Power Dispatch and Load Control with Generation Uncertainty

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Outline

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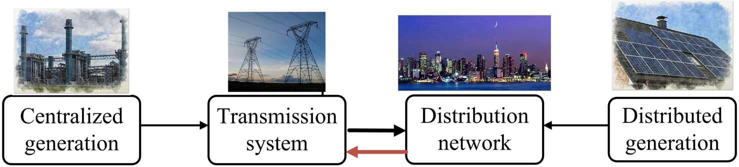
INTRODUCTION

- Using distributed generators (DGs) attracts more attention
  - DGs can provide environmental benefits by utilizing renewable energy resources (RERs) such as wind and solar
  - Install new transmission and bulk generation infrastructure is expensive
- In systems with high penetration of RERs
  - Generation may exceed the demand
  - The reverse power flow back to the substation can cause the voltage rise problem

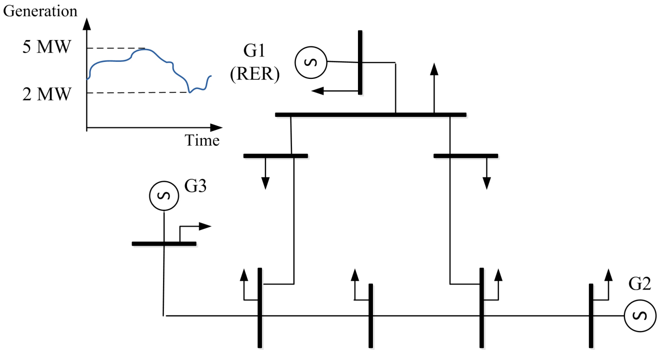


Motivations

- It is a challenge to cope with the undesirable variations of voltage due to the random nature of RERs
- It is a challenge to economically dispatch the output power of RERs and conventional DGs
  - RER generation may exceed the presumed level
  - Formulate an optimal power flow (OPF) problem such that a risk level of RER generation surplus will not exceed a certain threshold
- It is also a challenge to control flexible loads in the system to better match supply and demand
  - Adopt demand response (DR) programs to shape the load pattern of the users to provide voltage regulation services

EXAMPLE

- For conventional DGs, the best solution obtained from the OPF may be 2.5 MW, 4MW, 5MW for generator 1, 2 and 3, respectively
- If generator 1 is RER and its output power varies between 2 MW and 5 MW randomly, then it may have generation surplus
- To minimize the risk level of the RER generation surplus, the OPF solution may change to 3.5 MW, 3.5 MW, 4.5 MW for instance



Related Work

- Ruiz *et al.* (2010) proposed a direct load control algorithm to select the start time and the duration of the residential appliances' control actions
- Lavaei *et al.* (2012) formulated the OPF problem as a SDP
  - Provided the sufficient conditions to ensure the existence of a global optimum for the OPF problem in ac grids
- Dall'Anese *et al.* (2015) studied an OPF problem
  - The risk level of PV generation surplus will not exceed a certain threshold
  - Adopted the concept of conditional value-at-risk (CVaR) to capture the risk of having over-voltage

CONTRIBUTIONS

- Consider the problem of power dispatch and load scheduling of the users
  - Distributed network operator (DNO) is responsible to determine the optimal generation level of the generators and control the flexible loads
- Adopt a semidefinite programming (SDP) relaxation to solve an OPF problem
  - Minimize the generation cost and the discomfort cost of the users
- Schedule the power consumption of the users to better match supply and demand
- Apply the concept of CVaR to minimize the risk of having generation deviation

System Model

- Consider a power system with $N \triangleq |\mathcal{N}|$ buses and $L \triangleq |\mathcal{L}|$ lines
  - Set of buses: $\mathcal{N}$
  - Set of lines: $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$
- The operation cycle is divided into $T \triangleq |\mathcal{T}|$ time slots
  - Set of time slots: $\mathcal{T} = \{1, \ldots, T\}$
- V_t^n is defined as the complex voltage of bus n at time slot t, and $\mathbf{V}_t \triangleq (V_t^1, \ldots, V_t^N)$
- Define $\mathbf{X}_t \triangleq \left[\text{Re}\{\mathbf{V}_t\}^{\text{T}}, \text{Im}\{\mathbf{V}_t\}^{\text{T}}\right]^{\text{T}}$
- Define $\mathbf{W}_t \triangleq \mathbf{X}_t \mathbf{X}_t^{\text{T}}$
- We have $\text{rank}(\mathbf{W}_t) = 1$

System Model (cont.)

