Microgrid Dispatch and Price of Reliability using Stochastic Approximation

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Motivation

- Grid efficiency vs. supply reliability

- Multi-stage economic dispatch of a microgrid
  - day-ahead economic dispatch
  - recourse actions (adjustments, re-dispatching)
  - emergency actions (e.g., load shedding)

- Uncertainty in demand, renewable generation, market prices

- Minimize *expected* network operation cost ensuring reliability guarantees

Work in context

- Worst-case approaches could be over-conservative [Y. Zhang-Gatsis-Giannakis'13]
- Stochastic two-stage dispatch [Bouffard-Galina'08]
  no network; no expected lost load constraints
- Multi-stage risk-limiting dispatch [Varaiya et al'11]
- Stochastic dynamic programming [Rajagopal et al'13]
- Network effects under restricting conditions [B. Zhang et al'14]
  small network, normal pdfs, small variance
- Our contributions
  - two-stage stochastic dispatch
  - constraint on expected lost load
  - efficient distribution-free solver

Day-ahead economic dispatch

- Microgrid with *N+1* buses; approximate DC power flow model
- Bus 0 as reference bus and connection point to external grid

$$\begin{aligned} \min_{p_0, \mathbf{g}} \quad & C(\mathbf{g}) + \beta p_0 \\ \text{s.to} \quad & \mathbf{p} = \mathbf{g} + \mathbf{w} - \mathbf{d} && \longleftarrow \text{nodal power injections} \\ & \mathbf{1}^\top \mathbf{p} + p_0 = \mathbf{0} && \longleftarrow \text{power balance equation} \\ & -\bar{\mathbf{f}} \leq \mathbf{H}\mathbf{p} \leq \bar{\mathbf{f}} && \longleftarrow \text{line thermal limits} \\ & \mathbf{0} \leq \mathbf{g} \leq \bar{\mathbf{g}} && \longleftarrow \text{generation limits} \end{aligned}$$

- Minimize generation and energy exchange cost
- Wind generation and load demand assumed deterministic

Real-time economic dispatch

real-time procurement cost

quadratic load-shedding penalty

spilling penalty

$$f(p_0, \mathbf{g}) := \min_{\{\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi})\}} \mathbb{E}_{\boldsymbol{\xi}} \left[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w \right]$$

$$\text{s.to} \quad \mathbf{p} = (\mathbf{g} + \boldsymbol{\delta}_g) + (\mathbf{w} - \boldsymbol{\delta}_w) - (\mathbf{d} - \boldsymbol{\delta}_d) \qquad \longleftarrow \boldsymbol{\lambda}$$

$$\mathbf{1}^\top \mathbf{p} + p_0 + \delta_0 = 0 \qquad \longleftarrow \lambda_0$$

$$-\bar{\mathbf{f}} \leq \mathbf{H}\mathbf{p} \leq \bar{\mathbf{f}}$$

$$\mathbf{0} \leq \boldsymbol{\delta}_w \leq \mathbf{w}$$

$$\mathbf{0} \leq \boldsymbol{\delta}_d \leq \mathbf{d}$$

$$\mathbf{0} \leq \boldsymbol{\delta}_g \leq \bar{\boldsymbol{\delta}}_g$$

$$\mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_d] \leq \eta$$

instantaneous constraints →

expected lost load couples real-time decisions →

Problem statement

Given samples from anticipated demand and wind, find the optimal first- and second-stage generation schedules

$$
\begin{aligned}
\min_{p_0,\mathbf{g},\{\boldsymbol{\delta}(\boldsymbol{\xi}),\mathbf{p}(\boldsymbol{\xi})\}} \quad & C(\mathbf{g}) + \beta p_0 + f(p_0, \mathbf{g}) \\
\text{s.to} \quad & (\text{inst. constraints}) \ \forall\, \boldsymbol{\xi} \\
& \mathbf{0} \leq \mathbf{g} \leq \bar{\mathbf{g}} \\
& \mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_\delta] \leq \eta
\end{aligned}
$$

