Data rate maximization based power allocation for OFDM System in a High-Speed Train Environment

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Overview

Introduction

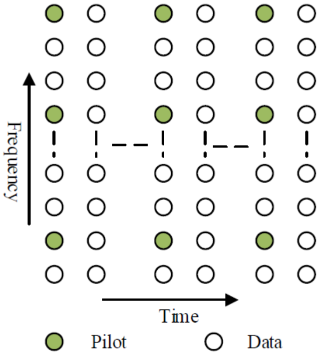
- The high-speed train (HST) has been widely developed in many countries.
- There are some limitations in the existing communication system GSM-R.
- A novel broadband wireless communication system for HST is needed.

Introduction

- OFDM has been adopted for high data rate services.
- The advantage of OFDM
  - more resistant to frequency selective fading
  - high spectral efficiency
- The HST channel is time variant and frequency selective within one OFDM symbol, which creates intercarrier interference (ICI).

Introduction

The pilot symbols are inserted into the transmitted data stream to learn the channel.



- The channel can be better estimated with allocating more power to the pilot but then the less power left for data transmission.
- Therefore, in this paper we consider a power allocation between pilot and data symbols to enhance this trade-off.

System model

- The SISO OFDM system contains K subcarriers. The HST channel is Rician fading with a dominant LOS path $h_0(n,m)$ expressed as

$$h_0(n,m) = h_0(n)e^{j(2\pi f_D \cos(\theta) m T_s)}, \tag{1}$$

where $h_0(n) = \alpha_{los}(n)e^{j\varphi_0}$, $\alpha_{los}(n)$ is channel amplitude determined by path loss during the $n\mathrm{th}$ OFDM symbol.

- The Rician factor of the channel is defined as

$$K_R = 10\log_{10}(\sigma_0^2/(1-\sigma_0^2)). \tag{2}$$

System model

- The received signal over the kth subcarrier of the nth OFDM symbol is

$$\tilde{y}(n,k) = \frac{1}{K}\sum_{m=0}^{K-1}[\sum_{l=0}^{L-1}\sum_{k'=0}^{K-1}\tilde{x}(n,k')e^{j\frac{2\pi m(k'-k)}{K}} \cdot e^{-j\frac{2\pi k' l}{K}} h_l(n,m)] + \tilde{w}(n,k), \quad (3)$$

where $\tilde{w}(n,k)$ is additive white Gaussian noise.

System model

- The eq. (3) can be written in vector form

$$\begin{aligned} \tilde{y}_n &= FH_nF^H\tilde{x}_n + \tilde{w}_n \\ &= \tilde{H}_n\tilde{x}_n + \tilde{w}_n, \end{aligned} \tag{4}$$

  where H_n is the time-domain channel matrix and F is the FFT matrix.

- The eq. (3) can be rewritten as

$$\begin{aligned} \tilde{y}(n,k) &= \tilde{H}_n(k,k)\tilde{x}(n,k) + \sum_{k'=0,\neq k}^{K-1} \tilde{H}_n(k,k')\tilde{x}(n,k') \\ &\quad + \tilde{w}(n,k). \end{aligned} \tag{5}$$

System model

- The ICI at the received pilot subcarriers includes the ICIs from other pilot subcarriers and data subcarriers
- The received pilot subcarriers for the $p_g\mathrm{th}$ subcarrier using eq. (5) can be written as

$$\tilde{y}(n,p_g) = \tilde{H}_n(p_g,p_g)\tilde{x}(n,p_g) + \sum_{\substack{g'=0,\\ g'\neq g}}^{S-1} \tilde{H}_n(p_g,p_{g'})\tilde{x}(n,p_{g'}) + \sum_{g'=0}^{K-S-1} \tilde{H}_n(p_g,d_{g'})\tilde{x}(n,d_{g'}) + \tilde{w}(n,p_g). \quad (6)$$

System model

- The above equation can be written as

$$\tilde{y}_n^p = \text{diag}\{\tilde{x}_n^p\} F_p \bar{h}_n + \tilde{H}_n^{pp} \tilde{x}_n^p + \tilde{H}_n^{pd} \tilde{x}_n^d + \tilde{w}_n^p, \qquad (7)$$

where $S \times L$ matrix $F_p(g, l) = exp(-j2\pi p_g l/K)$ and $S \geq L$.

