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Waveform Encoding for Nonlinear Electromagnetic Tomographic Imaging

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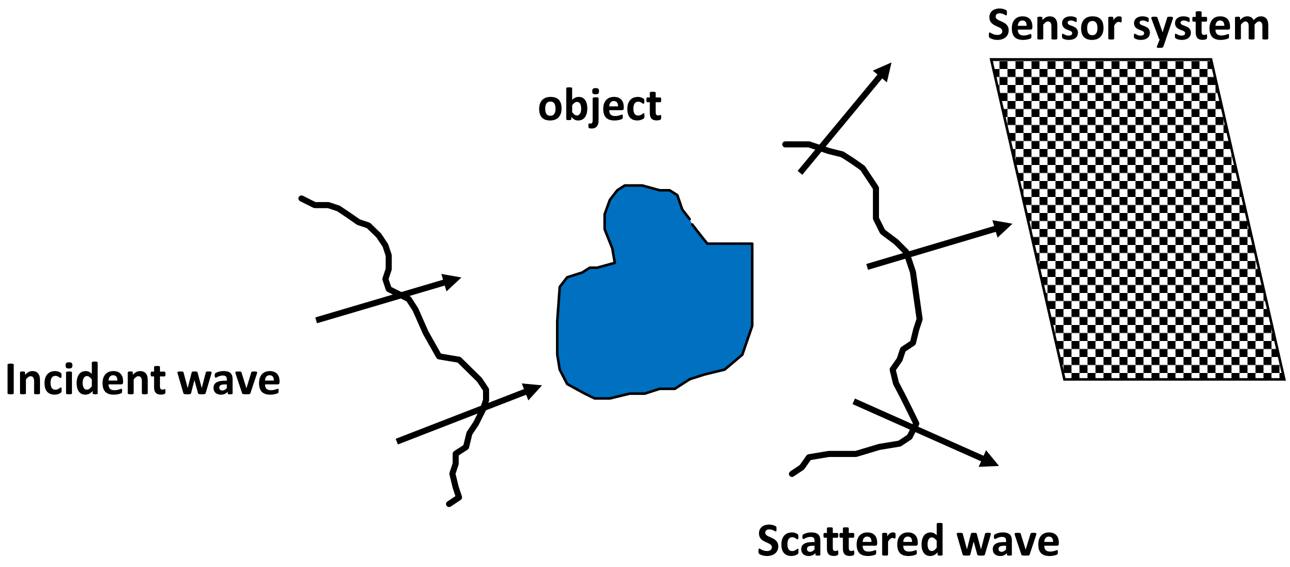
Outline

- Background and Motivation
- MIMO Tomographic Imaging Algorithm
- Numerical Simulations
- Conclusion

What is Electromagnetic (EM) Tomography?

- **EM tomography is an inverse scattering problem**
  - Source antenna transmits EM signals into a medium
  - Scattered signals are received
  - Inversion algorithms are applied to reconstruct material properties based upon Maxwell's equations
  - Applications: Large scale seismic imaging, medical imaging



Electromagnetic Tomography

- **Mathematically, EM tomography is an inverse problem**
  - Infer model parameters $\boldsymbol{p}(\boldsymbol{r})$ from measured data based upon underlying Maxwell's equations
  - Image to be reconstructed: a spatial distribution of $\boldsymbol{p}(\boldsymbol{r}), \boldsymbol{r} \in \boldsymbol{\Omega}$

$$\boldsymbol{y}_j = \boldsymbol{A}_j(\boldsymbol{p}(\boldsymbol{r}); \boldsymbol{s}_j) + \boldsymbol{\eta}_j$$

$\boldsymbol{y}_j \in \partial\boldsymbol{\Omega} \times [\boldsymbol{0}, \boldsymbol{T}]$	**measured data by receivers**
$\boldsymbol{p}(\boldsymbol{r}), \boldsymbol{r} \in \boldsymbol{\Omega}$	**material values (i.e., dielectric const.)**
$\boldsymbol{s}_j$	**$\boldsymbol{j}$-th excitation signal**
$\boldsymbol{A}_j$	**Nonlinear operator determined by wave model**
$\boldsymbol{\eta}_j$	**Noise and disturbance**