Second-stage optimization

- Suppose first-stage variables are fixed and given

$$f(p_0, \mathbf{g}) := \min_{\{\boldsymbol{\delta}, \mathbf{p}\}} \mathbb{E}_{\boldsymbol{\xi}}[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w]$$

$$\text{s.to} \quad \text{(inst. constraints)} \; \forall \boldsymbol{\xi};$$

$$\mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_d] \leq \eta$$

optimal value of real-time dispatch

coupling across realizations of $\boldsymbol{\xi}$

- Solve using dual approach

$$\mathscr{D}(\nu; p_0, \mathbf{g}) := \min_{\{\boldsymbol{\delta}, \mathbf{p}\}} \mathbb{E}_{\boldsymbol{\xi}}\left[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w + \nu(\mathbf{1}^\top \boldsymbol{\delta}_d - \eta)\right]$$

$$\text{s.to} \quad \text{(inst. constraints)} \; \forall \boldsymbol{\xi}$$

- If ν^* is known, the problem decomposes across realizations of $\boldsymbol{\xi}$

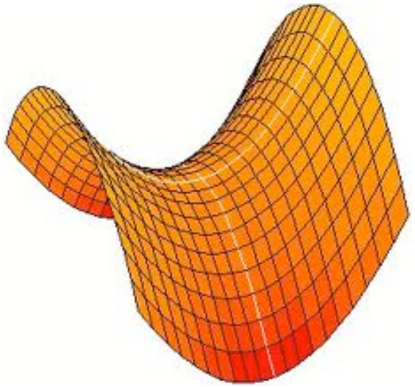
First-stage optimization

- From strong duality $f(p_0, \mathbf{g}) = \max_{\nu} \mathscr{D}(\nu; p_0, \mathbf{g})$

- First-stage schedules minimize the problem

$$\min_{p_0, \mathbf{g} \in \mathcal{G}} C(\mathbf{g}) + \beta p_0 + f(p_0, \mathbf{g}) = \min_{p_0, \mathbf{g} \in \mathcal{G}} \max_{\nu \geq 0} C(\mathbf{g}) + \beta p_0 + \mathscr{D}(\nu; p_0, \mathbf{g})$$

- Stochastic saddle-point problem



- Solved via stochastic saddle-point mirror algorithm [Nemirovski et al, 2012]

Stochastic subgradient iterations

- Dual variable update

$$\nu^{k+1} = \left[\nu^k + \mu_k\left(\mathbf{1}^\top \boldsymbol{\delta}_d{}^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k) - \eta\right)\right]_+$$

- Primal variable updates

LMP at energy exchange bus

$$p_0^{k+1} = p_0^k - \varepsilon_k\left(\beta - \lambda_0^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k)\right)$$

$$\mathbf{g}^{k+1} = \left[\mathbf{g}^k - \varepsilon_k\left(\partial_{\mathbf{g}} C(\mathbf{g}^k) + \boldsymbol{\lambda}^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k)\right)\right]_{\mathcal{G}}$$

LMPs at buses with conv. gen.

