Parameter Estimation of Polynomial Phase Signal Based on Low-complexity LSU-EKF Algorithm in Entire Identifiable Region

Zhenmiao Deng, Rongrong Xu, Yixiong Zhang, Pingping Pan, Rujia Hong

College of Information Science and Engineering, Xiamen University, Xiamen 361005, China

Abstract

Fast implementation of parameter estimation for polynomial phase signal (PPS) is considered in this paper. A method which combines the least squares unwrapping (LSU) estimator and the extended Kalman filter (EKF) is proposed. A small number of initial samples are used to estimate the PPS's parameters and then these coarse estimates are used to initial the EKF. The proposed LSU-EKF estimator greatly reduces the computation complexity of the LSU estimator and can work in entire identifiable region which inherits from the LSU estimator. Meanwhile, in the EKF stage its output is in point-by-point wise which is useful in real applications.

Motivations

- Polynomial phase signals are common in fields including radar, sonar, geophysics, radio communication and biology
- The existing methods can loosely be grouped into two classes: estimators based on 'multilinear transforms', such as HAF, PHAF, CPF; and estimators based on phase unwrapping, such as LSU
- The main limitation of the estimators based on 'multilinear transforms' is the small identifiable region on which they can operate
- The major drawback of the LSU estimator is that computing a nearest lattice point is, in general, computationally difficult
- The proposed method can work in entire identifiable region which is inherent from the LSU estimator and the estimation is output in point-by-point wise in the EKF stage with small computation

Theoretical Analysis

1.Signal Model

A received polynomial phase signal can written as

$$y(n) = Ae^{2\pi j\theta(n)} + \omega(n)$$

where A is the amplitude of the signal and $\omega(n)$ is a complex additive white Gaussian noise (AWGN) with zero mean and variance σ^2.

2.The Extend Kalman Filter Algorithm

we use the extended Kalman filter algorithm to estimate the parameters of polynomial phase signals:

EKF Algorithm

Initial Conditions ($k=0$):

$$\hat{X}_0 = E(X_0)$$
$$\hat{P}_0 = E\left[(X_0-\hat{X}_0)(X_0-\hat{X}_0)^T\right] \quad (16)$$

Predict Equations:

$$X'_k = F_k\hat{X}_{k-1}$$
$$P'_k = F_k\hat{P}_{k-1}F_k^T + Q_{k-1}$$
$$Y'_k = H_kX'_k \quad (17)$$
$$H_k = \left.\frac{\delta h}{\delta X}\right|_{X'_k}$$

Update Equations:

$$K_k = P'_kH_k^T\cdot\left[H_kP'_kH_k^T + R_k\right]^{-1}$$
$$\hat{X}_k = X'_k + K_k\left[Y_k - Y'_k\right] \quad (18)$$
$$\hat{P}_k = \left[I - K_kH_k\right]P'_k$$

The extended Kalman filter is sensitive to the initial conditions and if the filter is improperly initialized, the filtering may be failed. In this paper, in order to make the filter work effectively, we utilize the estimates of the least squares unwrapping estimator to initialize the EKF.

3.The Least Squares Unwrapping (LSU) Estimator

The phase θ_n of polynomial phase signals is written as

$$\theta_n = \frac{\angle y(n)}{2\pi} = \emptyset_n + \theta(n)$$

Where $\angle$ denotes the angle of a complex number, and $\emptyset_n$are random variables representing the phase noise induced by $\omega(n)$.

According to the fundamental of the least square method, the parameters $u_0,\cdots,u_m$ can be evaluated by

$$SS(\boldsymbol{u}) = argmin\left(\sum_{n-1}^{N}\left(\theta_n - \sum_{k=0}^{m}u_kn^k\right)^2\right).$$

The LSU estimator is the minimum of $SS(\boldsymbol{u})$ over the identifiable region of PPSs, which can be represented as a nearest lattice point problem . When the number of samples is less than 60, the sphere decoder can exactly computes a nearest lattice point to evaluate the parameters $u_0,\cdots,u_m$. In the other way, if the number of samples is more than 60, we can apply the K-best algorithm having complexity $O(N^3\log N)$ for approximating the LSU estimator.

