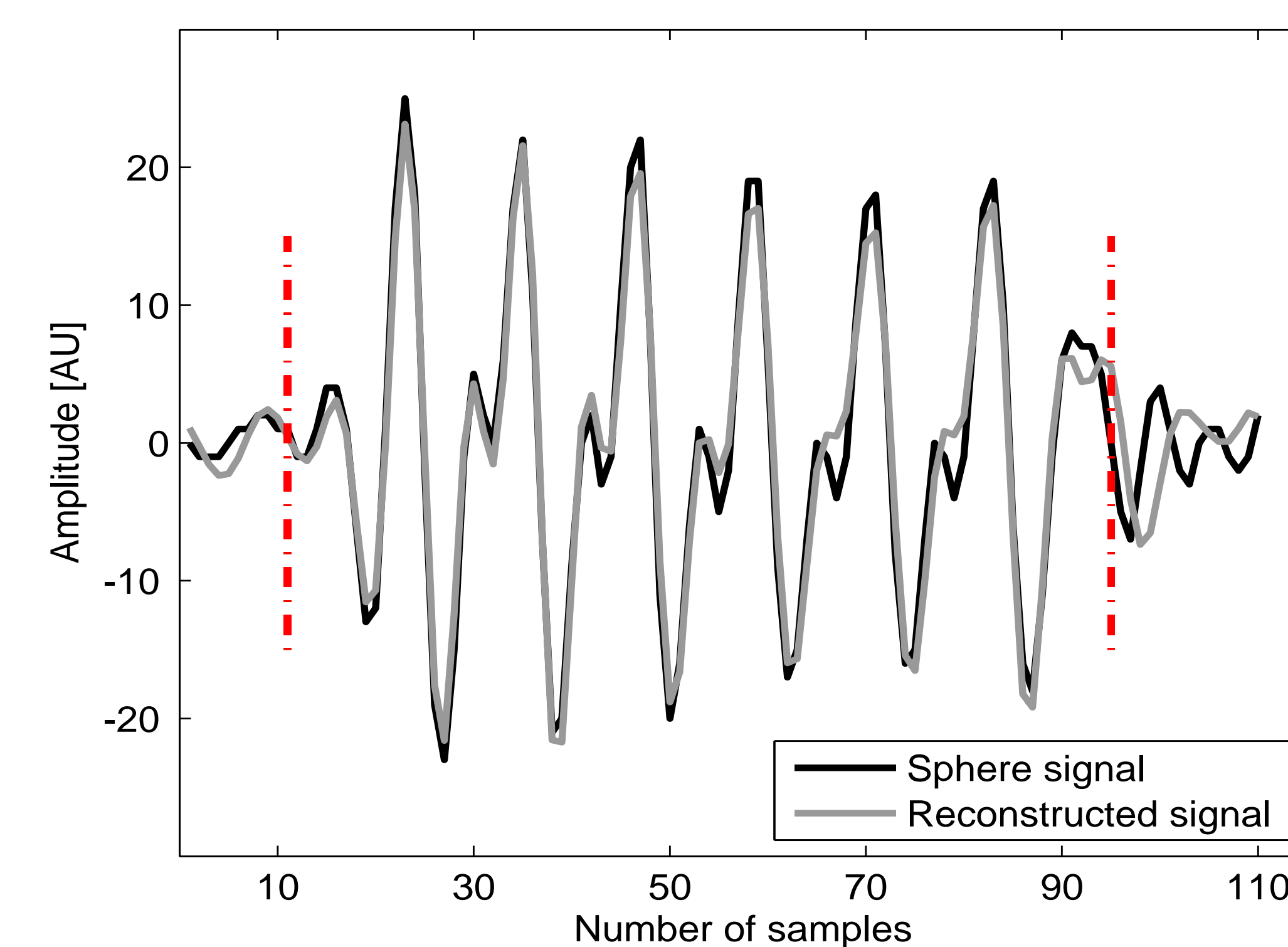
where  $\Psi = (k, \{\omega_k, a_k, \sigma_k^2\})$ ,  $p(\Psi)$  denotes the joint prior distribution and  $p(y | \Psi)$  the likelihood function.