The Challenges of EM Tomography

- **Challenge 1: The inverse problem is ill-posed**

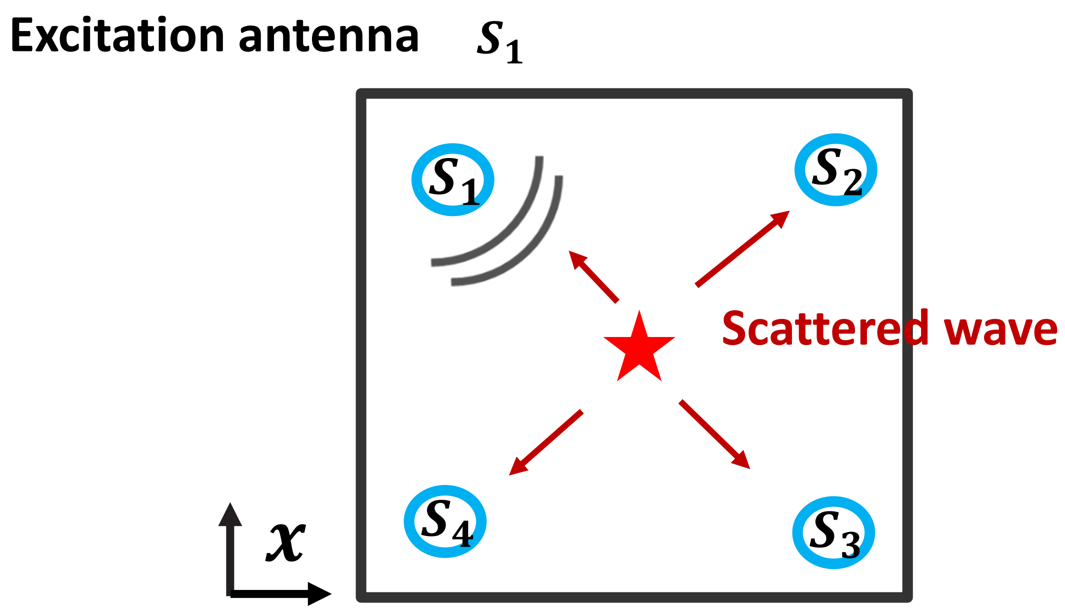
dimension of p (# of grids) $\gg$ dimension $|\partial\Omega|$ (# of receivers)

  - Classic approach: Regularization is required (e.g., sparsity constraint on $\boldsymbol{p}$) to reduce the dimension of solution space

- **Challenge 2: Nonlinear inversion method**
  - Classic approach: least squares optimization or the iterative Newton's method require iterative algorithms
  - The cost of computation depends on the size of the data volumes and on the discretization of the wave model

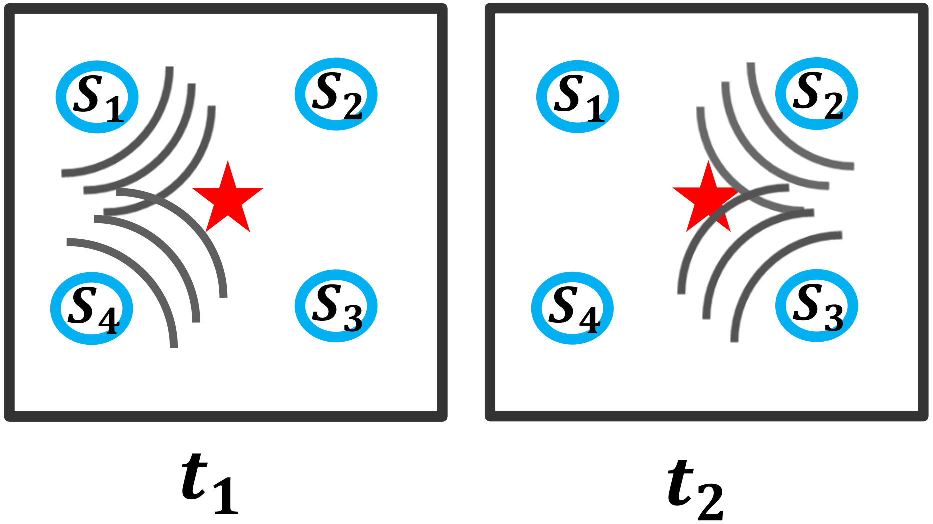
The SIMO Classic Data Collection Process