- The following relations hold for all $n \in \mathcal{N}$, $(n,m) \in \mathcal{L}$, and $t \in \mathcal{T}$

$$
\begin{aligned}
P_{n,t}^{G} - P_{n,t}^{D} &= \text{Tr}\{\mathbf{Y}_n \mathbf{W}_t\}, && \text{(1a)}\\
Q_{n,t}^{G} - Q_{n,t}^{D} &= \text{Tr}\{\bar{\mathbf{Y}}_n \mathbf{W}_t\}, && \text{(1b)}\\
P_{nm,t} &= \text{Tr}\{\mathbf{Y}_{n,m} \mathbf{W}_t\}, && \text{(1c)}\\
|S_{nm,t}|^2 &= \text{Tr}\{\mathbf{Y}_{nm} \mathbf{W}_t\}^2 + \text{Tr}\{\bar{\mathbf{Y}}_{nm} \mathbf{W}_t\}^2, && \text{(1d)}\\
|V_t^n|^2 &= \text{Tr}\{\mathbf{M}_n \mathbf{W}_t\}. && \text{(1e)}
\end{aligned}
$$

- Matrices $\mathbf{Y}_n$, $\bar{\mathbf{Y}}_n$, $\mathbf{Y}_{n,m}$, $\bar{\mathbf{Y}}_{nm}$ and $\mathbf{M}_n$ are determined from the elements of admittance matrix Y

System Model (cont.)

- The voltage and power values are subject to the power balance equations and physical constraints at all time slots

$$P_{n,t}^{G,\min} - P_{n,t}^{D} \leq \text{Tr}\{\mathbf{Y}_n \mathbf{W}_t\} \leq P_{n,t}^{G,\max} - P_{n,t}^{D}, \tag{2a}$$
$$Q_{n,t}^{G,\min} - Q_{n,t}^{D} \leq \text{Tr}\{\bar{\mathbf{Y}}_n \mathbf{W}_t\} \leq Q_{n,t}^{G,\max} - Q_{n,t}^{D}, \tag{2b}$$
$$\text{Tr}\{\mathbf{Y}_{nm} \mathbf{W}_t\} \leq P_{nm}^{\max}, \tag{2c}$$
$$\text{Tr}\{\mathbf{Y}_{nm} \mathbf{W}_t\}^2 + \text{Tr}\{\bar{\mathbf{Y}}_{nm} \mathbf{W}_t\}^2 \leq (S_{nm}^{\max})^2, \tag{2d}$$
$$(V_n^{\min})^2 \leq \text{Tr}\{\mathbf{M}_n \mathbf{W}_t\} \leq (V_n^{\max})^2. \tag{2e}$$

- DNO can remotely control the operation of some appliances of the user at bus n

$$P_{n,t}^{D,\min} \leq P_{n,t}^{D} \leq P_{n,t}^{D,\max}, \tag{3a}$$
$$Q_{n,t}^{D,\min} \leq Q_{n,t}^{D} \leq Q_{n,t}^{D,\max}, \tag{3b}$$
$$0 \leq E_n \leq \textstyle\sum_{t \in \mathcal{T}} P_{n,t}^{D}, \tag{3c}$$

where E_n denotes the total energy requirement of the load connected to bus n

System Model (cont.)

