

A METROPOLIS-WITHIN-GIBBS SAMPLER TO INFER TASK-BASED FUNCTIONAL BRAIN CONNECTIVITY

M Faizan Ahmad, James Murphy, Simon Godsill, Deniz Vatansever*, Emmanuel A Stamatakis*

Department of Engineering, University of Cambridge, UK

*Department of Clinical Neuroscience, Addenbrookes Hospital, Cambridge, UK

ABSTRACT

Present methods of analysing functional networks in brain during task-conditions mainly include concatenation followed by temporal correlation. We employ Markov Chain Monte Carlo methods, namely Metropolis within Gibbs sampling, on a stochastic model to infer dynamic functional connectivity. By using a Bayesian probabilistic framework, distributional estimates are obtained as opposed to point estimates, and the uncertainty of the existence of such links is accounted for. The methodology is applied to fMRI data from a finger opposition paradigm with task and fixation conditions, investigating the dynamics of the well characterised somato-motor network while using the visual network as a control case.

MODEL

The BOLD signal is modelled such that the change in the signal value is the result of a summation of forces, which are dependent on BOLD signals from other nodes:

$$\Delta x_{i,t} = \left[\sum_{j=1, i \neq j}^N S_{i,j} \varphi_{i,j} x_{j,t} - \gamma_i x_{i,t} \right] \Delta t + \sigma_i^2 W_{i,t}$$

where $x_{i,t}$ is the BOLD signal at node i at time t , $\varphi_{i,j}$ is the interaction parameter between nodes i and j , $S_{i,j}$ is an indicator variable to specify the existence of a link, $W_{i,t}$ represents Brownian motion, γ_i is a mean-reverting term, and N is the total number of nodes.

The system over all nodes can be described as a linear stochastic differential equation of the form:

$$\Delta X_t = AX_t + BW_t,$$

and the transition density is given by the following Normal distribution:

$$p(X_t | X_{t-1}, S, \Phi) = N(X_t | F(S, \Phi)X_{t-1}, Q),$$

where F and Q can be computed. Similarly, if the observation has random, additive noise, then the observation density is given by:

$$p(Z_t | X_t) = N(Z_t | X_t, \sigma_z^2 I)$$

ALGORITHM

An MCMC algorithm is used to infer the networks in different task conditions via sampling from the joint distribution $p(S, \Phi | Z_{1:T})$ without the need to sample the BOLD signals $X_{1:T}$ as these are marginalised out. We would like to sample each $\varphi_{i,j}$ from its full conditional

distribution:

$$p(\varphi_{i,j} | S, \varphi_{all \setminus i,j}, Z_{1:T})$$

As it is difficult to sample directly from this distribution, Metropolis within Gibbs sampling is employed in order to make inference. A proposal is made and accepted with probability given by the ratio:

$$K = \frac{p(Z_{1:T} | S, \varphi_{all \setminus i,j}, \varphi_{i,j}^*)}{p(Z_{1:T} | S, \varphi_{all \setminus i,j}, \varphi_{i,j}^{n-1})}$$

The observation likelihood $p(Z_{1:T} | S, \Phi)$ can be found from Kalman filtering such that:

$$p(Z_{1:T} | S, \Phi) = p(Z_1) \prod_{h=2}^{h=T} p(Z_h | \mu_{h|1:h-1}, C_{h|1:h-1})$$

This distribution is an integration over the hidden state $X_{1:T}$ since:

$$p(Z_{1:T} | S, \Phi) = p(Z_1 | S, \Phi) \prod_{h=2}^{h=T} p(Z_h | Z_{h-1}, S, \Phi)$$

and

$$p(Z_t | Z_{t-1}, S, \Phi) = \int p(Z_t | X_t) p(X_t | Z_{1:t-1}, S, \Phi) dX_t$$

As the indicator variables can only take one of two possible values, the probability of both possibilities can be calculated in a Gibbs Sampling framework. The posterior over $S_{i,j} \in \{0, 1\}$ is given by:

$$p(S_{i,j} | Z_{1:T}, \Phi, S_{all \setminus i,j}) =$$

$$p(Z_{1:T} | S_{i,j}, \Phi, S_{all \setminus i,j}) p(S_{i,j} | \Phi, S_{all \setminus i,j})$$

The prior $p(S_{i,j} | S_{all \setminus i,j}, \Phi)$ can, in the simplest case, be given by an independent Bernoulli distribution for each case of $S_{i,j}$.