- **Single-Input Multiple Output (SIMO)**
  - An image is reconstructed from measured data in response to a single excitation antenna source.
  - The reconstruction process continues till all the sources are excited



Single-input multiple-output (SIMO) configuration

Our approach: MIMO Excitation and Waveform Encoding



- **How to remove cross-talk induced by simultaneous waveform excitation due to wave interference?**
  - **Time delays**
  - **Random weights**

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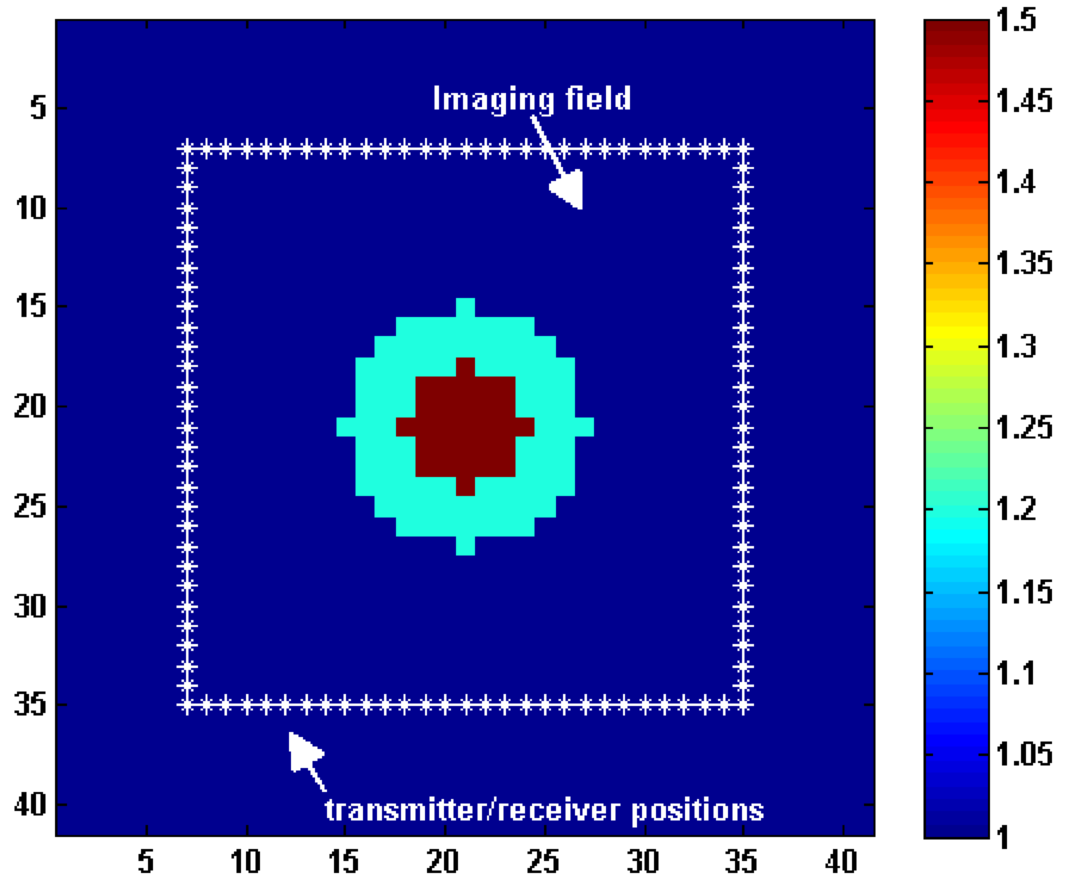
MIMO Imaging Problem Formulation

- **Imaging Configuration**



The 2D transverse magnetic (TM) model

- **Maxwell's equations**

$$\frac{\partial E_z}{\partial t}(t) = -\frac{\sigma}{\epsilon}E_z(t) + \frac{1}{\epsilon}\left(\frac{\partial H_y}{\partial x} + x\frac{\partial H_x}{\partial y}\right)(t) - \frac{1}{\epsilon}J_z(t)$$

$$\frac{\partial H_y}{\partial t}(t) = \frac{1}{\mu}\frac{\partial E_z}{\partial x}(t)$$

$$\frac{\partial H_x}{\partial t}(t) = -\frac{1}{\mu}\frac{\partial E_z}{\partial y}(t)$$