## rjMCMC Algorithm

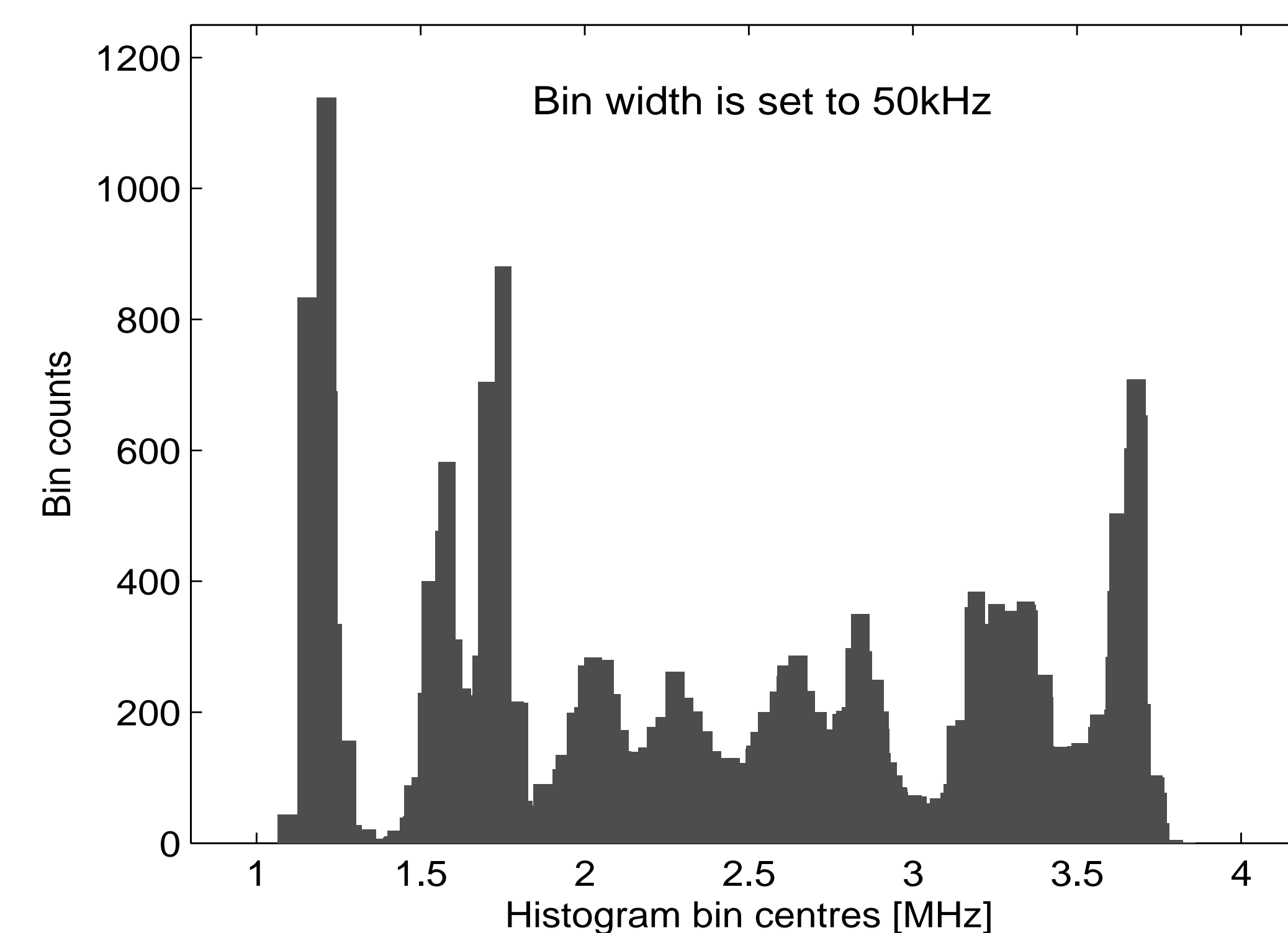
- 1: Insert the input signal to the algorithm
- 2: Employ the MUSIC method to provide an initial spectrum estimate:  $(k^{(0)}, \{\omega_k, a_k, \sigma_k^2\}^{(0)})$
- 3: **for**  $i = 1$  to numIteration **do**
- 4:   Sample  $\Lambda, \delta^2$
- 5:   Sample  $u$  from  $U_{(0,1)}$  (uniform distribution)
- 6:   Calculate the probabilities  $b_k$  and  $d_k$  based on  $\frac{p^{(k+1)}}{p^{(k)}}$  and  $\frac{p^{(k-1)}}{p^{(k)}}$  respectively
- 7:   **if**  $u \leq b_{k(i)}$  **then**
- 8:     Propose a new frequency randomly on  $[0, \pi)$  and accept it with a probability of  $\alpha_B$  (birth move)
- 9:   **else if**  $u \leq b_{k(i)} + d_{k(i)}$  **then**
- 10:     Remove an existing frequency randomly from  $\omega_k$  and accept it with a probability of  $\alpha_D$  (death move)
- 11:   **else**
- 12:     Update for all  $k$  frequencies according to a proposal distribution and accept it with a probability  $\alpha_U$  (update move)
- 13:   **end if**
- 14:   Sample nuisance parameters  $a_k$  and  $\sigma_k^2$
- 15: **end for**
- 16: Separate estimates based on  $k$
- 17: Remove outliers from  $a_k$
- 18: Extract mean for  $\omega_k, a_k$  and  $\sigma_k^2$
- 19: Reconstruct signal