RESULTS

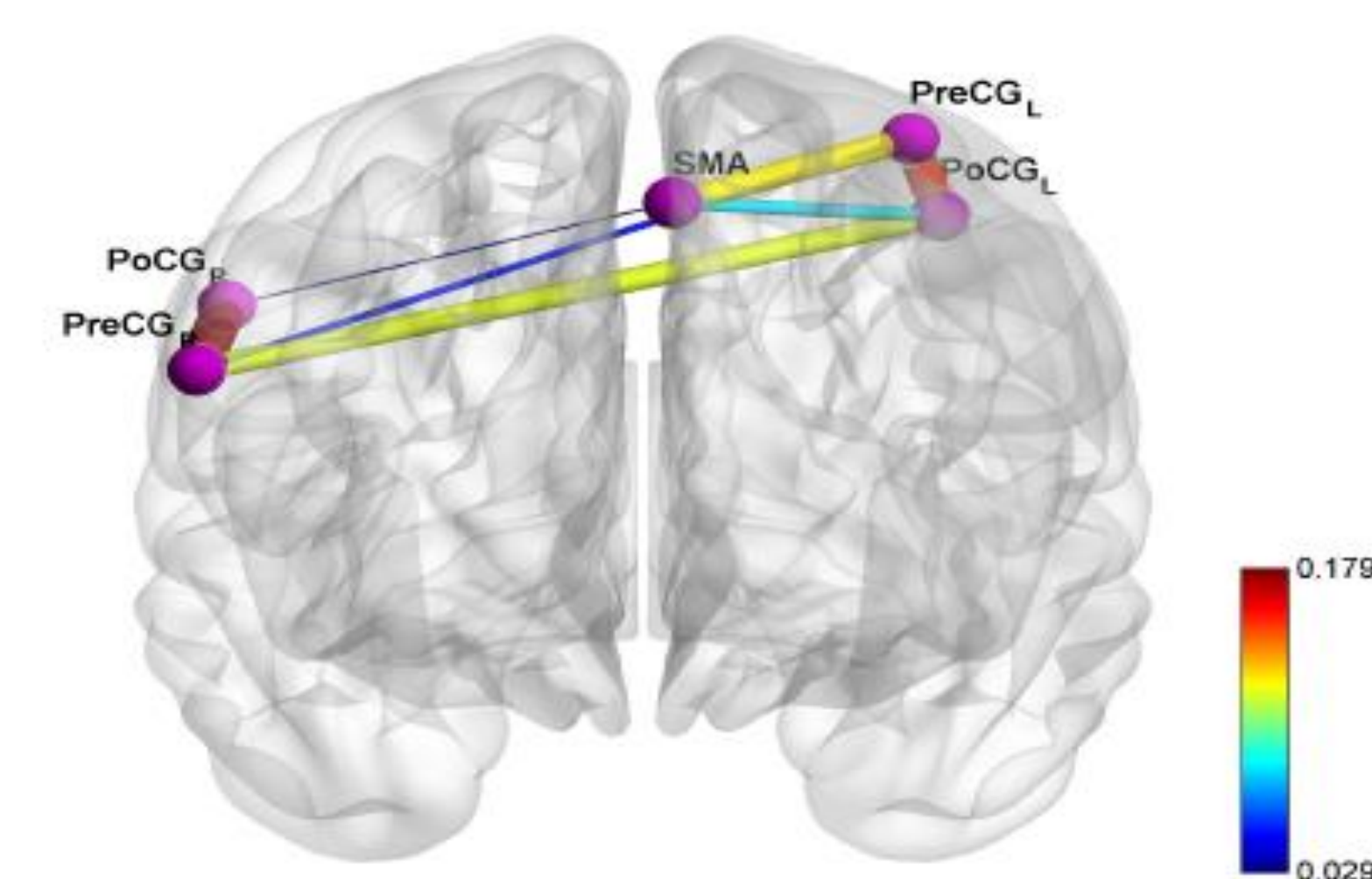
22 subjects participated in a self-paced, right-handed finger opposition task-based, boxcar design experiment with five alternating cycles of task and fixation blocks. BOLD time-series data acquired for five regions of interest (ROIs) corresponding to motor function and five regions that relate to visual activity, as shown in Table 1, was analysed.

Figure 1 shows the results of applying our algorithm to fMRI measurements from one subject. For the entire cohort, the average change in all links is displayed in Table 2. The absolute change in the motor network is found to be greater on average than that in the visual network, and the mean proportional change in the interaction strengths matches very well to the one found by correlation analysis.