Simulation Results

Fig.1 shows the MSEs of the HAF estimator , the PHAF estimator, and the proposed estimator, where the values of coefficients are $\boldsymbol{u} = \left[\frac{1}{4},\frac{1}{4},\frac{1}{8N},\frac{1}{24N^2},\frac{1}{96N^3},\frac{1}{480N^4}\right]^T$ for the 5-order PPSs and $\boldsymbol{u} = \left[\frac{1}{4},\frac{1}{4},\frac{1}{8N},\frac{1}{24N^2}\right]^T$ for the 3-order PPSs.

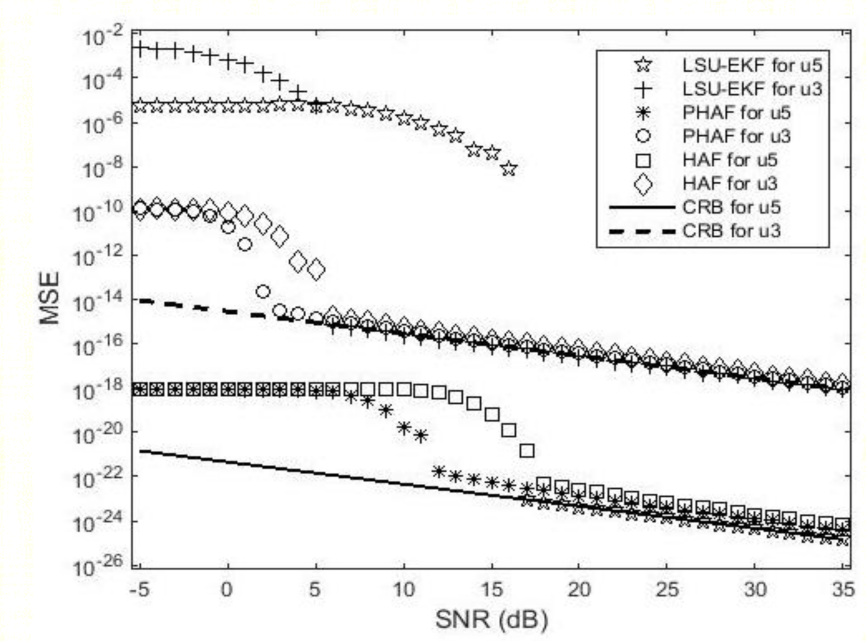

Fig. 1: Sample MSEs for the coefficient u_3 for PPSs of order 3 and the coefficient u_5 for PPSs of order 5

Fig.2 shows the MSEs of the LSU estimator, the HAF estimator , the PHAF estimator , the ZW estimator, and the proposed estimator, where the values of coefficients are $\boldsymbol{u} = \left[\frac{1}{4},\frac{1}{4},\frac{1}{8},\frac{1}{24},\frac{1}{96},\frac{1}{480}\right]^T$ for the 5-order PPSs and $\boldsymbol{u} = \left[\frac{1}{4},\frac{1}{4},\frac{1}{8},\frac{1}{24}\right]^T$ for the 3-order PPSs.

