A PARAMETER-FREE MAP IMAGE RECONSTRUCTION ALGORITHM FOR IMPULSE-BASED UWB GROUND PENETRATING RADAR

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Overview

- Motivation
- Detection Techniques for Landmines and IEDs
- Introduction to GPR
- Existing Methods for UWB GPR Reconstruction
- Overall Methodology
- GPR Linear Model
- Maximum a Posteriori Estimation
- Majorize-Minimize Optimization Technique
- Parameter-Free Map (PFM) Algorithm
- Conclusions and Acknowledgement

Motivation

- Landmines are self-contained explosive devices that detonate when triggered by a person or vehicle
- Factors that can trigger landmines
  - *Pressure*
  - *Movement*
  - *Sound*
  - *Vibration*
  - *Passage of time*
  - *Signals*
- Types of landmines
  - *Anti-tank*
  - *Anti-personnel*
- Different shapes, casings and materials

Motivation

- Currently more than 50-70 million uncleared landmines in at least 70 countries
- It will take about 1,100 years to remove all landmines at the current clearance rate
- Over 26,000 people are killed or maimed every year by landmines
- Over 1,000,000 casualties reported since 1980 [1]
- Half of all casualties in the Iraq and Afghanistan wars were attributed to land mines and improvised explosive devices (IEDs) [2]

[1] U.S. Department of State, "Hidden Killers: The Global Landmine Crisis", US Department of State Publication

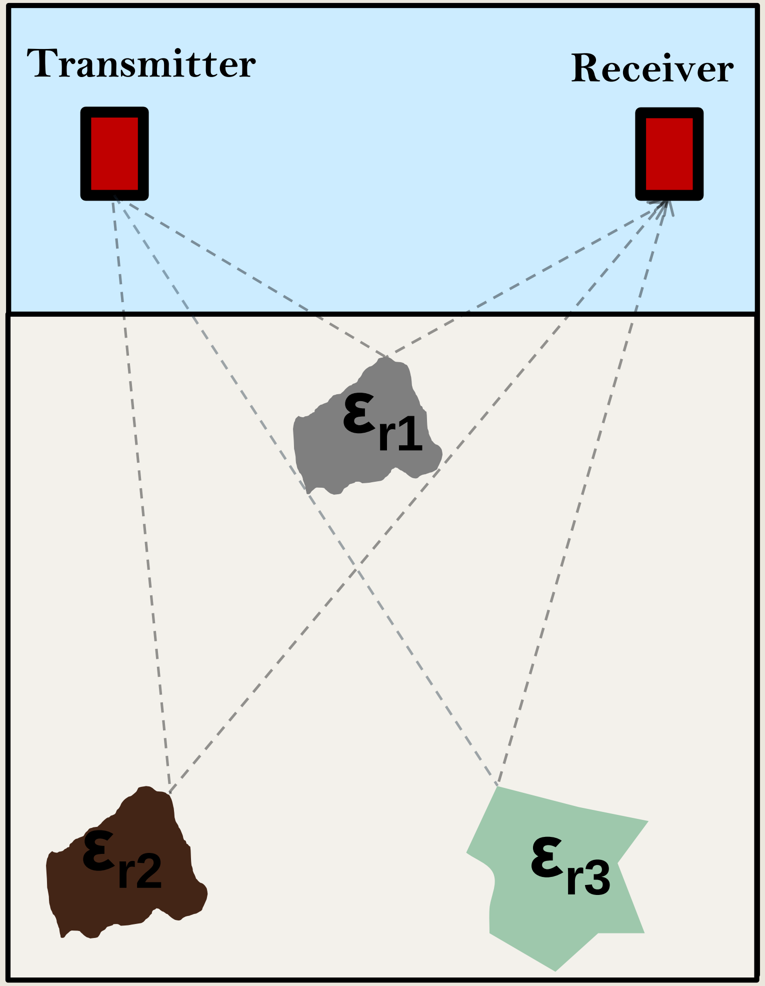
[2] C. Wilson, "Improvised explosive devices (IEDs) in Iraq and Afghanistan: effects and countermeasures". CRS Report for Congress, 2007