- The LS estimate of $\bar{h}_n$ can be obtained as $\hat{\bar{h}}_n = (\text{diag}\{\tilde{x}_n^p\} F_p)^\dagger \tilde{y}_n^p$.
- The MSE is given by

$$\begin{aligned} MSE &= tr\{(\text{diag}\{\tilde{x}_n^p\} F_p)^\dagger R (\text{diag}\{\tilde{x}_n^p\} F_p)^{\dagger H}\} \\ &= T_n^{pp} + (\sigma_d^2/\sigma_p^2) T_n^{pd} + (\sigma_{\tilde{w}}^2/\sigma_p^2) T_{Fp}. \end{aligned} \qquad (8)$$

Power allocation

- Since $\hat{\tilde{H}}_n^{d\_diag} = \tilde{H}_n^{d\_diag} - e_n^{d\_diag}$, the output of the equalization over the nth OFDM symbol is

$$\hat{\tilde{x}}_n^d = \tilde{x}_n^d + (\hat{\tilde{H}}_n^{d\_diag})^{-1}(e_n^{d\_diag}\tilde{x}_n^d + \tilde{H}_n^{d\_ICI}\tilde{x}_n + \tilde{w}_n^d). \qquad (9)$$

- Then, the symbol error after the equalization is

$$e_n^d = (\hat{\tilde{H}}_n^{d\_diag})^{-1}(e_n^{d\_diag}\tilde{x}_n^d + \tilde{H}_n^{d\_ICI}\tilde{x}_n + \tilde{w}_n^d). \qquad (10)$$

Power allocation

- From the above equation, the SINR of the $k\text{th}$ output data subcarrier can be obtained as follows

$$\gamma_k = \sigma_d^2 / [R_e]_{k,k}, \tag{11}$$

where R_e is the covariance matrix of e_n^d and can be written as

$$\begin{aligned} R_e = E\{ & \hat{\tilde{H}}_n^{d\_diag^{-1}} (\sigma_d^2 e_n^{d\_diag} e_n^{d\_diag^H} \\ & + \tilde{H}_n^{d\_ICI} \tilde{x}_n \tilde{x}_n^H \tilde{H}_n^{d\_ICI^H} + \sigma_w^2 I) \hat{\tilde{H}}_n^{d\_diag^{-H}} \}. \end{aligned} \tag{12}$$

Power allocation

- Here, we adopt the sum rate for symbol detection as the objective function, the optimization problem is

$$\max_{\bar{\sigma}_d, \bar{\sigma}_p} \sum_{i=1}^{K-S} \log_2(1+\gamma_k) : \; S\bar{\sigma}_p + (K-S)\bar{\sigma}_d = K. \qquad (13)$$

- The optimization problem with $x = (1 - S/K)\bar{\sigma}_d$ can be written as

$$\max_{x} f(x) := \sum_{i=1}^{K-S} \log_2(1 + f_k(x)) : \; 0 \leq x \leq 1, \qquad (14)$$

where

$$f_k(x) := \frac{a_{\mathrm{N}} x^2 + b_{\mathrm{N}} x}{a_{\mathrm{D},k} x^2 + b_{\mathrm{D},k} x + c_{\mathrm{D},k}}. \qquad (15)$$

Power allocation

- On the other hand, from the above discussion

$$\frac{df_k(x)}{dx} = \frac{A_k x^2 + B_k x + C_k}{(a_{\mathrm{D},k} x^2 + b_{\mathrm{D},k} x + c_{\mathrm{D},k})^2},$$

where $A_k = b_{\mathrm{N}} a_{\mathrm{D},k} + 2a_{\mathrm{N}} b_{\mathrm{D},k} - a_{\mathrm{N}} b_{\mathrm{D},k} - 2a_{\mathrm{D},k} b_{\mathrm{N}}$

$$B_k = 2a_{\mathrm{N}} c_{\mathrm{D},k}, \; C_k = b_{\mathrm{N}} c_{\mathrm{D},k},$$

which can be changed in its sign at one time on $[0, 1]$ at the unique root $0 < \alpha_k < 1$ of the equation

$$A_k x^2 + B_k x + C_k = 0.$$

Power allocation

- By defining $\alpha_{\min} = \min_k \alpha_k$ and $\alpha_{\max} = \max_k \alpha_k$.
- It is sufficient to consider the problem on $[\alpha_{\min}, \alpha_{\max}]$, which is divided on subsegments $[\alpha_{k_\ell}, \alpha_{k_{\ell+1}}]$ where α_k are arranged in $\alpha_\ell, \ell = 1, 2, ..., K - S$ such that $\alpha_{k_\ell} < \alpha_{k_{\ell+1}}$.
- For convenience of notation, set $\gamma_\ell = \alpha_{k_\ell}$ so each function $f_k(x)$ is either monotonically increased or monotonically decreased on each $[\gamma_\ell, \gamma_{\ell+1}]$.
- Define

$$\begin{aligned} I_\ell^+ &= \{k \in \{1, 2, ..., K - S\} \; f_k(x) \\ &\qquad \text{is monotonically increased}\} \\ I_\ell^- &= \{1, 2, ..., K - S\} \setminus I_\ell^+. \end{aligned} \tag{16}$$