- We consider a quadratic cost function $C_n(P^G_{n,t}) = a_n(P^G_{n,t})^2 + b_n(P^G_{n,t}) + c_n$
- Substituting $P^G_{n,t} - P^D_{n,t} = \text{Tr}\{\mathbf{Y}_n\mathbf{W}_t\}$

$$C_n(\mathbf{W}_t, P^D_{n,t}) = a_n(\text{Tr}\{\mathbf{Y}_n\mathbf{W}_t\} + P^D_{n,t})^2 + b_n(\text{Tr}\{\mathbf{Y}_n\mathbf{W}_t\} + P^D_{n,t}) + c_n. \quad (4)$$

- $L_{n,t}$ denotes the *desired level* of power consumption at bus n at time slot t
- We assume a quadratic dissatisfaction cost function as

$$H_{n,t}(P^D_{n,t}) = \theta_{n,t}(P^D_{n,t} - L_{n,t})^2, \quad (5)$$

where $\theta_{n,t}$ is a non-negative constant

Problem Formulation

- The SDP relaxation form of the OPF problem obtained by relaxing the rank constraint

$$\underset{\substack{\mathbf{W}_t,\, S^D_{n,t},\\ \lambda_{n,t},\, \gamma_{n,t},\\ t\in\mathcal{T},\, n\in\mathcal{N}}}{\text{minimize}} \quad \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \lambda_{n,t}+\gamma_{n,t} \tag{6a}$$

$$\text{subject to} \quad C_n(\mathbf{W}_t, P^D_{n,t}) \le \lambda_{n,t}, \tag{6b}$$

$$H_{n,t}(P^D_{n,t}) \le \gamma_{n,t}, \tag{6c}$$

$$P^{G,\min}_{n,t} - P^D_{n,t} \le \mathrm{Tr}\{\mathbf{Y}_n\mathbf{W}_t\} \le P^{G,\max}_{n,t} - P^D_{n,t}, \tag{6d}$$

$$Q^{G,\min}_{n,t} - Q^D_{n,t} \le \mathrm{Tr}\{\bar{\mathbf{Y}}_n\mathbf{W}_t\} \le Q^{G,\max}_{n,t} - Q^D_{n,t}, \tag{6e}$$

$$\mathrm{Tr}\{\mathbf{Y}_{nm}\mathbf{W}_t\} \le P^{\max}_{nm}, \tag{6f}$$

$$\mathrm{Tr}\{\mathbf{Y}_{nm}\mathbf{W}_t\}^2 + \mathrm{Tr}\{\bar{\mathbf{Y}}_{nm}\mathbf{W}_t\}^2 \le (S^{\max}_{nm})^2, \tag{6g}$$

$$(V^{\min}_n)^2 \le \mathrm{Tr}\{\mathbf{M}_n\mathbf{W}_t\} \le (V^{\max}_n)^2, \tag{6h}$$

$$P^{D,\min}_{n,t} \le P^D_{n,t} \le P^{D,\max}_{n,t}, \tag{6i}$$

$$Q^{D,\min}_{n,t} \le Q^D_{n,t} \le Q^{D,\max}_{n,t}, \tag{6j}$$

$$0 \le E_n \le \textstyle\sum_{t\in\mathcal{T}} P^D_{n,t}. \tag{6k}$$

Problem Formulation (cont.)

- Constraint $\text{Tr}\{\mathbf{Y}_{nm}\mathbf{W}_t\}^2 + \text{Tr}\{\bar{\mathbf{Y}}_{nm}\mathbf{W}_t\}^2 \leq (S_{nm}^{\max})^2$ can be replaced by

$$\begin{bmatrix} (S_{nm}^{\max})^2 & \text{Tr}\{\mathbf{Y}_{nm}\mathbf{W}_t\} & \text{Tr}\{\bar{\mathbf{Y}}_{nm}\mathbf{W}_t\} \\ \text{Tr}\{\mathbf{Y}_{nm}\mathbf{W}_t\} & -1 & 0 \\ \text{Tr}\{\bar{\mathbf{Y}}_{nm}\mathbf{W}_t\} & 0 & -1 \end{bmatrix} \preceq 0. \quad (7)$$