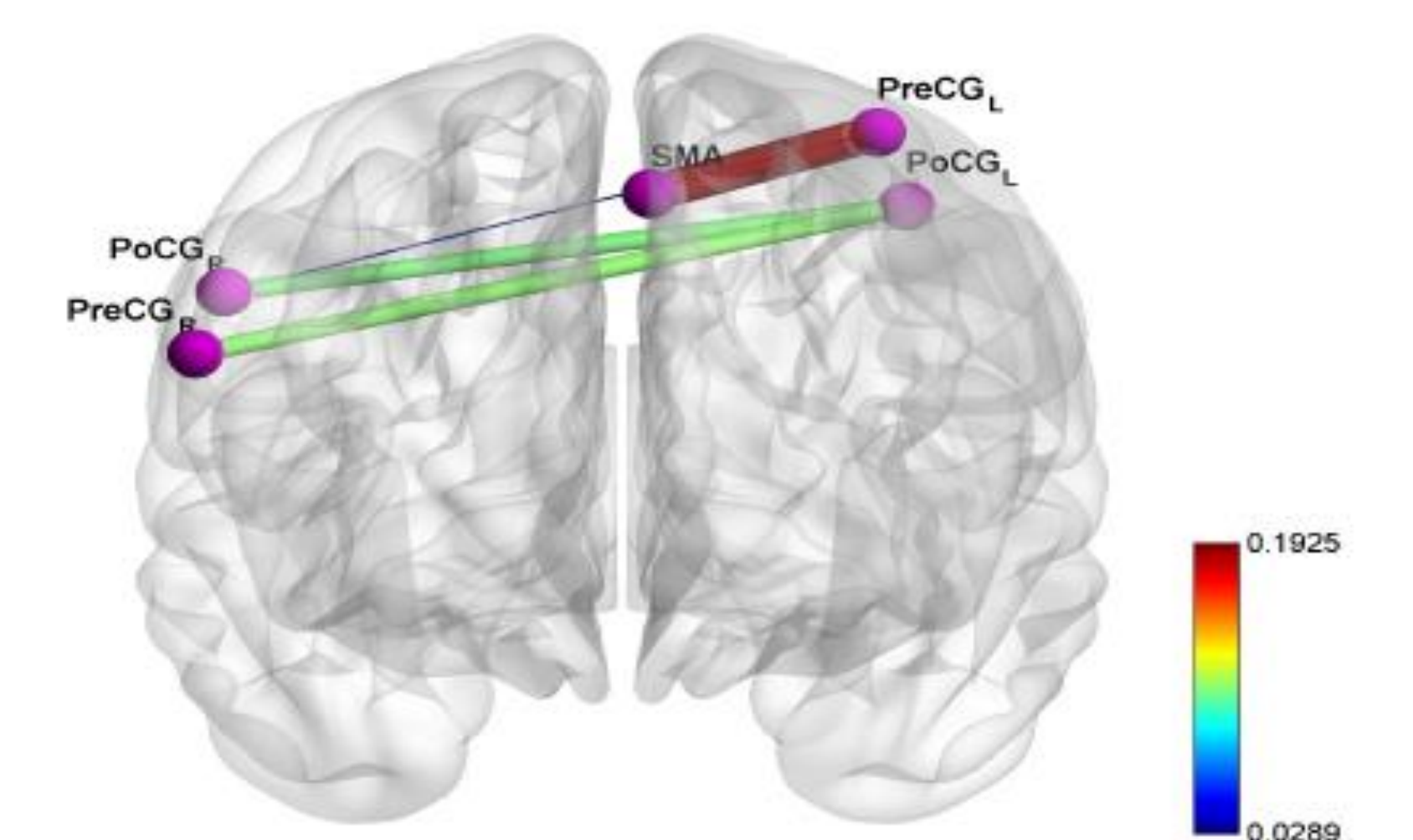
In addition, paired t -test was applied at a 5% significance level: unlike for the visual case, the p -value for the motor network indicated that it undergoes a statistically significant change between the activated and fixated periods.