Power allocation

- Therefore on $[\gamma_\ell, \gamma_{\ell+1}]$,

$$\sum_{k=1}^{K-S} \log_2(1 + \frac{a_{\mathrm{N}} x^2 + b_{\mathrm{N}} x}{a_{\mathrm{D},k} x^2 + b_{\mathrm{D},k} x + c_{\mathrm{D},k}}) = F_\ell(x) + G_\ell(x), \quad (17)$$

where $F_\ell(x)$ is monotonically increased, while $G_\ell(x)$ is monotonically decreased

$$F_\ell(x) = \sum_{k \in I_\ell^+} \log_2(1 + f_k(x)), \quad (18)$$

$$G_\ell(x) = \sum_{k \in I_\ell^-} \log_2(1 + f_k(x)). \quad (19)$$

Power allocation

- Global Optimization Algorithm.

  Step 0. Given a tolerance $\epsilon > 0$. Start with $M_0 = \{[\gamma_\ell, \gamma_{\ell+1}] \ : \ \ell = 0, 1, ...K - S - 1\}$ and current best value (CBV) $CBV = \max_{\ell=0,1,\ldots,K-S-1} f(\gamma_\ell)$ and current best solution (CBS) $x_{opt} = \arg\max_{\gamma_\ell} f(\gamma_\ell)$. Set $\mathcal{S}_1 = \mathcal{N}_1 = M_0$. Set $\kappa = 1$.

  Step 1. For each $M = [a, b] \in \mathcal{N}_\kappa$ define $\beta(M) = F_\ell(b) + G_\ell(a)$, where ℓ is such that $M \subset [\gamma_\ell, \gamma_{\ell+1}]$. If $f((a+b)/2) > CBV$ update $x_{opt} = (a+b)/2$ and $CBV = f((a+b)/2)$.

Power allocation

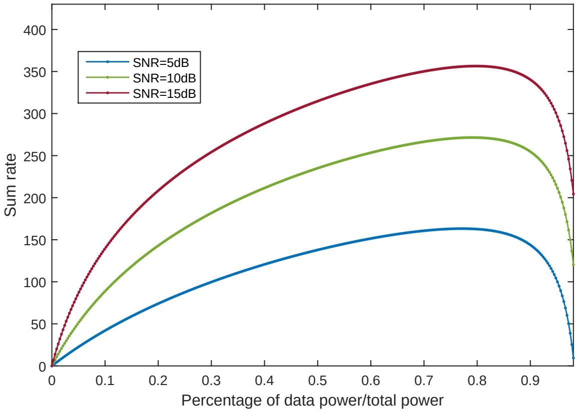
- Global Optimization Algorithm (Continue).

Step 2. In $\mathcal{S}_\kappa$ delete all M such that $\beta(M) \leq CBV + \epsilon$. Let $\mathcal{R}_\kappa$ be the set of remaining segments. If $\mathcal{R}_\kappa = \emptyset$ terminate: CBV is the ϵ-optimal value with the corresponding optimal solution x_{opt}.

Step 3. Choose $M_\kappa = [a_\kappa, b_\kappa] \in \arg\ \max\{\beta(M)\ :\ M \in \mathcal{R}_\kappa\}$ and subdivide it into two smaller segments $M_{\kappa,1} = [a_\kappa, (a_\kappa + b_\kappa)/2]$ and $M_{\kappa,2} = [(a_\kappa + b_\kappa)/2, b_\kappa]$. Let $\mathcal{N}_{\kappa+1} = \{M_{\kappa,1}, M_{\kappa,2}\}$ and $\mathcal{S}_{\kappa+1} = (\mathcal{R}_\kappa \setminus M_\kappa) \cup \mathcal{N}_{\kappa+1}$. Set $\kappa \leftarrow \kappa + 1$ and go back to step 1.

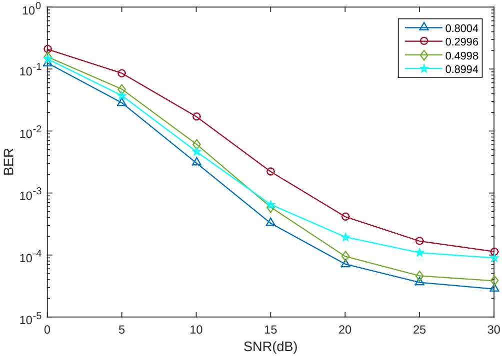
Simulation result

- For each channel SNR value, we can find that there is an optimal data percentage that maximizes the sum rate.



Simulation result

- The optimal data percentage 0.8004 at the $SNR = 25$dB is compared with other percentages of data power, namely 0.2996, 0.4998, 0.8994. We can get the conclusion that spending more power to data symbols maybe not obtain better BER.



Thank you