$E_z(t) \in \Omega \times [0, T]$	Electric field intensity in z-direction
H_y, H_x	Magnetic field intensity in x-, y-direction
$p = [\mu, \sigma, \epsilon]$	Parameter set
ϵ	Dielectric constant is to be reconstructed
$J_z(t)$	Excitation source

Approach to solve $y_j = A_j(p(r); s_j) + \eta_j$

- **Newton's method: iteration starts from an initial guess f^0**

$$p^{k+1} = p^k + \lambda \ \delta p^k$$

λ: **Relaxation factor**

δp^k: **Increment value**

- **The increment value δp^k can be solved by adjoint method**

Waveform Encoding Schemes

- **Random phase encoding: weights w_j are phase coded**

$$J_{z,m}^{(1)}(r^t, t) = \sum_{j=1}^{L_m} w_j s_j(t)\delta(r^t - r_j^t)$$

- **Time-delay encoding: delay τ_j**

$$J_{z,m}^{(2)}(r^t, t) = \sum_{j=1}^{L_m} s_j\big(t - \tau_j\big)\delta(r^t - r_j^t)$$

- **Uniform weight encoding**

$$J_{z,m}^{(3)}(r^t, t) = \sum_{j=1}^{L_m} s_j(t)\delta(r^t - r_j^t)$$

Impact of Excitation Sources

- **Re-write the 2D TM Maxwell's equations**

$$\frac{\partial^2 E_z}{\partial t^2} - c^2 \frac{\partial^2 E_z}{\partial z^2} = f(z,t)$$

The forcing term

$$f(z,t) = \frac{1}{\epsilon} \frac{\partial}{\partial t} (J_z + \sigma E_z)$$

- **The solution (general solution + particular solution)**

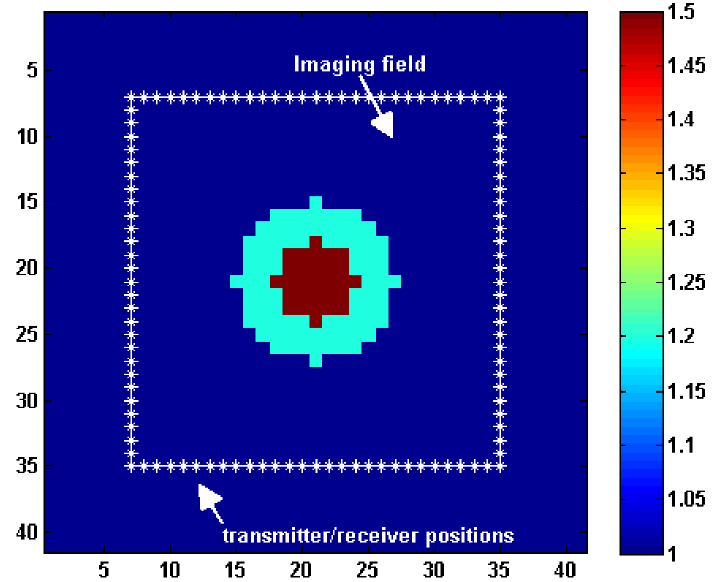
$$E_z = E_{z,g} + E_{z,p}$$

$$E_{z,p} = \frac{\sqrt{\mu\epsilon}}{c} \int_0^t \int_{z-(t-t')/\sqrt{\mu\epsilon}}^{z+(t-t')/\sqrt{\mu\epsilon}} f(z,t) dz dt$$

- Electric field computed in the forward model depends on the excitation source, which also affects the reconstruction procedure implicitly

