MUTUAL INFORMATION BASED RADAR WAVEFORM DESIGN FOR JOINT RADAR AND CELLULAR COMMUNICATION SYSTEMS

Aalto University
School of Electrical Engineering

USC Viterbi
School of Engineering

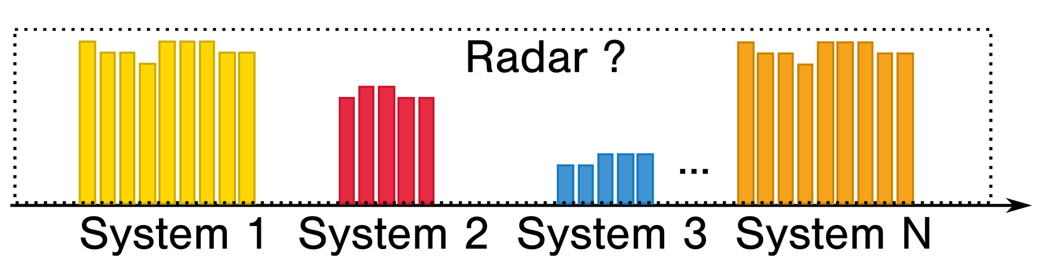
Marian Bica* **Kuan-Wen Huang†** **Visa Koivunen*** **Urbashi Mitra†**

*Department of Signal Processing and Acoustics, Aalto University, Espoo, Finland
†Department of Electrical Engineering, University of Southern California, Los Angeles, CA, USA

INTRODUCTION

A joint radar/communication system is considered, where:

- the two systems co-exist in the same spectrum
- the multicarrier radar waveform is optimized such that the interference caused to the cellular systems is strictly controlled
- the radar waveform optimization is done using different Mutual Information (MI) based criteria



CONTRIBUTIONS

- It is shown which of the proposed MI based optimization criteria provides better radar waveforms for target detection task.
- It is shown that a larger maximized MI does not guarantee an optimal detection performance.
- The importance of exploiting the communication signals for target detection is demonstrated.

The goal is to optimize the multicarrier radar waveform for spectral co-existence with communication systems.

SYSTEM MODEL

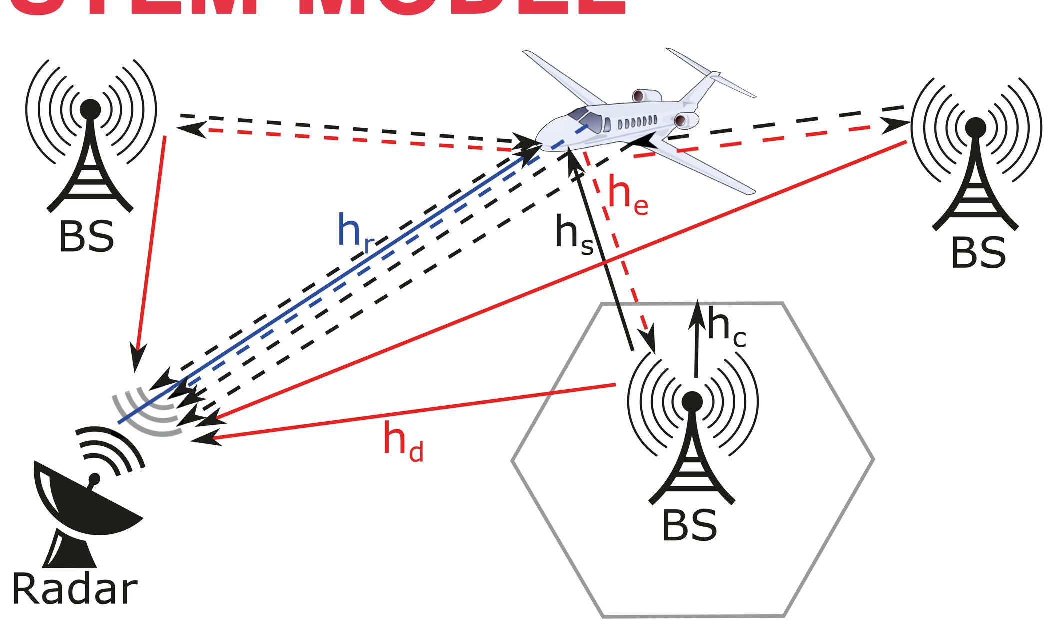


Fig 1. System model composed of the radar and communication base stations

$$\underbrace{y(t) = x_r(t) * h_r(t)}_{\text{radar target return}} + \underbrace{x_s(t) * h_s(t)}_{\text{comm. target return}} + \underbrace{x_s(t) * h_d(t)}_{\text{comm. other paths}} + \underbrace{n(t)}_{\text{noise \& clutter}}$$