Landmines Detection Techniques

- Metal Detector
  - *Based on disturbances from time-varying magnetic field*
  - *Most popular but risky and limited to metallic detection*
- Acoustic/Seismic methods
  - *Based on vibration of materials subjected to sound waves*
  - *Unaffected by moisture and weather but limited by depth of penetration and interference*
- Biological Methods
  - *Use of trained dogs, rats, pigs, birds and bees*
  - *Training required, false alarms common, distraction inevitable*
- Mechanical Methods
  - *Includes prodding and use of mine clearing machines*
  - *Efficient but risky and costly*
- Electromagnetic Methods
  - *Use of microwaves, infrared, X-ray , GPR etc*
  - *Microwaves produce ambiguous results, infrared-based algorithms are not well developed, X-ray results are poor, GPR is promising*

Introduction to GPR

- Electromagnetic pulses are directed at the physical scene-of-interest (SOI)
- Backscatter occurs when the transmitted pulse encounters dielectric constant changes within an SOI
- Backscattered echoes capture the information needed to map the SOI onto the reconstructed imaging space
- Buried targets are expected to have higher dielectric constant values than surrounding material, such as soil and rocks
- GPR data are echoes recorded by receivers for each transmitted pulse



Existing Methods for UWB-GPR Reconstruction

- Delay-and-Sum (DAS)
  - *Fast and straightforward implementation*
  - *Produces images with poor resolution and large side lobes*
- Recursive Sidelobe Minimization (RSM)
  - *Reconstructs images with reduced clutter and has been applied in other SAR applications*
  - *Does not incorporate the a priori information that SOI is sparse*
- Least Absolute Selection and Shrinkage Operator (LASSO)
  - *No straightforward way to choose parameter*
- Sparsity Learning Iterative Method (SLIM)
  - *Involves matrix inversion that may be too computationally intensive for real applications*

Motivation for a Parameter-Free Algorithm

- Determining a suitable choice for the parameter of the prior probability density function is not straightforward

- Cross-validation is an off-line procedure that is time-consuming and sacrifices measurement for validation

- L-curve is an off-line procedure that is computationally expensive for large-size large scale estimation problems

Overall Methodology

- Use the MAP method to incorporate the a priori knowledge that the SOI contains few scatterers
- Use “integrate-out” approach to obtain a hyper-parameter-free prior probability density function
- Solve the resulting MAP objective functions using the majorize-minimize optimization technique
- Jointly estimate noise power

GPR Linear Model

Output of the j^{th} receiver at the i^{th} transmit position

$$s_{ij}(t) = \sum_{l=1}^{L} x_l \alpha_{ijl} p(t - \tau_{ijl}) + w(t)$$

- x_l: Unknown reflection coefficient at l^{th} terrain pixel
- $p(t)$: Transmitted pulse
- α_{ijl}: Round trip pulse attenuation
- τ_{ijl}: Round trip pulse travel time
- $w(t)$: Noise
- $\{\boldsymbol{y}_{ij}\}$: sampled GPR data where

$$\boldsymbol{y}_{ij} = \left[s_{ij}(0), s_{ij}(T), \ldots, s_{ij}((N-1)T)\right]^{\mathrm{T}}$$

GPR Linear Model (Cont.)

- Model:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w}$$

- Vector $\boldsymbol{y}$ contains sampled GPR data for I transmit positions and J receivers.
- Vector $\boldsymbol{x}$ contains L unknown reflection coefficients
- System matrix $\boldsymbol{A}$ is $(IJN) \times L$
- Problem Statement: Given $\mathbf{y}$ and $\boldsymbol{A}$, estimate $\boldsymbol{x}$

Maximum A Posteriori Estimation

- MAP estimate:

$$\hat{x}_{MAP} = arg \max_{\boldsymbol{x}} f_{\boldsymbol{X}|\boldsymbol{Y}}(\boldsymbol{x}|\boldsymbol{y})$$

- Assumptions:
  - *Noise is WGN with variance* σ^2
  - *Reflection coefficients are independent and identically distributed*

$$\hat{x}_{MAP} = arg \min_{\boldsymbol{x}} \frac{1}{2\sigma^2} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2^2 + \frac{K}{2} \log \sigma^2 - \log f_{\mathbf{X}}(x)$$

- Prior density function is Laplacian distribution

$$f(a;\lambda) \triangleq \frac{\lambda}{2} \exp(-\lambda|a|)$$

Maximum A Posteriori Estimation (Cont.)