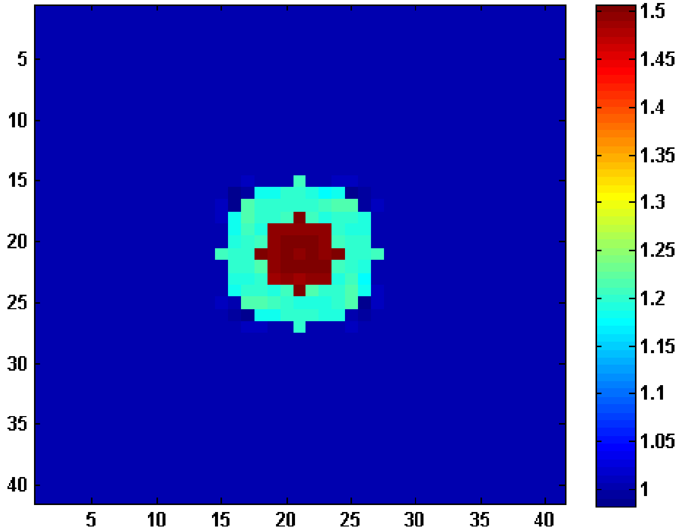
Numerical Simulations

- Simulation configuration

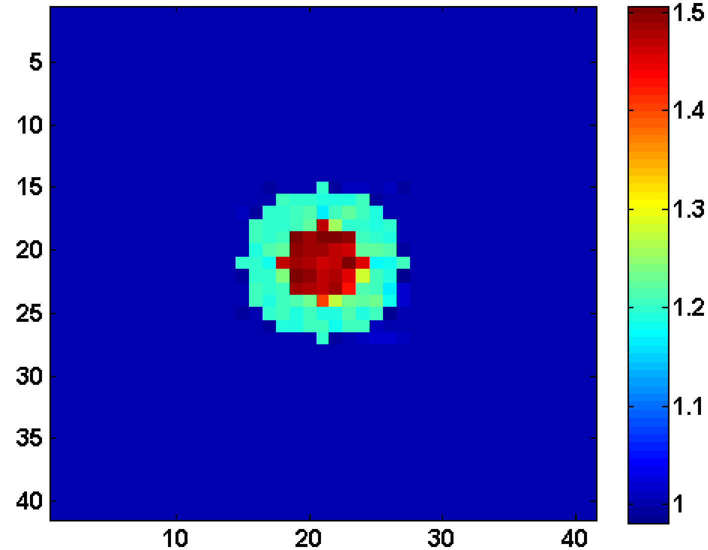


Excitation signal	Gaussian modulated pulse
Target dielectric values	$\epsilon_1 = 1.2\epsilon_0, \epsilon_2 = 1.5\epsilon_0$
Computational region	12 cm by 12 cm
Mesh grids	40 by 40
# of antennas	90
Noise level	5%

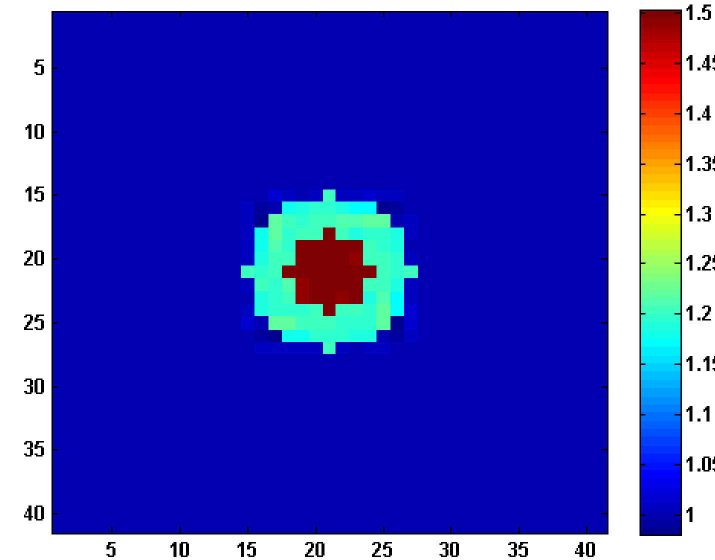
Reconstructed Images



(a)



(b)