The equivalent baseband matrix formulation can be obtained as:

$$\mathbf{y} = \mathbf{X}_r\mathbf{h}_r + \mathbf{X}_s\mathbf{h}_s + \mathbf{X}_s\mathbf{h}_d + \mathbf{n}$$

- where $\mathbf{X}_s$ and $\mathbf{X}_r$ are well approximated to circulant matrices
- channels $\mathbf{h}_r$, $\mathbf{h}_s$, $\mathbf{h}_d$ and noise and clutter $\mathbf{n}$ are zero mean Gaussian random vectors with known covariance matrices.

RADAR WAVEFORM OPTIMIZATION

Find the radar waveform that maximizes the MI between the received signal and the target impulse response.

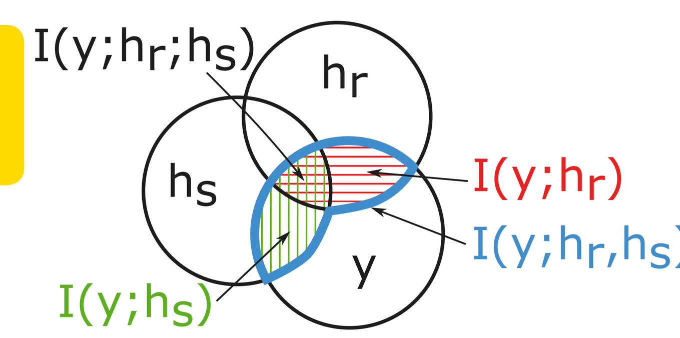


Fig 2. Venn diagram of information theoretic measures for $\mathbf{y}$, $\mathbf{h}_r$ and $\mathbf{h}_s$

Different MI based optimization criteria are proposed. These differ by how the scattering due to the communication signals is considered:

- As useful energy

$I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r,\mathbf{h}_s)$

$$\max_{X_r[l]} \sum_{l=0}^{L-1} \log\left(1 + \frac{|X_r[l]|^2\sigma^2_{h_r}[l] + |X_s[l]|^2\sigma^2_{h_s}[l]}{|X_s[l]|^2\sigma^2_{h_d}[l] + \sigma^2_n[l]}\right)$$
$$\text{s.t.} \quad \log\left(1 + \frac{|X_s[l]|^2\sigma^2_{h_c}[l]}{|X_r[l]|^2\sigma^2_{h_e}[l] + \sigma^2_n}\right) \geq t_l$$
$$\sum_{l=0}^{L-1} |X_r[l]|^2 \leq P_T$$

Re-written

$$\max_{x_l} \sum_{l=0}^{L-1} \log\left(1 + \frac{x_l}{a_l} + b_l\right) \quad \text{s.t.} \quad \mathbf{0} \preceq \mathbf{x} \preceq \mathbf{c}, \quad \mathbf{1}^T\mathbf{x} \leq P_T$$

$$x_l^* = \begin{cases} 0, & a_l(1+b_l) > \frac{1}{\lambda_3} \\ \frac{1}{\lambda_3} - a_l(1+b_l), & \frac{1}{\lambda_3} - c_l < a_l(1+b_l) < \frac{1}{\lambda_3} \\ c_l, & a_l(1+b_l) < \frac{1}{\lambda_3} - c_l \end{cases}$$

- As interference

$I(\mathbf{y};\mathbf{h}_r) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r)$

$$\max_{X_r[l]} \sum_{l=0}^{L-1} \log\left(1 + \frac{|X_r[l]|^2\sigma^2_{h_r}[l]}{|X_s[l]|^2\sigma^2_{h_s}[l] + |X_s[l]|^2\sigma^2_{h_d}[l] + \sigma^2_n[l]}\right)$$
$$\text{s.t.} \quad \log\left(1 + \frac{|X_s[l]|^2\sigma^2_{h_c}[l]}{|X_r[l]|^2\sigma^2_{h_e}[l] + \sigma^2_n}\right) \geq t_l$$
$$\sum_{l=0}^{L-1} |X_r[l]|^2 \leq P_T$$

Re-written

$$\max_{x_l} \sum_{l=0}^{L-1} \log\left(1 + \frac{x_l}{a_l(1+b_l)}\right) \quad \text{s.t.} \quad \mathbf{0} \preceq \mathbf{x} \preceq \mathbf{c}, \quad \mathbf{1}^T\mathbf{x} \leq P_T$$

The same radar waveform maximizes both $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$ and $I(\mathbf{y};\mathbf{h}_r)$. This is due to the optimization being done only for the radar waveform.