- Integrate-out approach places a noninformative hyperprior over a parameter [3] to give a hyperparameter-free probability density function

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = \int_0^\infty f_{\boldsymbol{X}|\boldsymbol{\Lambda}}(\boldsymbol{x}|\lambda)\, f_{\boldsymbol{\Lambda}}(\lambda) d\lambda$$

- Conditional probability density function is the Laplacian distribution

$$f_{X|\Lambda}(x|\lambda) = \frac{\lambda}{2} \mathrm{e}^{-\lambda|x_l|}$$

- Hyperprior is the Jeffreys' prior for the Laplacian distribution

$$f_{\Lambda}(\lambda) \triangleq [I(\lambda)]^{\frac{1}{2}} \quad \text{(in terms of Fisher's information)}$$

$$I(\lambda) = -E_{\lambda}\left[\frac{\partial^2 \log f_{X|\Lambda}(\boldsymbol{x}|\lambda)}{\partial \lambda^2}\right] = \frac{1}{\lambda^2}$$

$$f_{\Lambda}(\lambda) = \frac{1}{\lambda} \quad (\lambda > 0)$$

[3] G.C. Cawley, N.L. Talbot, and M. Girolami, "Sparse multinomial logistic regression via bayesian L1 regularization ,``Bioinformatics, 10, pp. 209-216, 2007.

Maximum A Posteriori Estimation (Cont.)

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = \int_0^\infty f_{\boldsymbol{X}|\boldsymbol{\Lambda}}(\boldsymbol{x}|\lambda)\, f_{\boldsymbol{\Lambda}}(\lambda) d\lambda \;= \int_0^\infty \prod_{l=1}^{L} \left(\frac{\lambda}{2} \cdot e^{-\lambda |x_l|} \right) \cdot \frac{1}{\lambda}\, d\lambda$$

- Simplifying and using the gamma integral

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = \left[\frac{1}{2^L} \cdot \frac{\Gamma(L)}{\left(\sum_{l=1}^{L} |x_l|\right)^L} \right]$$

- Resulting MAP objective function

$$\phi(x, \sigma^2) = \frac{1}{2\,\sigma^2} \phi_{LS}(\boldsymbol{x}) + + \frac{K}{2} log(\sigma^2) + L \cdot \gamma\,(\boldsymbol{x})$$

where

$$\phi_{LS}(\boldsymbol{x}) \triangleq \left|\left|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\right|\right|_2^2$$

$$\gamma(\boldsymbol{x}) \triangleq log \left(\sum_{l=1}^{L} |x_l| \right)$$

Majorize-Minimize Optimization Technique

- $g(\boldsymbol{x}, \boldsymbol{y})$ is a majorizing function of f if
  a) $g(\boldsymbol{x}, \boldsymbol{y}) \geq f(\boldsymbol{x})$ for all $\boldsymbol{x}, \boldsymbol{y}$
  b) $g(\boldsymbol{x}, \boldsymbol{x}) = f(\boldsymbol{x})$ for all $\boldsymbol{x}$

- MM algorithm
  $$\boldsymbol{x}^{(n+1)} = \arg\min_{\boldsymbol{x}} g\left(\boldsymbol{x}, \boldsymbol{x}^{(n)}\right)$$

- Property of MM algorithm
  $$f\left(\boldsymbol{x}^{(n+1)}\right) \leq f\left(\boldsymbol{x}^{(n)}\right) \text{ for all } n$$