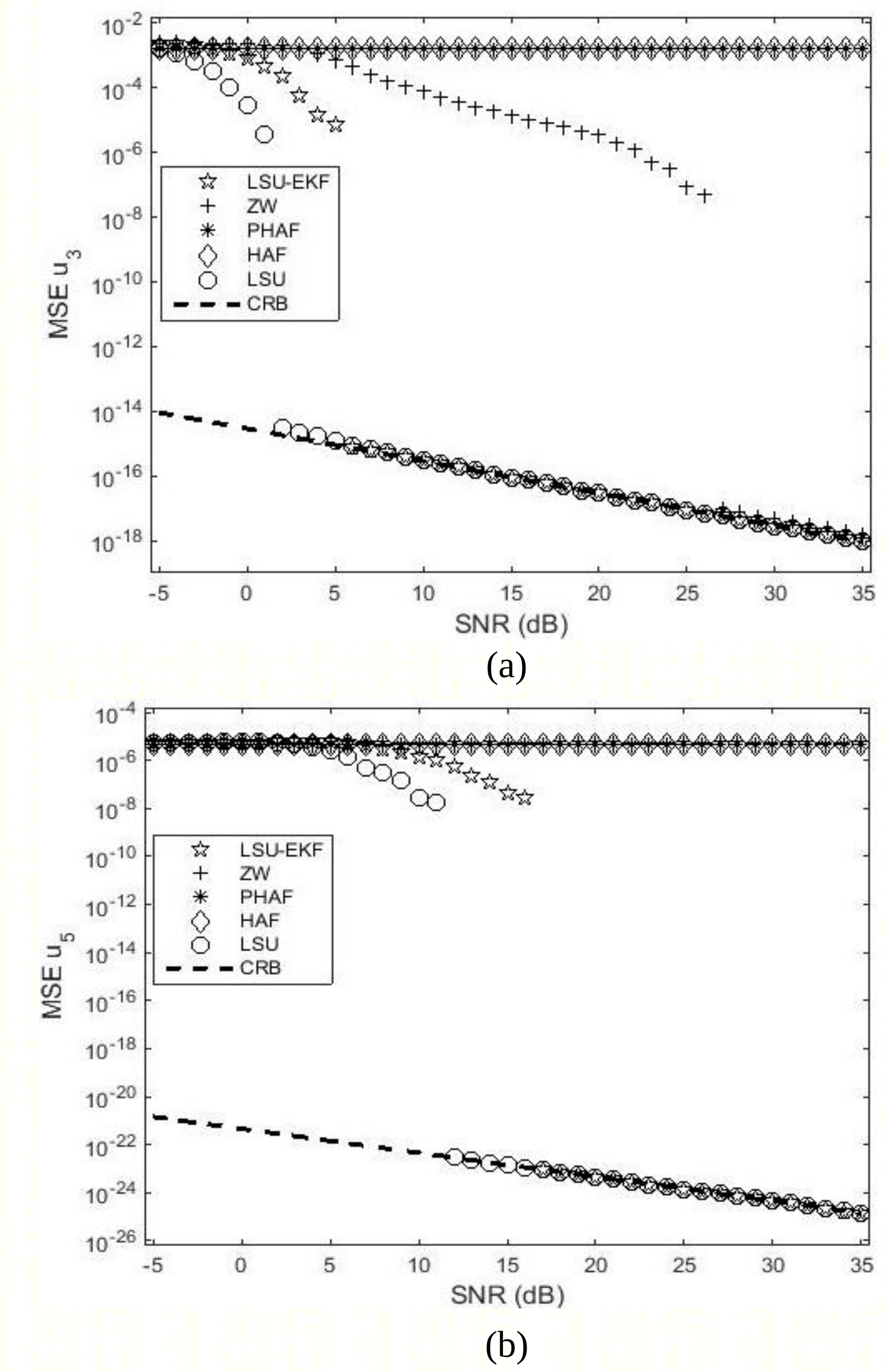

Fig. 2: Sample MSEs for the coefficient u_3 for PPSs of order 3 and the coefficient u_5 for PPSs of order 5

	HAF	PHAF	ZW	LSU	LSU-EKF
$m=3$	100.4s	180.5s	119.6s	25624.1s	1063.7s
$m=5$	114.3s	174.1s	177.4s	44883.4s	3113.7s

Table 1: Running time in seconds with different methods for PPSs of order 3 and PPSs of order 5

Conclusion

The proposed LSU-EKF estimator greatly reduces the computation complexity of the LSU estimator while at the same time it can still work in entire identifiable region. Since the number of samples to initial the EKF is relative with the threshold SNR and the computation complexity, the size of the initial sample number should be careful chosen according to the tradeoff between these two factors.