- Features of the algorithm
  - distribution-free
  - asymptotic convergence

Economic interpretations

- Optimality condition on p_0 implies

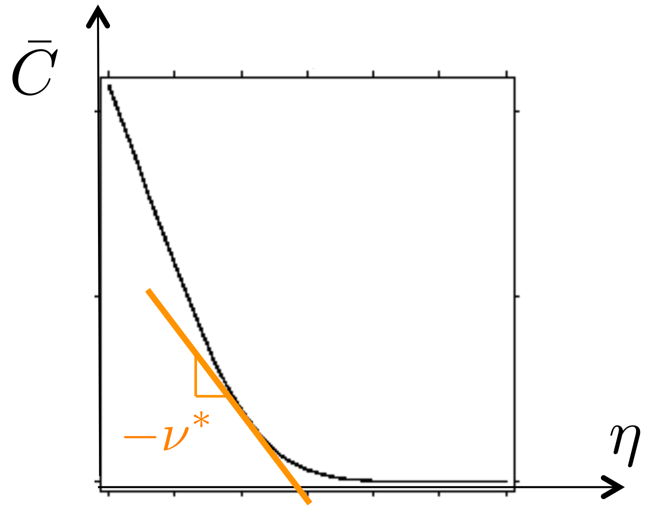
$$\mathbb{E}\left[\lambda^*(\boldsymbol{\xi}_k; \nu^*, p_0^*, \mathbf{g}^*)\right] = \beta$$