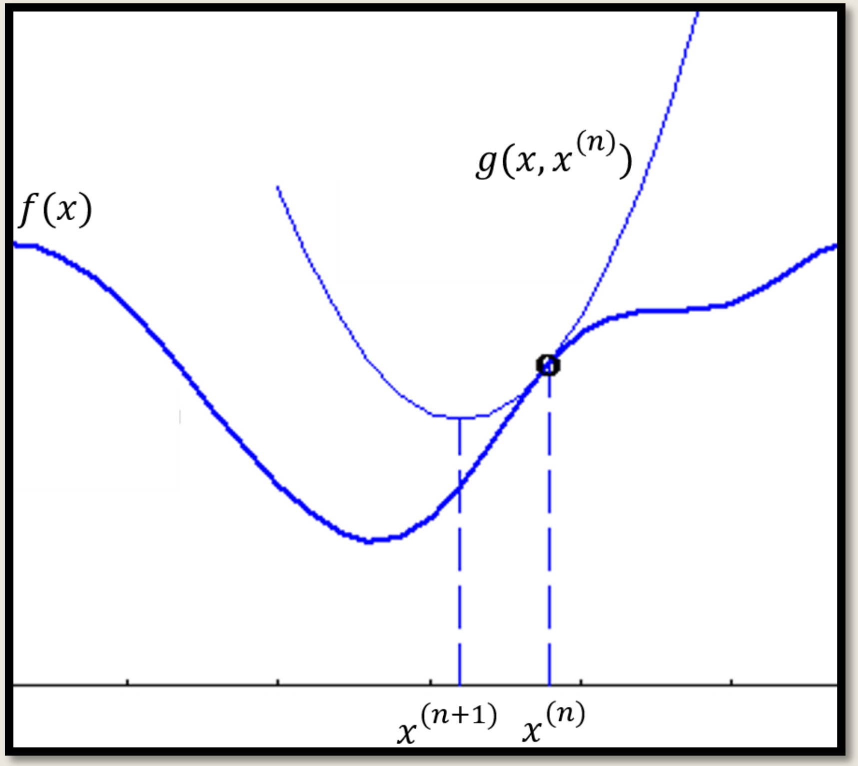


Illustration of MM Concept

PFM Algorithm

Overview:

Suppose Q is a majorizing function for the MAP objective function, ϕ, at the point $\boldsymbol{x}^{(m)}$. Then,

$$\boldsymbol{x}^{(m+1)} \triangleq arg \min_{\boldsymbol{x}} Q\left(\boldsymbol{x}, \sigma^{2(m)}; \boldsymbol{x}^{(m)}\right)$$

$$\sigma^{2(m+1)} \triangleq arg \min_{\sigma^2>0} \phi\left(\boldsymbol{x}^{(m+1)}, \sigma^2\right)$$
$$= \frac{1}{K}\left|\left|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}^{(m+1)}\right|\right|_2^2$$

PFM Algorithm (Cont.)

What is a suitable choice for Q?

- For any $\boldsymbol{x} \epsilon R^n$ and function $h(\boldsymbol{x})$,

$$log(h(\boldsymbol{x})) \leq log\left(h(\boldsymbol{x}^{(m)})\right) + \frac{h(\boldsymbol{x})}{h(\boldsymbol{x}^{(m)})} - 1$$

$$\gamma(x) = log\left(\sum_{l=1}^{L} |x_l|\right) \leq log\left(\sum_{l=1}^{L} \left|x_l^{(m)}\right|\right) + \frac{\sum_{l=1}^{L} |x_l|}{\sum_{l=1}^{L} \left|x_l^{(m)}\right|} - 1$$

- Using De Leeuw and Lange's [4] majorization function for $|a|$

$$q(\boldsymbol{x}; \boldsymbol{x}^{(m)}) = log\left(\sum_{l=1}^{L} \left|x_l^{(m)}\right|\right) + \frac{\sum_{l=1}^{L} \frac{\gamma'\left(x_l^{(m)}\right)}{2x_l^{m}}\left(x_l^2 - x_l^{(m)}\right) + \gamma(x_l^{(m)})}{\sum_{l=1}^{L} \left|x_l^{(m)}\right|} - 1$$

[4] J. de Leeuw and K. Lange, "Sharp Quadratic Majorization in One Dimension" Computational Statistics and Data Analysis, vol. 53 no.1 pp 2478 February 2004.

PFM Algorithm (Cont.)

- Majorizing function for $\phi_{LS}(\boldsymbol{x})$

$$\phi_{LS}(\boldsymbol{x}) = \sum_{k=1}^{K} y_k^2 - 2\sum_{k=1}^{K} y_k[\boldsymbol{Ax}]_k + \sum_{k=1}^{K} \left(\left[\boldsymbol{Ax}^{(m)}\right]_k\right)^2$$

- De Pierro [5] developed a majorizing function for $\left(\left[\boldsymbol{Ax}^{(m)}\right]_k\right)^2$

$$r_k\left(\boldsymbol{x};\boldsymbol{x}^{(m)}\right) = \sum_{l=1}^{L} c_{kl}\left(n_k A_{kl} x_l - n_k A_{kl} x_l^{(m)} + \left[\boldsymbol{Ax}^{(m)}\right]_k\right)^2$$

- Therefore,

$$q_{LS}\left(\boldsymbol{x};\boldsymbol{x}^{(m)}\right) = \sum_{k=1}^{K} y_k^2 - 2\sum_{k=1}^{K} y_k[\boldsymbol{Ax}]_k + \sum_{k=1}^{K} r_k\left(\boldsymbol{x};\boldsymbol{x}^{(m)}\right)$$

[5] A.R De Pierro , " A modified expectaction Maximization algorithm for penalized likelihood estimation in emission tomography", IEEE transactions, medical imagery, pp 132-137, 1995

PFM Algorithm (Cont.)