- Constraints $C_n(\mathbf{W}_t, P_{n,t}^D) \leq \lambda_{n,t}$ and $H_{n,t}(P_{n,t}^D) \leq \gamma_{n,t}$ can be replaced by

$$\begin{bmatrix} b_n\delta_{n,t} - \lambda_{n,t} + c_n & \sqrt{a_n}\delta_{n,t} \\ \sqrt{a_n}\delta_{n,t} & -1 \end{bmatrix} \preceq 0, \quad (8a)$$

$$\begin{bmatrix} -2L_{n,t}P_{n,t}^D + L_{n,t}^2 - \gamma_{n,t}/\theta_{n,t} & P_{n,t}^D \\ P_{n,t}^D & -1 \end{bmatrix} \preceq 0, \quad (8b)$$

where $\delta_{n,t} \triangleq \text{Tr}\{\mathbf{Y}_n\mathbf{W}_t\} + P_{n,t}^D$

Problem Formulation (cont.)

- Therefore, OPF problem can be reformulated as

$$\underset{\substack{\mathbf{W}_t,\, S^D_{n,t},\\ \lambda_{n,t},\, \gamma_{n,t},\\ t\in\mathcal{T},\, n\in\mathcal{N}}}{\text{minimize}} \quad \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \lambda_{n,t}+\gamma_{n,t} \tag{9a}$$

$$\text{subject to} \quad \text{linear matrix inequality constraints} \tag{9b}$$

- The question is how to tackle the uncertainty in the RERs' generation?

Problem Formulation (cont.)

- To tackle the voltage variation problem, one option is to add a barrier term to the objective function which penalizes any voltage deviation
- Examples of such barrier functions include value at risk (VaR) and CVaR
- Voltage variation can be indirectly related to the fluctuations in the RERs' power generation
- $\hat{\mathbf{P}}_t \triangleq (\hat{P}_{1,t}, \ldots, \hat{P}_{N,t})$ is the vector of presumed values of the output power generation obtained from solving the OPF
- $\mathbf{P}_t^G \triangleq (P_{1,t}^G, \ldots, P_{N,t}^G)$ is the vector of the actual power generation
- We define $R(\cdot)$ as

$$R(\mathbf{P}_t^G, \hat{\mathbf{P}}_t) = \sum_{n \in \mathcal{N}} \left[P_{n,t}^G - \hat{P}_{n,t} \right]^+, \tag{10}$$

where $[\cdot]^+ \triangleq \max\{\cdot, 0\}$

Problem Formulation (cont.)

- $R(\cdot)$ is a random variable with the following cumulative distribution function

$$\Psi(\hat{\mathbf{P}}_t, \alpha) \triangleq \Pr\{R(\mathbf{P}_t^G, \hat{\mathbf{P}}_t) \leq \alpha\}. \tag{11}$$

- For the probability level $\beta \in (0, 1)$, the corresponding VaR, α_β, is defined as

$$\alpha_\beta(\hat{\mathbf{P}}_t) \triangleq \min\{\alpha : \Psi(\hat{\mathbf{P}}_t, \alpha) \geq \beta\}. \tag{12}$$

- It is the minimum threshold α for which the probability of voltage deviation from its nominal value being less than α is at least β
- For example, when $\beta = 0.95$, then VAR $= 0.1$ pu means with probability of 0.95 the voltage deviation is less than 0.1 pu
- CVaR is defined as

$$\phi_\beta(\hat{\mathbf{P}}_t) = \mathbb{E}\{R(\mathbf{P}_t^G, \hat{\mathbf{P}}_t) : R(\mathbf{P}_t^G, \hat{\mathbf{P}}_t) \geq \alpha_\beta(\hat{\mathbf{P}}_t)\}. \tag{13}$$

Problem Formulation (cont.)