LMP at exchange bus

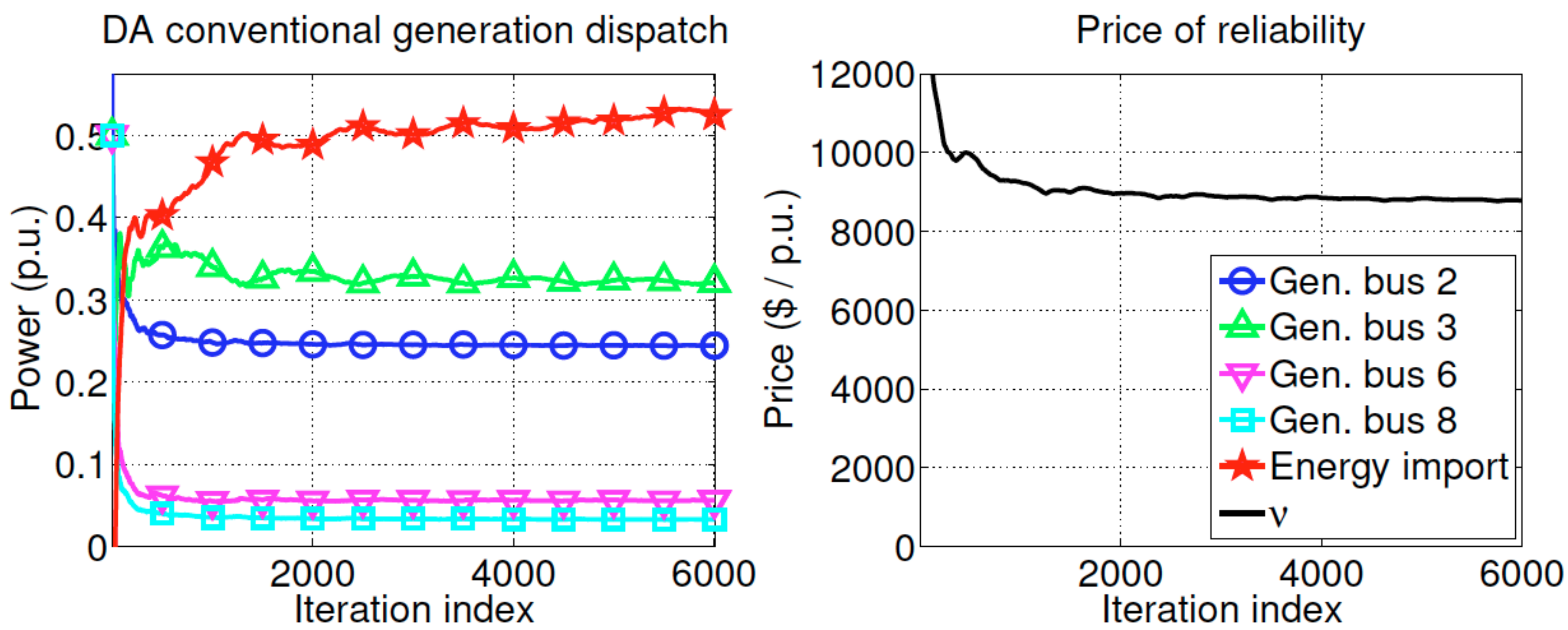
day-ahead energy price

- Multiplier ν^* is the *price of reliability*

  optimal cost sensitivity wrt upper bound on ELNS η

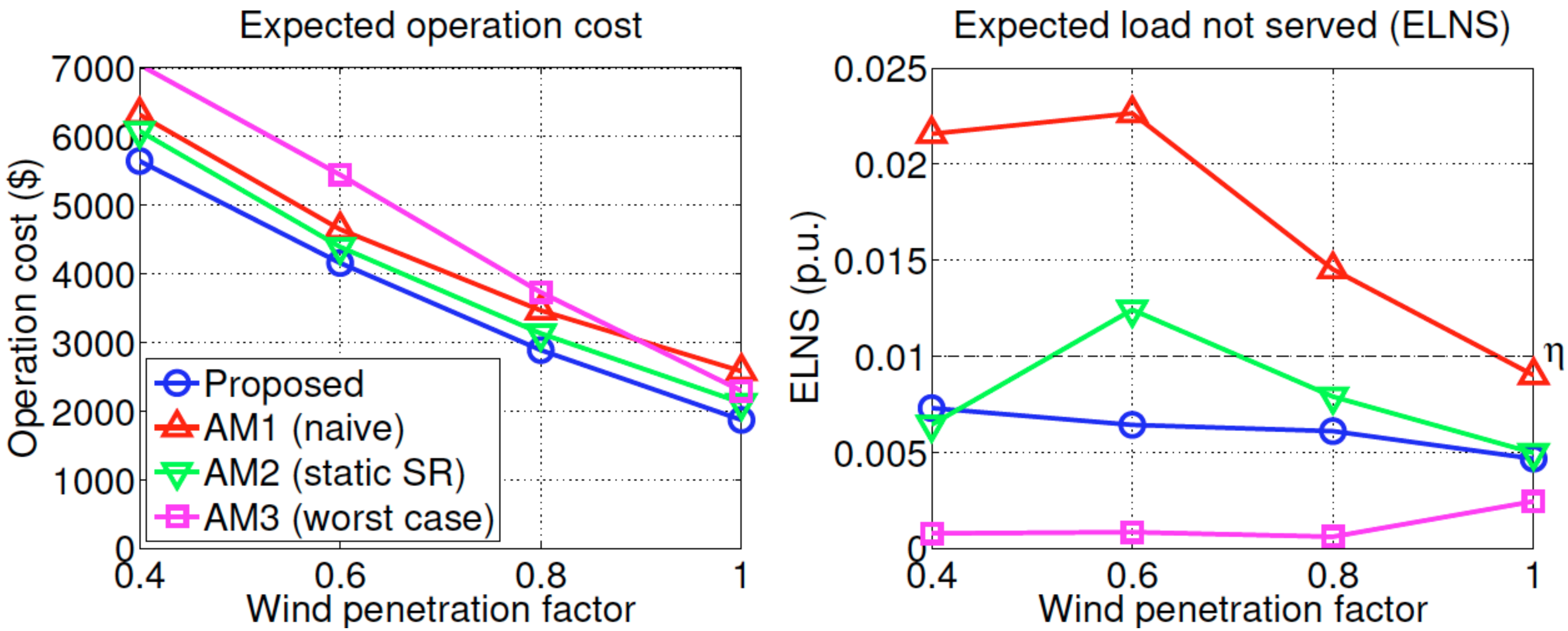


Convergence



- Convergence of primal and dual variables
- Constant step size and averaging iterates

Numerical comparison



- Increasing wind penetration levels

Conclusions

- Two-stage dispatch under load and wind uncertainty
- Minimum cost; recourse actions; upper bounded ELNS
- Stochastic approximation approach
  - efficient, sample-based, distribution-free
  - asymptotically optimal
- Future work
  - probability constraints (non-convex)
  - additional time-scales to incorporate voltage control

Thank you!