Majorizing function for ϕ:

$$Q\left(\boldsymbol{x};\boldsymbol{x}^{(m)}\right) = \frac{1}{2\sigma^2}\sum_{k=1}^{K}\left(y_k^2 - 2y_k[\boldsymbol{Ax}]_k + \sum_{l=1}^{L} c_{kl}\left(n_k A_{kl} x_l - n_k A_{kl} x_l^{(m)} + \left[\boldsymbol{Ax}^{(m)}\right]_k\right)^2\right) + \frac{k}{2}\log(\sigma^2) + L \cdot log\left(\sum_{l=1}^{L}\left|x_l^{(m)}\right|\right) + L \cdot \frac{\sum_{l=1}^{L} \frac{\gamma'\left(x_l^{(m)}\right)}{2x_l^{m}}\left(x_l^2 - x_l^{(m)}\right) + \gamma(x_l^{(m)})}{\sum_{l=1}^{L}\left|x_l^{(m)}\right|} - L$$

- Taking the derivative of the majorizing function Q with respect to x_l and setting it to zero yields the desired update for the l^{th} reflectance coefficient

PFM Algorithm (Cont.)

- Updates for reflection coefficients and noise power:

$$x_l^{(m+1)} = \frac{D_l^{(m)}\left(G_l^{(m)} + x_l^{(m)} H_l\right)}{D_l^{(m)} + L \cdot \sigma^{2(m)}}, \qquad l = 1, 2, \dots, L$$

$$H(l) = \sum_{k=1}^{K} n_k A_{kl}^2, \quad G^{(m)}(l) = \sum_{k=1}^{K} A_{kl}\left(y_k - \left[A x^{(m)}\right]_k\right)$$

$$D_l^{(m)} = \left|x_l^{(m)}\right| \sum_{l=1}^{L} \left|x_l^{(m)}\right|$$

$$\boldsymbol{\sigma}^{2(m+1)} = \frac{1}{K}\left\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}^{(m+1)}\right\|_2^2$$

PFM Algorithm in Practice

- $\gamma(\boldsymbol{x})$ is defined as

$$\gamma(\boldsymbol{x}) \triangleq log\left(\sum_{l=1}^{L} |x_l| + c\right)$$

- *C>0* insures the parameter-free MAP objective function has a minimizer

- *C* is chosen such that

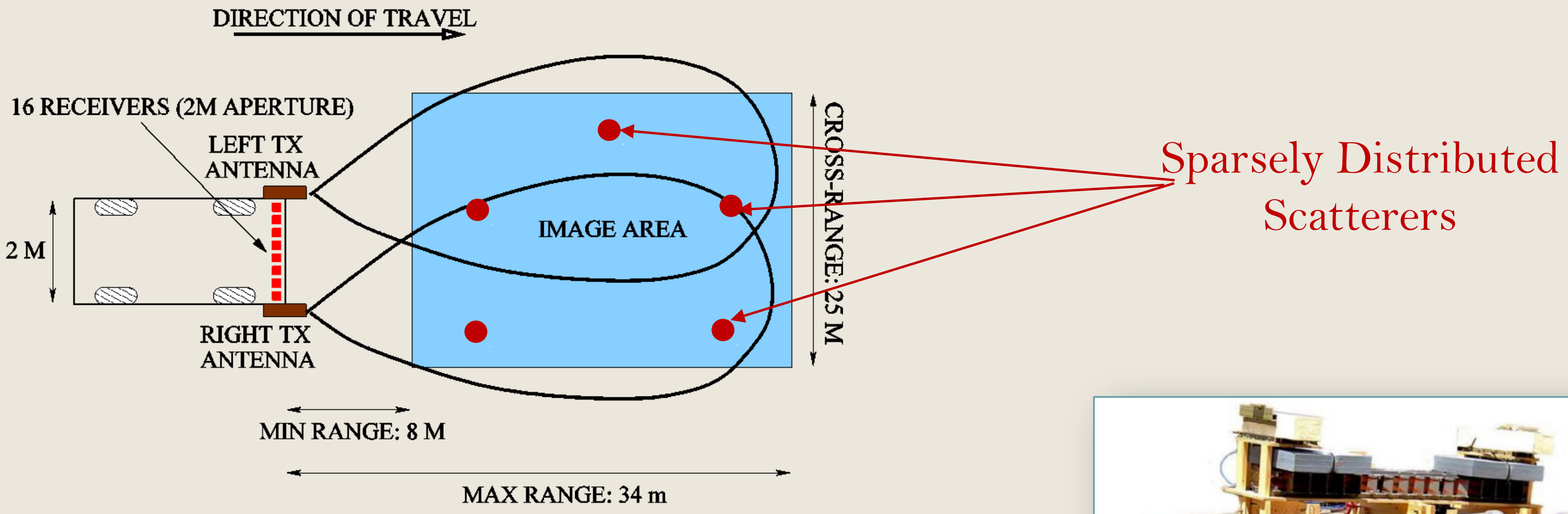
$$\phi(\hat{\boldsymbol{x}}) < \phi(\mathbf{0})$$