- CVaR can also be represented as $\phi_\beta(\hat{\mathbf{P}}_t) = \min_{\alpha\in\mathbb{R}} \Gamma_\beta(\alpha, \hat{\mathbf{P}}_t)$, where

$$\Gamma_\beta(\alpha, \hat{\mathbf{P}}_t) \triangleq \alpha + \frac{1}{1-\beta}\int \Big[R(\mathbf{P}_t^G, \hat{\mathbf{P}}_t) - \alpha\Big]^+ \rho(\mathbf{P}_t^G) d\mathbf{P}_t^G, \tag{14}$$

  and $\rho(\mathbf{P}_t^G)$ is the probability density function of random vector $\mathbf{P}_t^G$

- CVaR is convex in $\hat{\mathbf{P}}_t$, and for any threshold α, it is always greater than or equal to the VaR
  - Minimizing the CVaR results in having a low VaR as well

- It is possible to estimate the CVaR by adopting sample average technique

- Considering the set $\mathcal{K} \triangleq \{1, \ldots, K\}$ of K samples of the random vector $\mathbf{P}_t^G$

$$\hat{\Gamma}_\beta(\alpha, \hat{\mathbf{P}}_t) = \alpha + \frac{1}{K(1-\beta)} \sum_{k\in\mathcal{K}} \Big[R(\mathbf{P}_t^{G,k}, \hat{\mathbf{P}}_t) - \alpha\Big]^+. \tag{15}$$

Problem Formulation (cont.)

- Taking into account the uncertainty about the generation

$$\underset{\substack{\mathbf{W}_t,\, S^D_{n,t},\, \lambda_{n,t}, \\ \gamma_{n,t},\, \alpha,\, \hat{\mathbf{P}}_t, \\ t \in \mathcal{T},\, n \in \mathcal{N}}}{\text{minimize}} \quad \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \left(\lambda_{n,t} + \gamma_{n,t}\right) + \eta_t \, \hat{\Gamma}_\beta(\alpha, \hat{\mathbf{P}}_t) \tag{16a}$$

$$\text{subject to} \quad \text{linear matrix inequality constraints}, \tag{16b}$$

where η_t is a positive weighting coefficient

- Auxiliary vector $\boldsymbol{\mu}_t \in \mathbb{R}^K$ is introduced for $\left[R(\mathbf{P}_t^{G,k}, \hat{\mathbf{P}}_t) - \alpha\right]^+$
- The vector of auxiliary variables $\mathbf{u}_t^k \in \mathbb{R}^N$ are introduced for each sample k

$$\underset{\substack{\mathbf{W}_t,\, S^D_{n,t},\, \lambda_{n,t}, \\ \gamma_{n,t},\, \alpha,\, \hat{\mathbf{P}}_t, \\ \boldsymbol{\mu}_t,\, \mathbf{u}_t^k,\, k \in \mathcal{K}, \\ t \in \mathcal{T},\, n \in \mathcal{N}}}{\text{minimize}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} \left(\lambda_{n,t} + \gamma_{n,t}\right) + \eta_t \alpha + \frac{\eta_t}{K(1-\beta)} \mathbf{1}_K^{\mathrm{T}} \boldsymbol{\mu}_t \tag{17a}$$

$$\text{subject to} \quad \text{linear matrix inequality constraints}, \tag{17b}$$

$$\mathbf{1}_N^{\mathrm{T}} \mathbf{u}_t^k \le \alpha + \mu_t^k, \tag{17c}$$

$$P_{n,t}^{G,k} - \hat{P}_{n,t}^k \le u_{n,t}^k. \tag{17d}$$

Algorithm

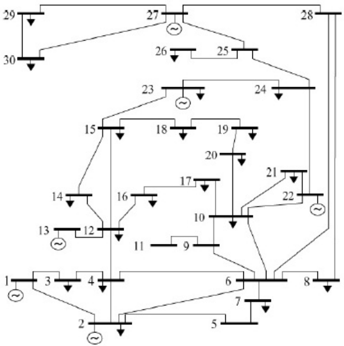
- $\mathbf{W}_t^{\text{opt}}$ is at most rank two for practical ac grids such as the IEEE test systems

Algorithm 1 determines the voltage of buses.