ROI	Abbreviation	Co-ordinates
Motor	SMA	[-4 -2 54]
	PRECG_L	[-36 -22 64]
	PRECG_R	[60 8 28]
	POCG_L	[-40 -26 52]
	POCG_R	[56 -16 38]
Visual	LING_L	[-15 -72 -8]
	LING_R	[18 -47 -10]
	LCAL_L	[-18 -68 5]
	LCAL_R	[8 -72 -8]
	FFG_R	[27 -59 -9]

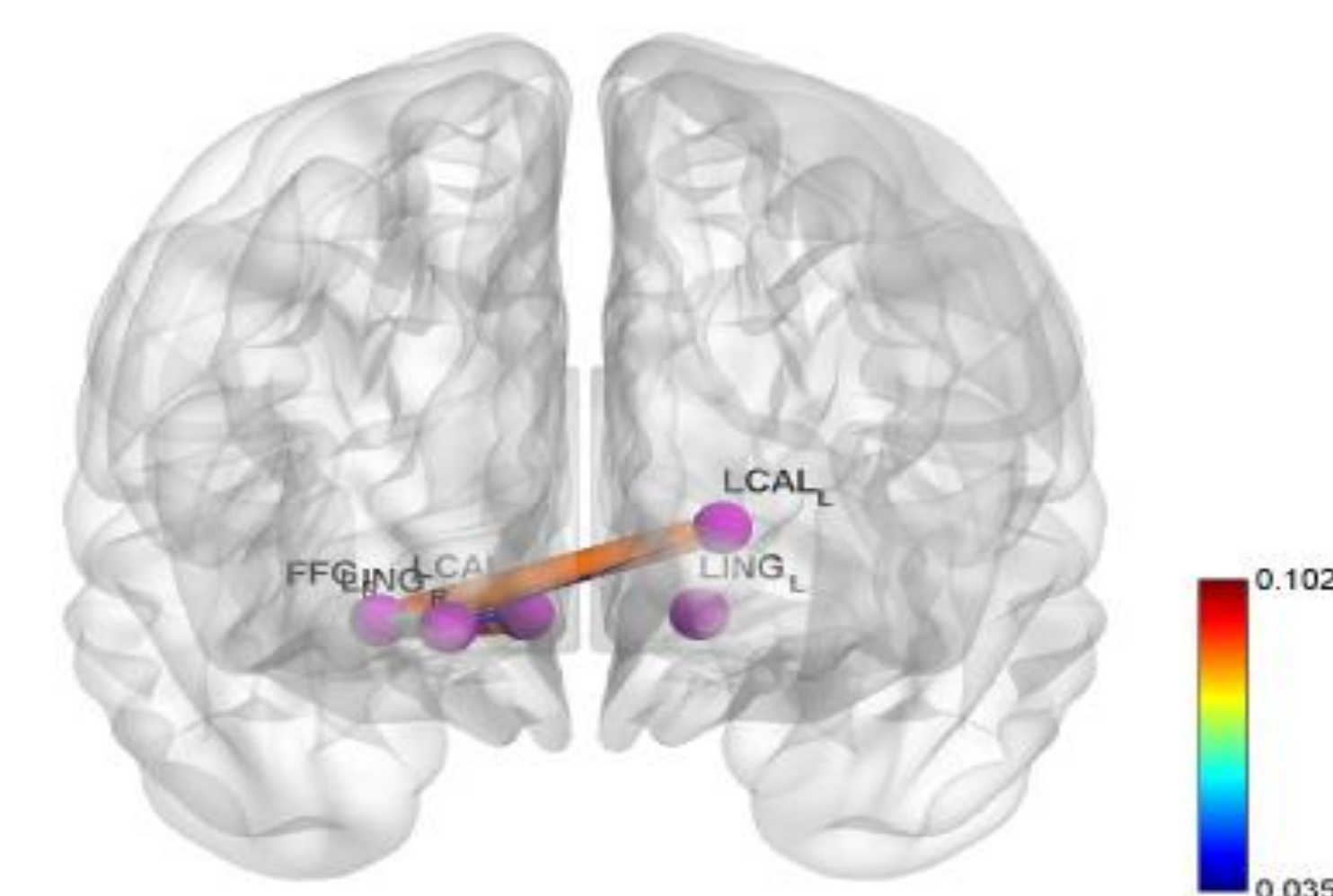
Table 1



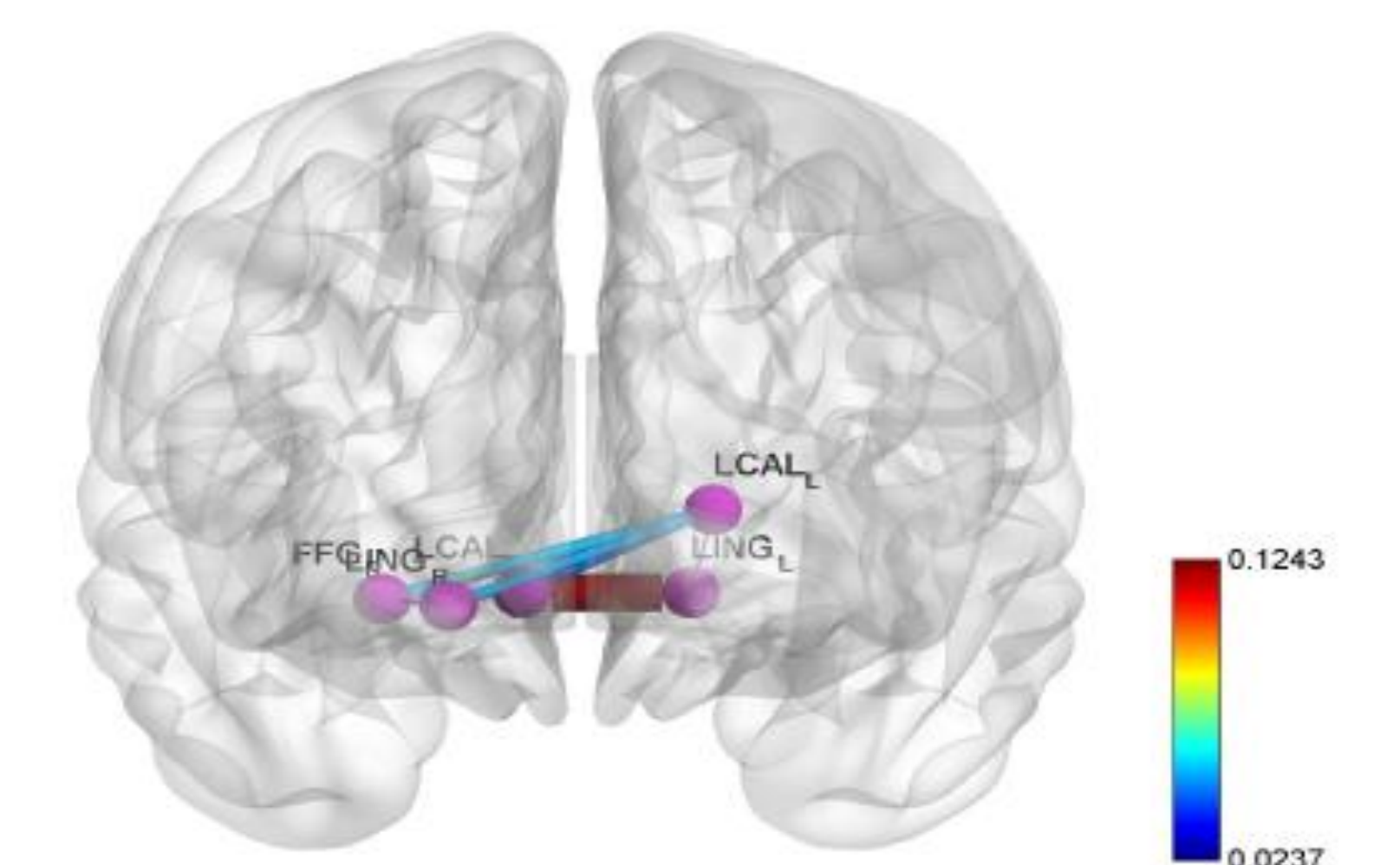
(a) Activation motor network



(b) Fixation motor network



(c) Activation visual network



(d) Fixation visual network

Fig. 1: Sample results obtained from experimental data of one subject.

Mean absolute Change	Correlation analysis	Metropolis with Gibbs sampler
Motor network	0.0376	0.0707
Visual network	0.0245	0.0497
Visual/Motor	0.70	0.65

Table 2

REFERENCES

- 1) R. M. Hutchison, T. Womelsdorf, E. a. Allen, P. a. Bandettini, V. D. Calhoun, M. Corbetta, S. Della Penna, J. H. Duyn, G. H. Glover, J. Gonzalez-Castillo, D. a. Handwerker, S. Keilholz, V. Kiviniemi, D. a. Leopold, F. de Pasquale, O. Sporns, M. Walter, and C. Chang, "Dynamic functional connectivity: promise, issues, and interpretations.," *NeuroImage*, vol. 80, pp. 360–78, oct 2013.
- 2) E. Allen, E. Damaraju, S. M. Plis, E. B. Erhardt, T. Eichele, and V. D. Calhoun, "Tracking whole-brain connectivity dynamics in the resting state.," *Cerebral cortex (New York, N.Y. : 1991)*, vol. 24, pp. 663–76, mar 2014.
- 3) C. G. Robert C, *Monte Carlo Statistical Methods*. Springer Texts in Statistics, Springer, 2 ed., 2004.