- DAS image is used as initial estimate

- Acceleration techniques for computing $H(l)$ and $G^{(m)}(l)$ from previous work [6] are used for a fast memory-efficient implementation.

[6] M. Ndoye and J.M.M. Anderson, "An MM-Based Algorithm for L1"-Regularized Least-Squares Estimation with an application to Ground Penetrating Radar Image Reconstruction", IEEE Trans. Image Process.

Simulation Results: ARL SIRE System



- A monocycle UWB pulse (300 -3000 MHz)
- 2 transmit antennas
- 1 active transmit antenna per shot
- 16 receive antennas

ARL SIRE System Prototype

Simulation Results: Real Data

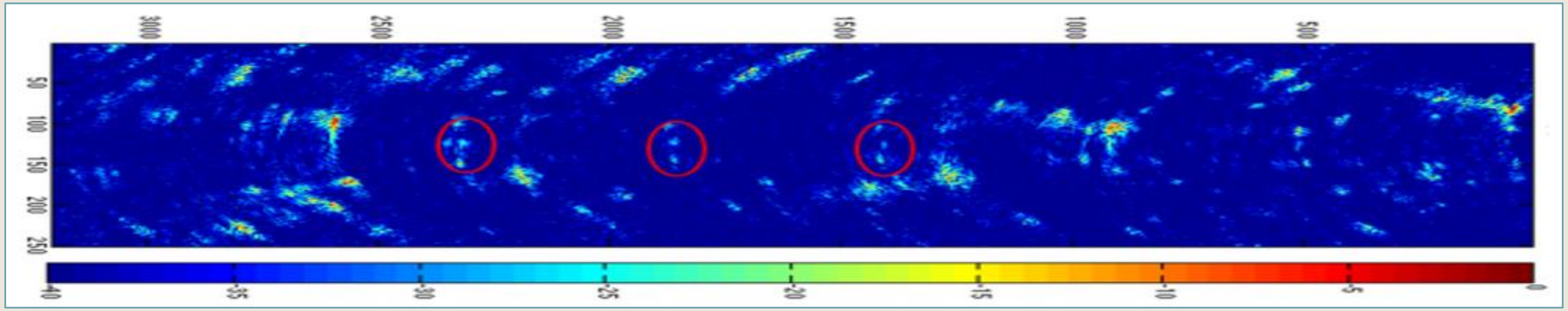


Image formed using DAS algorithm

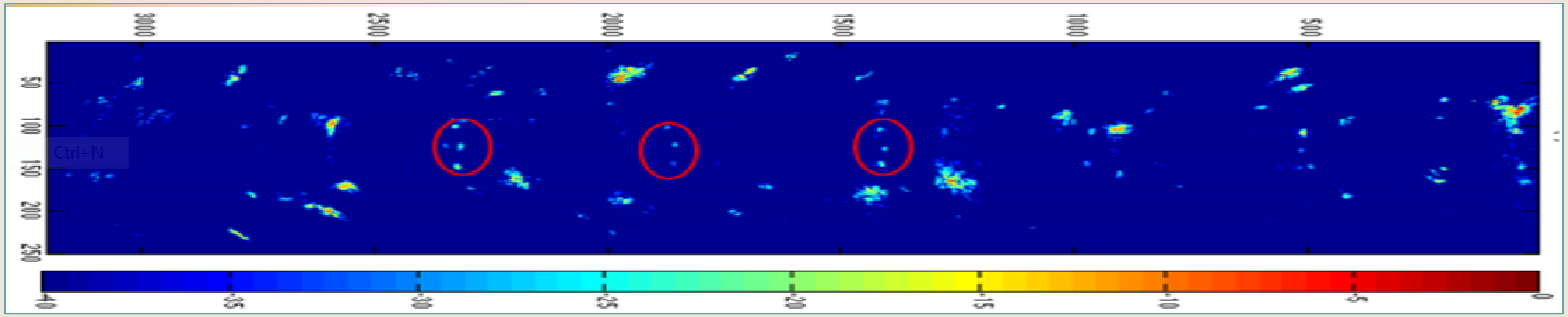


Image formed using LMM algorithm

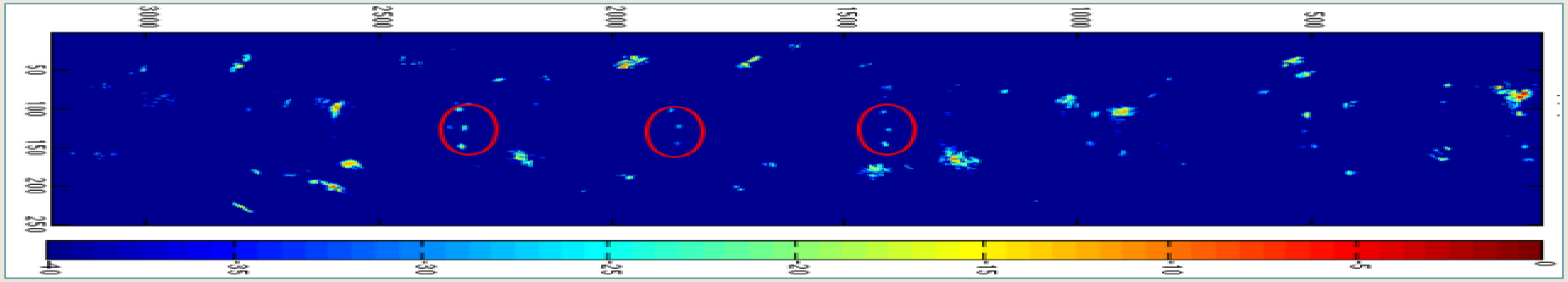
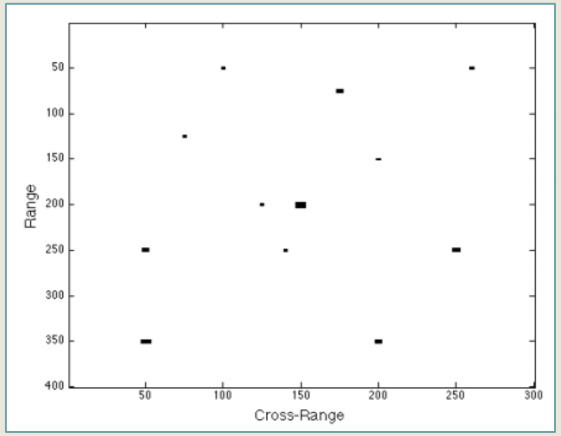
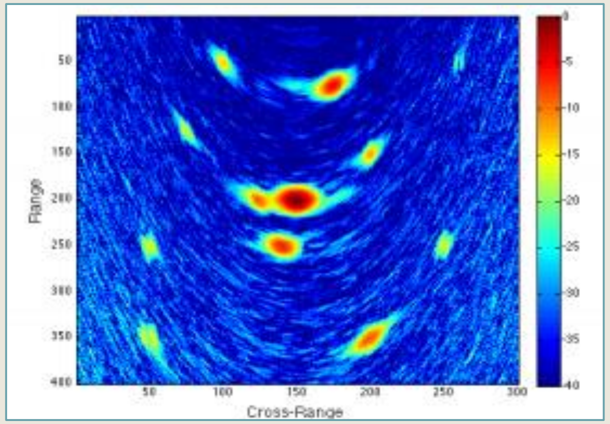