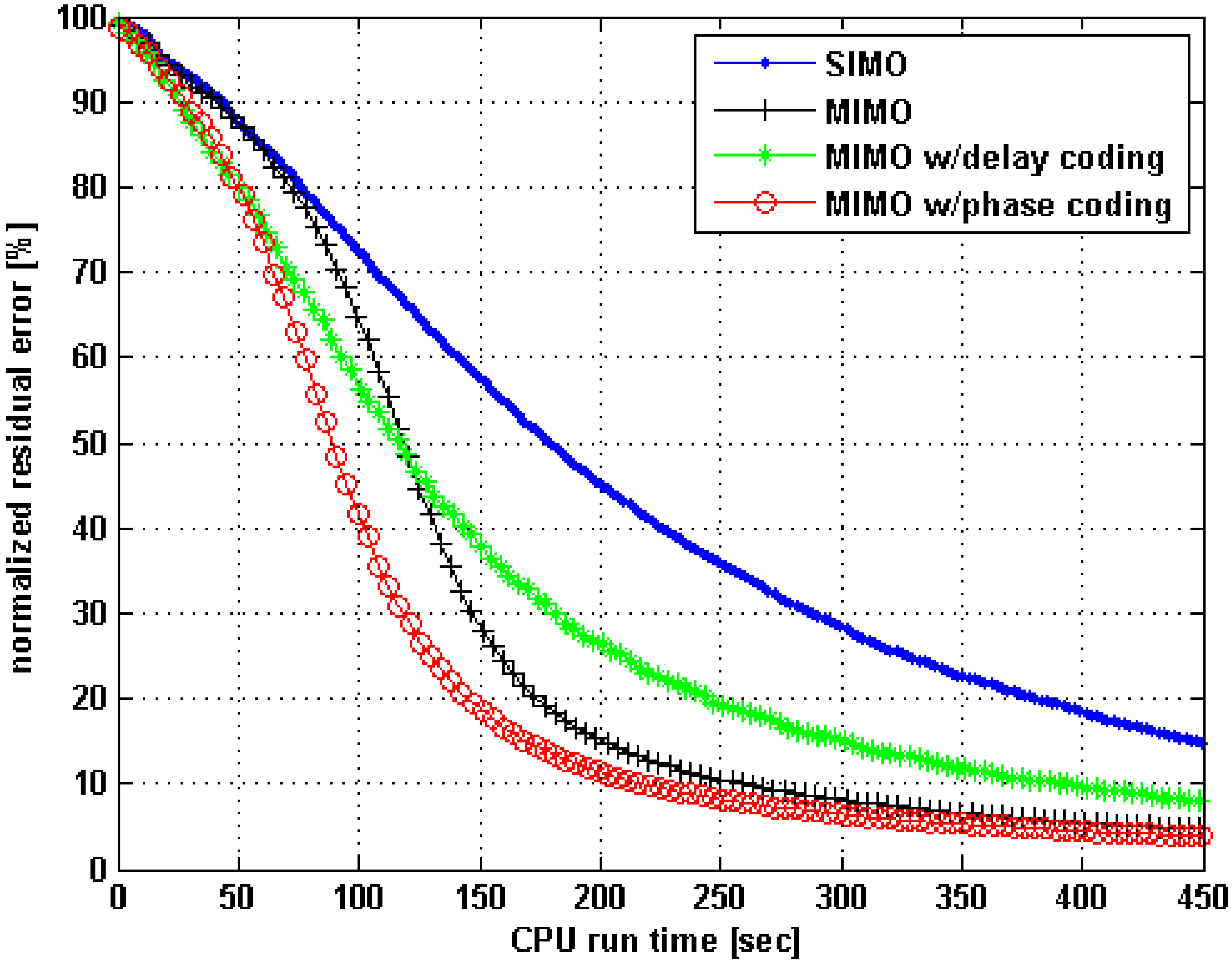


(c)

(a)	Random encoding
(b)	Time delay
(c)	Uniform weight

Convergence History

$$\beta = \frac{\left\|\hat{\epsilon}^k - \epsilon_{true}\right\|^2}{\left\|\hat{\epsilon}^0 - \epsilon_{true}\right\|^2}$$



Conclusions

- **Proper encoding techniques accelerate convergence of iterative inverse methods for nonlinear EM tomography**
- **Demonstrate the power of signal processing techniques for improving computational efficiency for solving nonlinear inverse problems**
- **Suitable for large-scale high resolution imaging applications**

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