## Measurement Setup

- A modified ultrasound transducer (Sonos5500 Philips Medical Systems) is used to acquire echo signals from solid copper spheres (SCSs).
- The experimental setup consists of a water tank and tubing that allows the drop of SCSs by gravity.
- The received response of a 6-cycle SCS signal, where the transmit frequency is 1.62 MHz and the sampling frequency is 20 MHz is shown below:

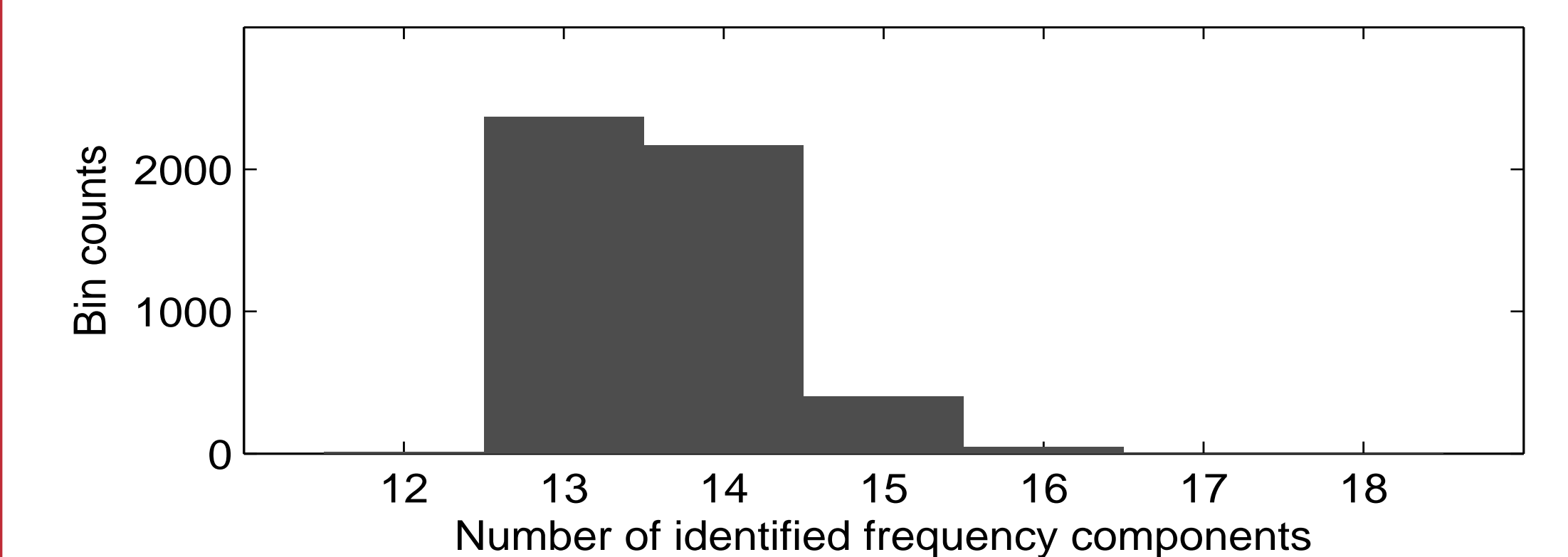


## Frequency Analysis

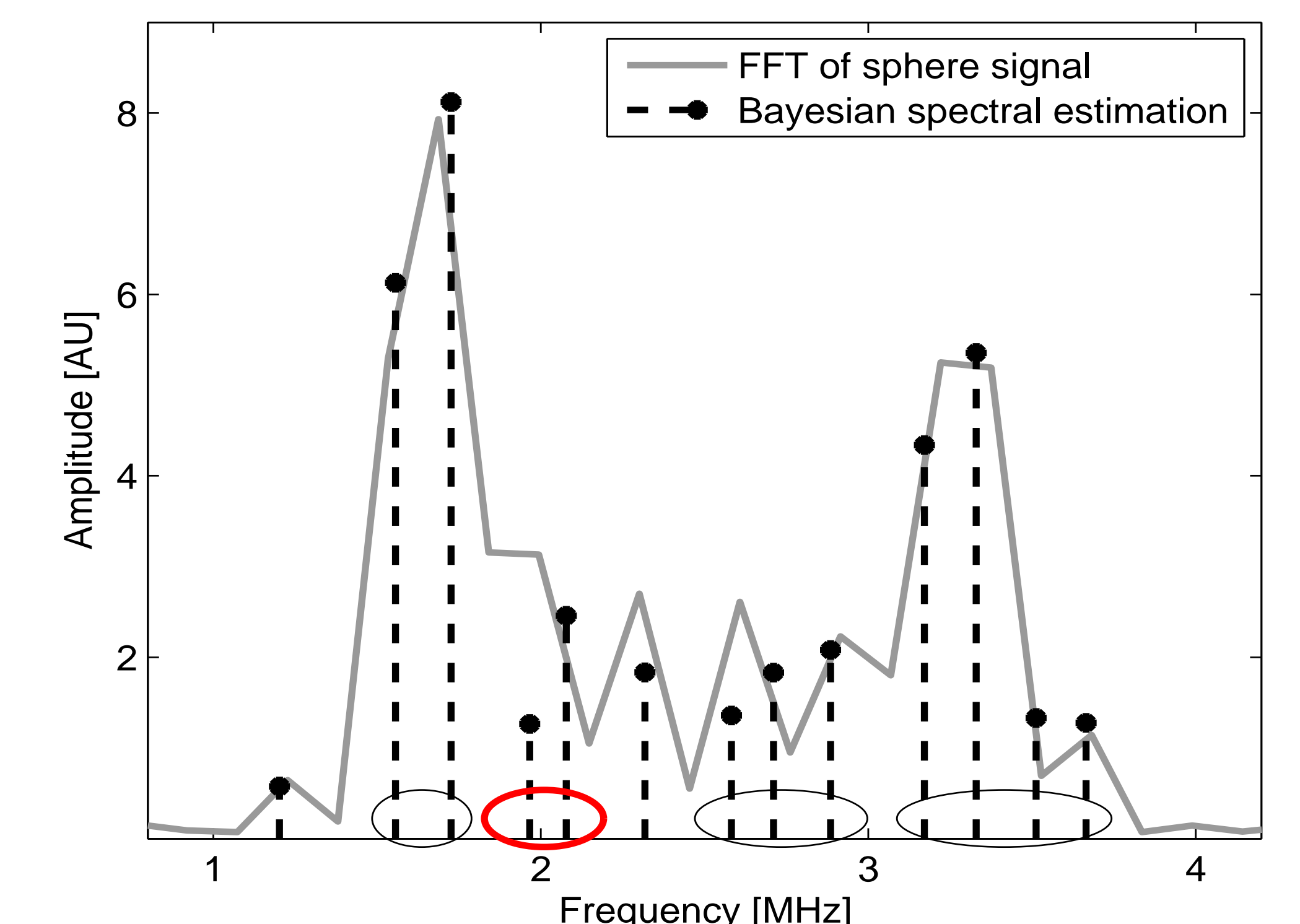


- Histogram displaying the cumulative distribution of the estimated frequency components from 5000 realizations.
- A 50 kHz bin width was used. Nine peaks can be clearly distinguished while four of them may correspond to two peaks merged to one.

## Comparison with the FFT



- Histogram showing number of identified frequency components for the 5000 realizations of the rjMCMC method.



- Comparison of the FFT of the initial sphere signal with the output of the parametric spectral estimation.
- The Bayesian method results in individual amplitude and frequency values instead of a spectrum.

## Conclusion

- The parametric spectral estimation provides a reconstructed signal with close resemblance to the echo signal.
- Minimum frequency separation is improved by a factor of two compared to FFT.
- The method may significantly improve the sensitivity and the specificity of existing diagnostic examinations.