1: Solve problem (17)
2: **if** $\mathbf{W}_t^{\text{opt}}$ is rank one with eigenvalue r and eigenvector $\boldsymbol{\nu}$
3: Calculate $\mathbf{X}_t^{\text{opt}} = \sqrt{r}\boldsymbol{\nu}$.
4: **else if** $\mathbf{W}_t^{\text{opt}}$ is rank two with two nonzero eigenvalues r_1 and r_2 and corresponding eigenvectors $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$
5: Calculate rank one matrix $\hat{\mathbf{W}}_t^{\text{opt}} = (r_1 + r_2)\boldsymbol{\nu}_1\boldsymbol{\nu}_2^{\text{T}}$.
6: Calculate eigenvalue $\hat{r}$ and eigenvector $\hat{\boldsymbol{\nu}}$ of $\hat{\mathbf{W}}_t^{\text{opt}}$.
7: Calculate $\mathbf{X}_t^{\text{opt}} = \sqrt{\hat{r}}\hat{\boldsymbol{\nu}}$.
8: **end if**

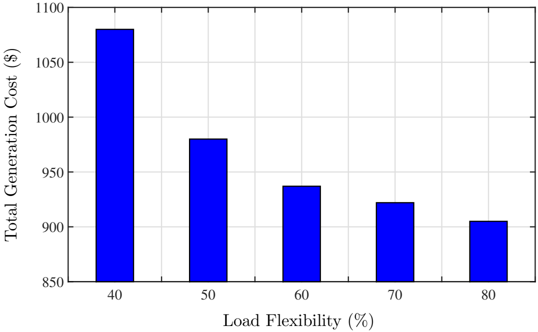
Performance Evaluation

- The IEEE 30-bus network is considered as a test case
- Some of the buses in the network are equipped with RERs (wind and solar)
- To estimate the CVaR, we use the sample average technique with $K = 100$ samples of power generation
- The operation period is divided into 3 time slots representing on-peak hours, off-peak hours, and mid-peak hours



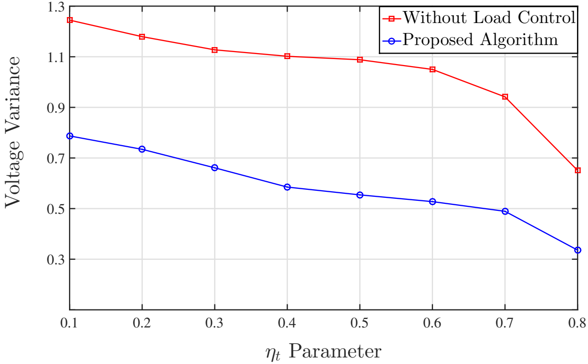
Performance Evaluation (cont.)

- Simulation results for the total generation cost of the system for different levels of load flexibility
- Load flexibility is defined as the percentage of the desired level of load in each time slot that can be reduced or increased. That is, $\chi_{n,t} \triangleq \Delta P^D_{n,t}/P^D_{n,t} \times 100\%$, where $\Delta P^D_{n,t}$ is the amount of power demand that can be adjusted
- The generation cost reduces when the DNO can shift more load from peak hours to off-peak hours



Performance Evaluation (cont.)

- Simulation results for the expected voltage values for different values of parameter η_t
- By increasing the parameter η_t, the voltage variations are reduced as more weights are put on minimizing the risk of having high voltage values



Conclusion

- We formulated an optimization problem to minimize the generation cost and the discomfort cost subject to power flow constraints for the equivalent circuit of the power system
- We adopted an SDP relaxation technique to solve the OPF problem
- The risk of having high voltage values was also minimized by including a barrier term based on CVaR in the objective function
- Our proposed algorithm reduces the generation cost and better eliminates the mismatch between the supply and demand

Thank you for your attention!