- Ignored

$I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s) = H(\mathbf{y}|\mathbf{h}_s) - H(\mathbf{y}|\mathbf{h}_r,\mathbf{h}_s)$

$$\max_{X_r[l]} \sum_{l=0}^{L-1} \log\left(1 + \frac{|X_r[l]|^2\sigma^2_{h_r}[l]}{|X_s[l]|^2\sigma^2_{h_d}[l] + \sigma^2_n[l]}\right)$$
$$\text{s.t.} \quad \log\left(1 + \frac{|X_s[l]|^2\sigma^2_{h_c}[l]}{|X_r[l]|^2\sigma^2_{h_e}[l] + \sigma^2_n}\right) \geq t_l$$
$$\sum_{l=0}^{L-1} |X_r[l]|^2 \leq P_T$$

Re-written

$$\max_{x_l} \sum_{l=0}^{L-1} \log\left(1 + \frac{x_l}{a_l}\right) \quad \text{s.t.} \quad \mathbf{0} \preceq \mathbf{x} \preceq \mathbf{c}, \quad \mathbf{1}^T\mathbf{x} \leq P_T$$

$$x_l^* = \begin{cases} 0, & a_l > \frac{1}{\lambda_3} \\ \frac{1}{\lambda_3} - a_l, & \frac{1}{\lambda_3} - c_l < a_l < \frac{1}{\lambda_3} \\ c_l, & a_l < \frac{1}{\lambda_3} - c_l \end{cases}$$

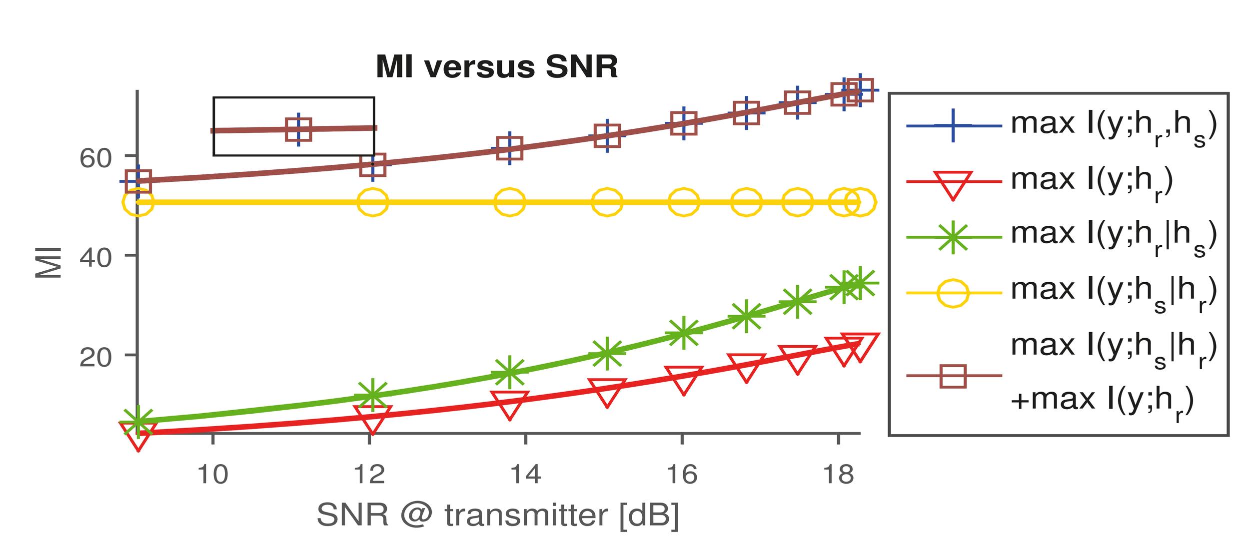


Fig 3. Discrete values for maximized MIs where $\max I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r)$

DETECTION PERFORMANCE

The detection performance of the optimized radar waveforms is analyzed using the Neyman-Pearson (NP) detector. The solutions to the optimization problems are plugged-in the NP detector and the receiver operating characteristic (ROC) curves are obtained.