Image formed using PFM algorithm

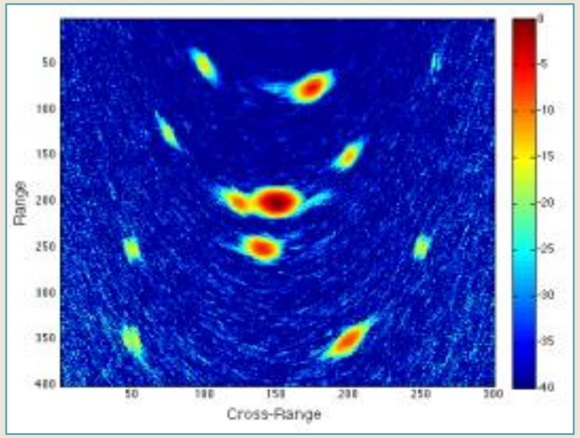
Comparison Using Threshold Detector



True scene-of-interest with 12 point-scatterers



DAS Image for 12 point-scatterers



RSM image for 12 point-scatterers

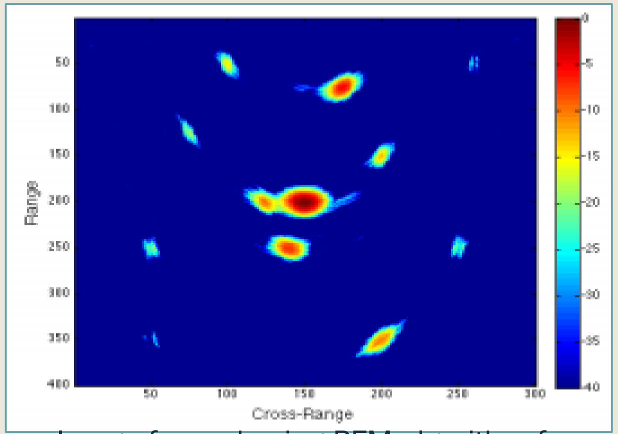
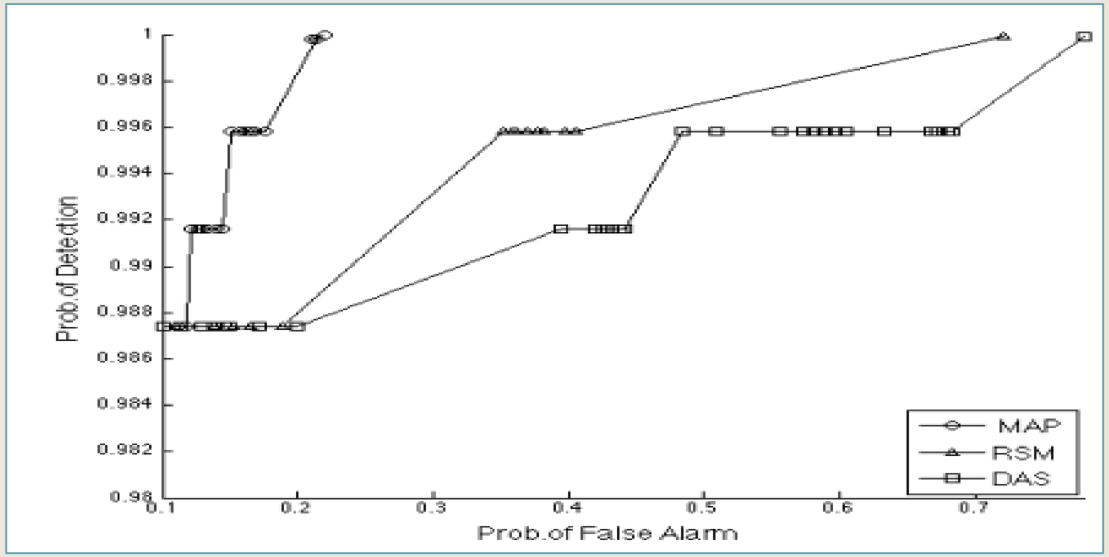


Image formed using PFM algorithm for 12 point-scatterers



ROC comparing DAS, RSM and PFM algorithms for 12 point-scatterers. SNR=20dB, I= 121, J=16

Conclusions

- Developed Parameter-Free MAP algorithms have been successfully applied to synthetic and real data from the impulse-based ARL SIRE system
- Algorithms produced images that are sparse with suppressed background noise while retaining known scatterers

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THANK YOU!