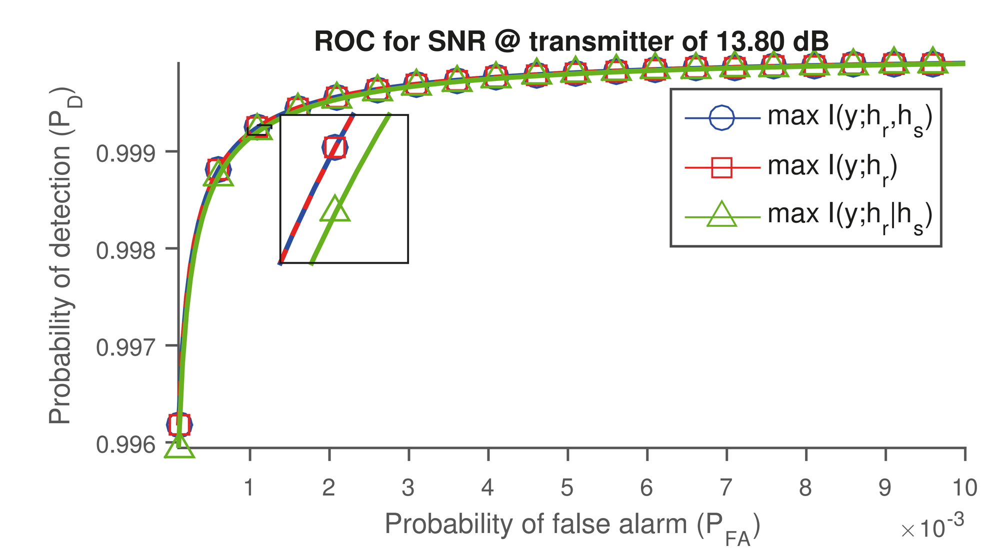


Fig 4. ROC curves showing that the radar waveforms that maximize $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$ or $I(\mathbf{y};\mathbf{h}_r)$ provide better detection performance, despite $\max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r)$

A larger maximized MI does not guarantee an optimal detection performance for NP detector.

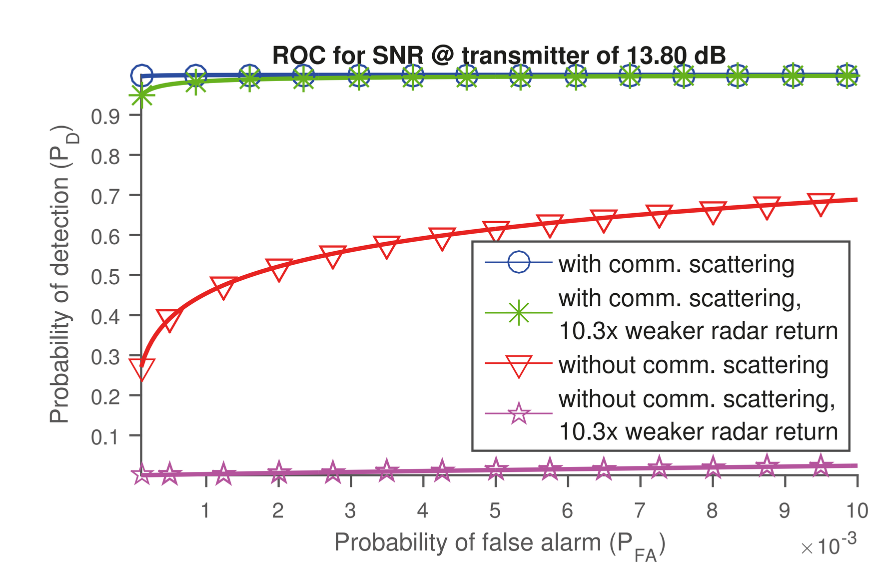


Fig 5. ROC curves showing that the detection capability decreases considerably when the communication signals scattered off the target are not exploited, especially when dealing with weak returns

Exploiting the scattering due to the communication signals improves the detection performance, especially for cases with weak radar returns.

CONCLUSIONS

- Three different MI based criteria for radar waveform optimization are proposed and it is established which criteria are better for waveforms that achieve better detection performance.
- It is shown that a larger maximized MI does not guarantee an optimal detection performance.
- It is shown that different optimization criteria provide the same optimized radar waveform.
- The importance of exploiting the scattering due to the communication signals for the detection performance is emphasized.